Analysis of forming –Slipline Field Method

1.1 Methodology of slipline field analysis:

Slab analysis of the forming process is considered approximate due to the assumption of homogeneous deformation of material. Slipline field analysis is more accurate as it considers the non-homogeneous deformation also. This method is widely applied for forming processes such as rolling, strip drawing, slab extrusion etc. Slipline field analysis is based on the important assumptions that the deformation of material is plane strain type, no strain hardening of the material, constant shear stress at interfaces, the material is rigid plastic.

The general methodology of this analysis can be described by the following steps:

First differential equations in terms of mean stress and deviatoric stress for plane strain deformation are formulated

Slipline field is constructed graphically out of orthogonal maximum and minimum shear lines.

From known stress at some point, the integral constants are determined. From this the forming load can be found.

Before we proceed to understand the methodology of the analysis a few definitions should be considered.

What are sliplines? They are planes of maximum shear, which are oriented at 45 degrees to the axes of principal stresses. Maximum and minimum slip lines are orthogonal.

What is plane strain deformation? It is a type of plastic deformation in which the material flow in one of the three principal directions is constrained. The material strain in the third direction is zero. This is possible by the application of a constraint force along the third direction. All displacements are restricted to xy plane, for example. Examples for this type of deformation include strip rolling, strip extrusion etc.

Constraint to deformation along the third axis could be introduced either through the die wall or through the rigid material adjacent to deforming material, which prevents the flow.

The basis for slipline field analysis is the fact that the general state of stress on a solid in plane strain deformation can be represented by the sum of two types of stresses, namely the mean stress and the pure shear stress.

For plane strain condition we have $\sigma_2 = \frac{\sigma_1+\sigma_3}{2}$

We can write the Tresca criterion for plane strain as: $\sigma_1 - \sigma_3$ = 2k

For plane strain deformation we have the equilibrium of stresses written in differential form as:

$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial x} = 0$ ------------------- 1

$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$ ------------------2

These two differential equations will be transformed into two algebraic equations along a changed coordinate system, namely, along two directions of maximum shear. Then they can be solved subjected two suitable boundary conditions.

Consider the plane strain state of stress acting on x-y plane. Let σ_x ,σ_y and τ_{xy} be the stresses acting in this plane.

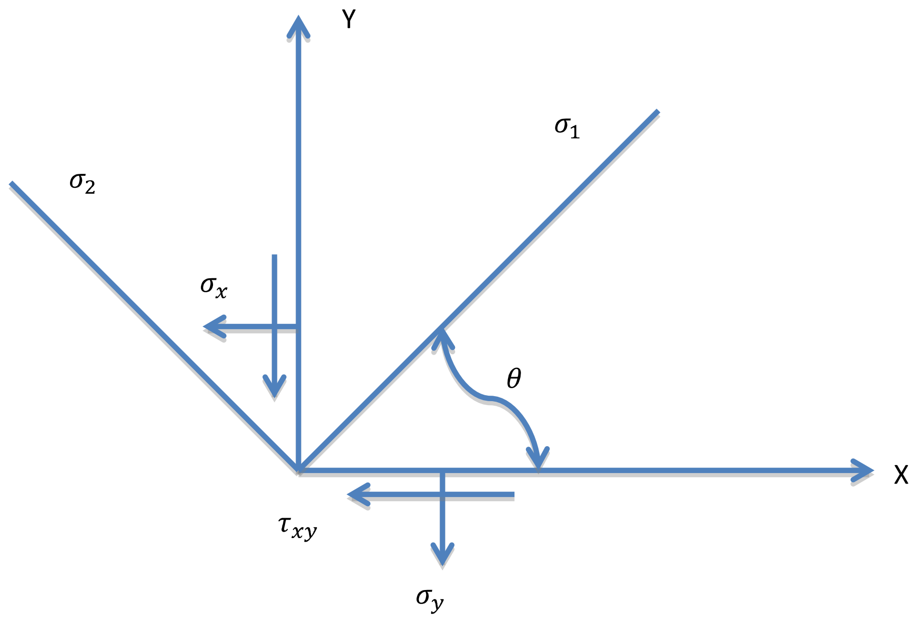

Fig. 6.1.1: Stresses in Plane strain condition

For plane strain condition, we have:

$\sigma_1 - \sigma_3$ = 2k ----------3

and

$\sigma_2 = \frac{\sigma_1+\sigma_3}{2}$ = -p (hydrostatic stress) ---- 4

For the stress conditions shown above, we can write:

$\sigma_x = \frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2} cos2\theta$ -----------------5

$\sigma_y = \frac{\sigma_1+\sigma_3}{2} - \frac{\sigma_1-\sigma_3}{2} cos2\theta$ ----------------6

$\tau_{xy} = \frac{\sigma_1-\sigma_3}{2} sin2\theta$ -----------------7

Now substituting 3 and 4 in 5, 6 and 7 we get:

σ_x = -p + kcos2θ -------------- 8

σ_y = -p - kcos2θ ---------- 9

τ_{xy} = ksin2θ ---------------- 10

Substituting the expressions for σ_x, σ_y and τ_{xy} from 8,9, 10 into the differential equations 1 and 2

We get:

$\frac{\partial(-p)}{\partial x} - 2ksin2\theta\frac{\partial\theta}{\partial x} + 2kcos2\theta\frac{\partial\theta}{\partial y} = 0$ ----- 11

$\frac{\partial(-p)}{\partial y} + 2kcos2\theta\frac{\partial\theta}{\partial x} + 2ksin2\theta\frac{\partial\theta}{\partial y} = 0$ ---------- 12

Let the x and y axes be rotated through 45°, that is, $\theta=\frac{\pi}{4}$

Then the equations 11 and 12 become:

$\frac{\partial(p+2k\theta)}{\partial x} = 0$ ------ 13

$\frac{\partial(p-2k\theta)}{\partial y} = 0$ --------- 14

If the directions x and y are taken to be directions of maximum shear, denoted as directions α *and* β,

Then we have:

$\frac{\partial(p+2k\theta)}{\partial\alpha} = 0$ ------ 15

$\frac{\partial(p-2k\theta)}{\partial\beta} = 0$ --------- 16

Equations 15 and 16 represent the two differential equations transformed to the directions of maximum shear, α *and* β.

Here the directions α *and* β are called slip lines (lines of maximum shear)

Therefore, we may now conclude from 15 and 16 that:

p+2kθ = constant along α line and = f(α) ------ 17

Similarly, p -2kθ = constant along β line and = f(β) ------18

Or we can write: p = -2k$\Delta\theta$ along α lines

And p = 2k$\Delta\theta$ along β slip lines

The above equations mean that the pressure p changes by an amount equivalent to change in the angle as one moves along the slip lines.

The following conditions are to be remembered while establishing the slip line field:

The stress normal to a free surface is a principal stress and hence the slip lines meet the free surface at 45^{o}.

α and βlines always meet at 45^{o} on a frictionless surface.

They meet at 0^{o} and 90^{o} on a surface with sticking friction

Slip happens along the slip lines as there is maximum shear along the slip lines. Further, along the tangent to the slip lines there is a discontinuity of velocity.

The angle between the intersection of one type of slip line with the other type of slip line remains the same all along the slip line.

The radii of curvature of the intersecting slip lines (β $lines$) along one type of slip lines (α $lines$) change by an amount equal to their distances traversed.

1.2 Illustration of the slip line field analysis:

Consider the extrusion of a strip through a square die. Assume a reduction of 50%. See figure below

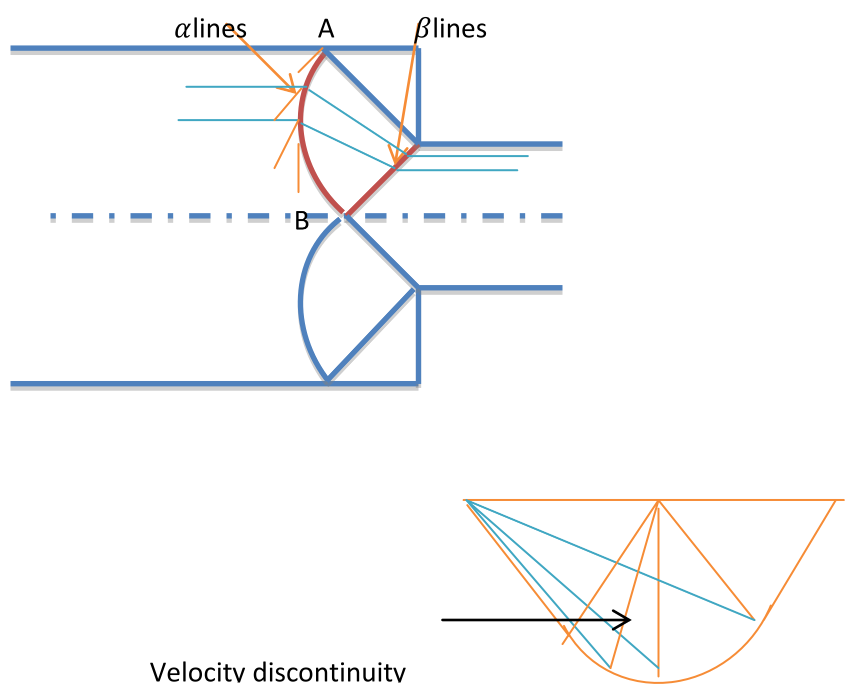

Fig.1.2.1: Slip lines and hodograph for axisymmetric extrusion

The slip lines are shown as radial and circular lines in the figure. The deformation field is symmetrical about center line. Therefore we may analyse one half of the deformation region. The hodograph – velocity diagram is also shown above.

Material undergoes velocity discontinuity along $\alpha\ lines$. The velocity discontinuities are shown in hodograph. Similarly the velocity vectors are shown in hodograph as lines radiating from top left corner of the hodograph.

The horizontal line in hodograph represents the velocity vector of the particles before they enter the $\alpha\ line$.The total length of the horizontal line in hodograph represents the exit velocity of the material, which is twice the initial velocity in this case – because we assume the reduction as 50%.

We need to find the punch pressure p. Stresses acting along the $\beta\ line$ are shear stress k and hydrostatic pressure p.

Along the $\alpha\ line$, we can write: $p+2k\theta$ = constant

Now the pressure at point A, is given by: $p_A - p_B = 2k(\theta_A - \theta_B) = 2k\frac{\pi}{2}$

We find that $p_B = k$ because, the $\beta\ line$ is inclined at 45° with the axis (No normal stress acts).

Therefore, the punch pressure, $p_A = k(1+\pi)$

The total extrusion pressure is given by: $p_e = k(1+\frac{\pi}{2})$

From the punch pressure and area of the billet we can calculate the punch force.

Source:
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