Analysis of forming –Upper bound analysis

Slip line field analysis has limited application in forming in view of its applicability to plane strain deformation only. A more accurate and general analysis for determination of forming load is the application of limit theorems. There are two limit theorems, upper bound and lower bound. The lower bound theorem is not widely used for forming because it under estimates the forming load. Upper bound analysis overestimates the forming load. Therefore, upper bound analysis is widely used for accurately predicting forming loads. It is applicable to almost all types of forming. One should get the solution to the forming problem so that the solution should be kinematically and statically admissible. Kinematically admissible means the velocity field chosen should satisfy the expected boundary conditions for the forming process as well as satisfy the requirement of incompressibility. In upper bound we expect the kinematically admissible condition to be satisfied by the solution. Rigid body motion is assumed for the deforming material –in the form of triangular elements. This could satisfy the requirement of kinematically admissible velocity field. The velocities of various parts of the deforming material are represented in diagrams called hodographs. One has to choose a trial velocity field such that it is closer to the actual velocity field expected in the forming process.

1.1 Upper-bound theorem:

It states that for a given set of velocity fields, the velocity field which minimizes the total energy is the nearest to the actual solution. In other words, this velocity field minimizes the function:

The upper-bound theorem can also be stated in a different way. It states that the estimate of the force obtained by equating the internal energy dissipation to external forces is equal to or greater than the correct force. We should assume a suitable flow field for the deformation.

In short, the field which minimizes the energy dissipation rate, given below, is the required field:

$$\dot{E} = \int_V \bar{\sigma}\,\dot{\bar{\varepsilon}}\,dV + \int_{Sd} k\,|\Delta V_t|\,dS + \int_{Sf} mk\,|\Delta V_s|\,dS = \text{Forming Load X Velocity of punch/die} \quad \text{-------- 1}$$

The first term on right hand side is the rate of work done due to plastic straining, the second term is the rate of energy dissipated in internal velocity discontinuity and the third term represents power consumed for friction. Generally for continuous velocity field the second term can be ignored.

In a nutshell, we could say that the rate of external work done in the process is equal to internal power required for homogeneous deformation plus rate of work done in shear or redundant deformation plus rate of work done for overcoming friction. In the following example we will illustrate the methodology for determination of work done in shear deformation of a material. Subsequesntly, we will know how forming load could be determined applying the upper bound theorem.

1.2 Example: Determination of shear work done

The general methodology of the analysis involves, first, assuming a flow filed within the deforming material that will suitable reflect the material flow. The flow field is otherwise called velocity field. Next step is to find the rate of energy consumption or the rate of work done for this flow field. Finally, the external work done is equated to the energy for the flow field. From this we can solve for the forming load.

The shear work or the rate of work in shearing a material can be determined easily from an assumed flow field for the forming process. Fist we must assume a suitable velocity field. We must draw the velocity vector diagram, called hodograph for the assumed flow field, which should be kinematically admissible. Once drawn, we can determine the shear work done or energy dissipated in shear. Equating the energy dissipated in shear to the rate of external work done, one can determine the forming force (under ideal condition). This analysis is based on the assumptions: deformation is homogeneous without work hardening, there is no friction or there is sticking friction at interface and the flow is two dimensional.

Let us consider a simple example to illustrate this approach. Consider a rigid element of the deforming material, pqrs, which is along the x axis. See figure below. Let this element have a velocity of V1. Let this element pass through a plane yy. After passing through the plane the element changes direction, attains a velocity of V_2. It also gets distorted to a new shape, p'q'r's'. The plane yy can be called shear plane, as it causes the shear of the material.

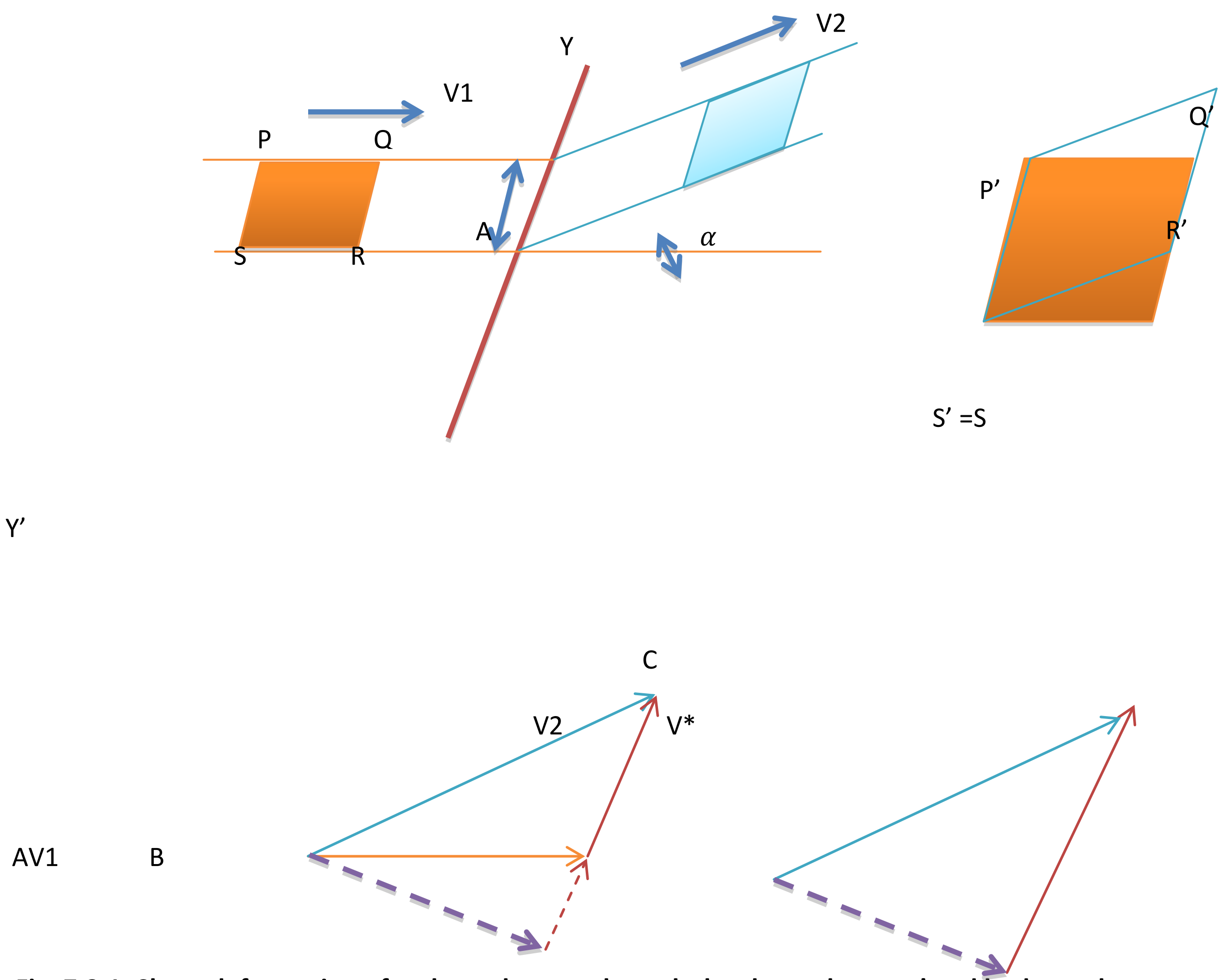

Fig. 7.2.1: Shear deformation of a plane element through the shear plane y-y' and hodograph

Let α be the angle by which the element gets sheared. Let the thickness of the element perpendicular to the plane of the paper be unity. A is the height of the element parellel to the plane of shear. V1 and V2 are the velocities before and after shear.

The hodograph or the velocity vector triangle is shown above. The velocities V1 and V2 can be resolved along the line of shear YY' and perpendicular to the line YY'. Note that the perpendicilar components are equal. The components of velocities along YY' are not equal. This gives raise to velocity discontinuity. The difference in the velocity components along the line YY- is called velocity discontinuity. It is denoted as V*.

The volume rate of flow of the material should remain constant through the process.

Therefore, we have the perpendicular velocity components equal.

Volume flow rate = V_1 X A (Unit depth) ------ 2

Shear work done per unit volume of the material = w = $\tau\gamma$ ---- 3

Let τ = k the shear yield strength of the material.

Shear strain of the deformed material can be written as γ = R'R/RS = V*/V1 (By similarity of the triangles ABC and S'RR')

Total power in the shear deformation is:

w X volumetric flow rate = $(kV^*/V_1)(V_1 \times A)$ = kV*A ------ 4

A can be called the length corresponding to the velocity discontinuity along the tangential direction to the plane of shear.

If multiple lines of velocity discontinuity are assumed for the deformation zone,

The rate of work done in shear deformation can be written as:

$\dot{w} = \sum kA_i {V^*}_i$ ------5

This method can be extended to complex flow geometries such as extrusion, by assuming triangular or polygonal elements of shear, each element assumed to move as a single rigid body.

1.3 Example to illustrate the general upper bound solution:

According to upper-bound theorem the following relation could be used for calculation of forming load for any forming operation, if we could assume a suitable velocity field or deformation field.

$\dot{W}$ = work done for homogeneous deformation + work done for shear deformation + work done in friction = $\dot{w}_h + \dot{w}_s + \dot{w}_f = \int_V \bar{\sigma}\,\dot{\bar{\varepsilon}} dV + \int_{Sd} k\,|\Delta V_t| dS + \int_{Sf} mk\,|\Delta V_s| dS$ ------ 6

Let us apply this principle for a simple forming process of plane strain compression of a rectangularplate. Let h be the height of the plate at any instance. Let Vd be the velocity of the punch or die.

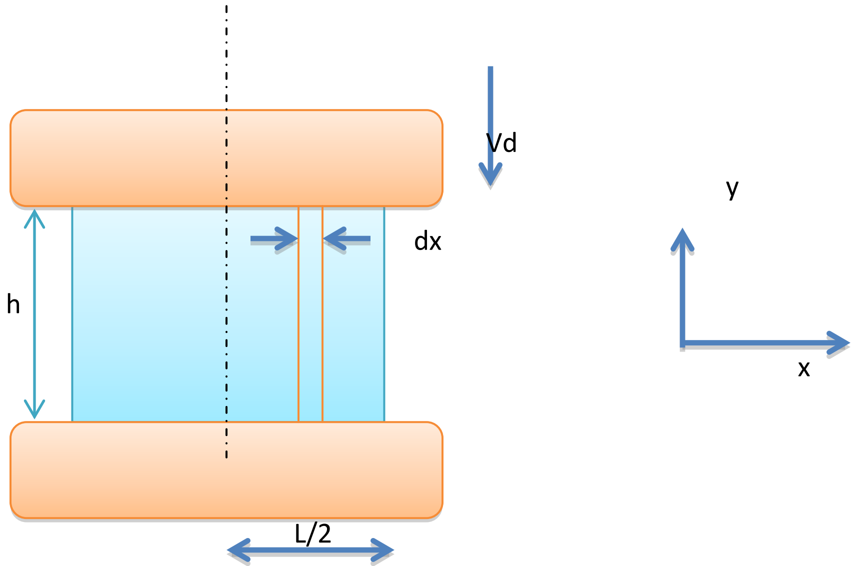

Figure 7.3.1: Plane strain upsetting

Consider an elemental strip of height h and thickness dx, width of unity. The upper die moves down with a velocity of Vd. The elemental strip is located at a distance x from the axis of compression. Assume the interfacial friction to be of sticking friction. So then the frictional shear stress is given by: τ = mk.

The height strain of the element can be written as: dh/h.

Strain rate along the height direction is given by: dh/h/dt = Velocity/h

Therefore, $\dot{\varepsilon}_h$ = Vd/h (For linear variation of velocity along the height)

Now, we have $\dot{\varepsilon}_h + \dot{\varepsilon}_x + \dot{\varepsilon}_z = 0$

However, we have $\dot{\varepsilon}_z = 0$ for plain strain compression

Therefore, we have: $\dot{\varepsilon}_x = -V_d/h$

Now the velocity of material along x direction, $V_x = \dot{\varepsilon}_x\, x = \frac{V_d}{h} x$

Let us now write down the individual terms in equation 6

Rate of work for homogeneous deformation:

$$\dot{w}_h = \int_V \bar{\sigma}\,\dot{\bar{\varepsilon}} dV = \bar{Y}\frac{V_d}{h}\,Lh = \frac{2}{\sqrt{3}}Y\frac{V_d}{h}\,Lh \qquad \text{---------} \; 7$$

Rate of shear work $\dot{w}_s = 0$

Rate of friction work $\dot{w}_f = \int_{Sf} mk\,|\Delta V_s|dS = 2\int_0^{L/2} mkdx\frac{V_d}{h}x = 2m\frac{Y}{\sqrt{3}}\frac{V_d}{h}\frac{L^2}{8}$

The total rate of work = $\frac{2}{\sqrt{3}}Y\frac{V_d}{h}\ Lh + 2m\frac{Y}{\sqrt{3}}\frac{V_d}{h}\frac{L^2}{8}$

We can equate the total rate of work to rate of external work.

Rate of external work done by the forming load F = F V_d

Therefor, the forming load = F =$(\frac{2}{\sqrt{3}}Y\ Lh + m\frac{Y}{\sqrt{3}}\frac{L^2}{4})\frac{1}{h}$

Now, we can write the average forming pressure or die pressure, p as:

$p_{av} = \frac{F}{L} = \frac{2}{\sqrt{3}}Y(1 + m\frac{L}{8h})$

Or $\boldsymbol{\frac{p_{av}}{\bar{Y}} = (1 + m\frac{L}{8h})}$

Thus we are able to apply the upperbound analysis for a simple upsetting problem.

Source:
http://nptel.ac.in/courses/112106153/12