Analysis of forming - Slab Method

Forming of materials is a complex process, involving either biaxial or triaxial state of stress on the material being formed. Analysis of the forming process, therefore is highly involved. Prediction of forming load in a particular process is rather empirical. However, fairly accurate methods have been developed in order to predict the forming process and process parameters. Some of the early methods of forming analysis include slab analysis, slip line field analysis, upper bound analysis etc. With the availability of high speed computers, we can depend on finite element method for accurate predictions of forming loads. Numerous metal forming softwarehave been developed based on finite element procedures for complex shapes with more realistic boundary conditions. In this lecture we will discuss the simple slab method of forming analysis, with a typical example.

Slab method is a simple analytical procedure based on principles of mechanics. We can assume a simple relation between forming load and material flow stress in the form: F = $k\bar{\sigma}A$, where k is an empirically determined constant which takes into account friction, redundant deformation etc. The general methodology involved in slab method can be stated as follows: First the material under deformation is sliced into infinitesimally small portions. Then force balance is made on the small element. From force balance a differential equation in terms of the forming stress, geometric parameters of the billet and friction coefficient is formulated. This differential equation is solved with suitable boundary conditions. The solution gives us the required forming stress. This method may involve some simplifying assumptions. Hence this method may be considered approximate. Moreover, it may not be easy to apply this method for more complex forming processes, such as impression die forging. Slab method is developed with the assumption that the material flow is homogeneous during forming.

1.1 Slab method - Upsetting of a ring

Let us try to understand the slab method of forming analysis with the help of a simple example. Sliding or Columbic friction often occurs at the material tool interface. As a result of friction the forming load is enhanced. The flow of material is also non-uniform due to friction. Another type of friction condition, namely, shear friction or sticking friction could be convenient to consider in the analysis. In shear friction model, we assume the frictional shear stress to be proportional to shear yield strength of the material. Thus we have: $\tau = mk$, where m is friction factor and k is shear yield strength. The following assumptions are the basis of the slab analysis:

1. The reference axes are in the directions of the applied stresses
2. Friction does not cause non-uniform deformation. Therefore material is assumed to deform homogeneously – a plane remains a plane after deformation.

Consider the homogeneous deformation of a ring shaped specimen subjected to upsetting force. Let us assume shear friction at tool-material interface. The ring compression process is widely used for finding the coefficient of friction for given condition of friction. Consider an elemental portion of the ring specimen and the various stresses on this element. The following diagram shows the stresses acting on the elemental part of the ring.

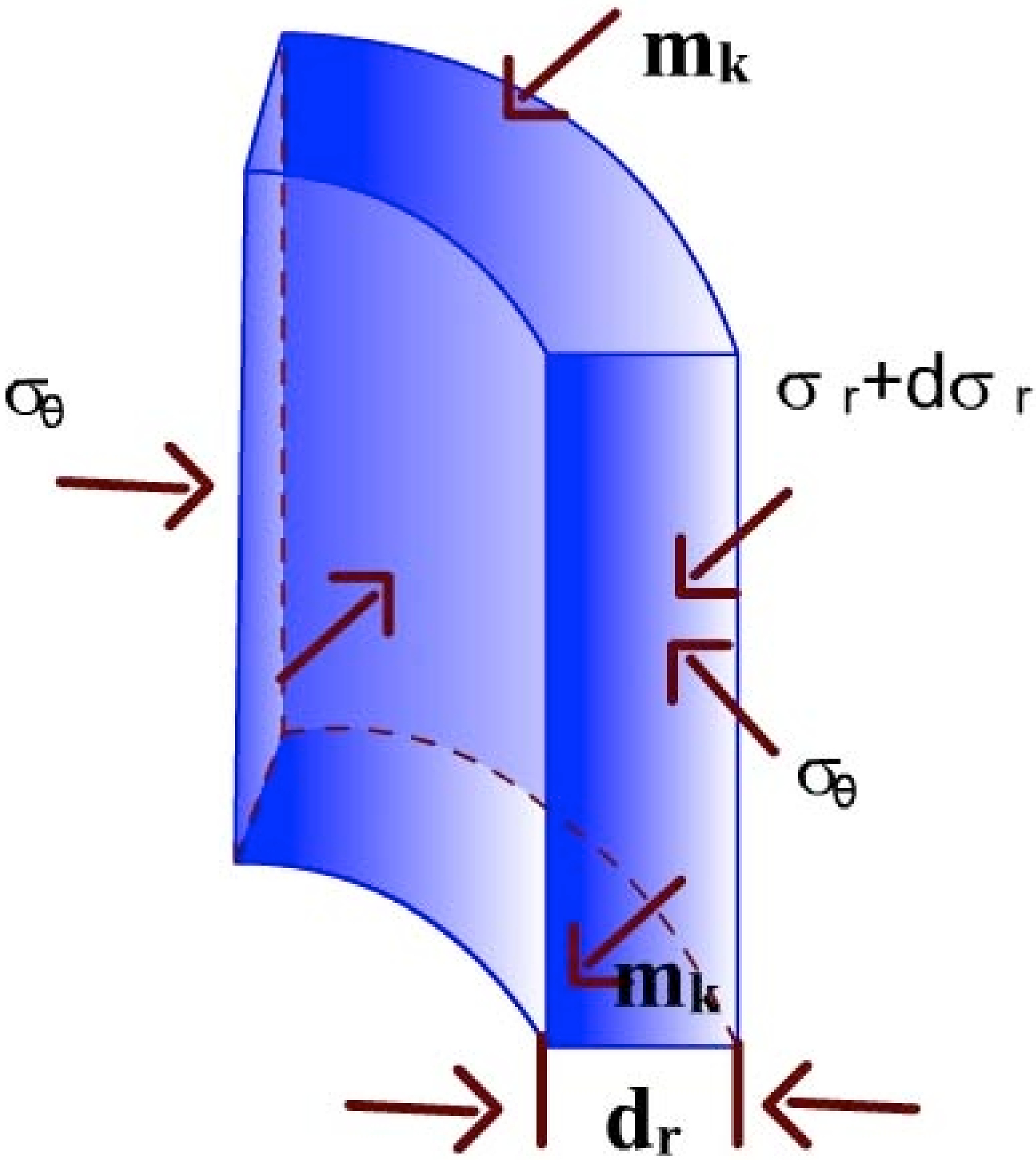

Fig. 5.1.1: Stresses acting on elemental ring subjected to upsetting

Consider a small sector of an elemental ring of radius r, radial thickness dr, height h and the angle of the sector as $d\theta$. The ring is subjected to upset force F, which is to be determined.

The various stresses acting on the sector are:

Radial stresses $\sigma_r, \sigma_r + d\sigma_r$ The corresponding forces are: σ_rrdθh and $(\sigma_r + d\sigma_r)$(r+dr)dθh

Hoop stress σ_θ The corresponding force is given by: σ_θ dr h sin$\frac{d\theta}{2}$

Frictional shear stress mk The corresponding force is: mk r drdθ

The arc length of the sector element is given by: rdθ

We can also note that for uniform deformation of the ring, $\sigma_\theta = \sigma_r$

There exists a neutral radius in the ring, such that the material deformation happens towards the axis for radii less than the neutral radius. There is a decrease in diameter of the ring.

For radii greater than the neutral radius, the material flow is away from the axis-axially outward. This condition exists because of friction. Therefore, the friction force is observed to act axially outward within the neutral section. It acts radially inward in sections beyond the neutral section.

The force balance along the radial direction gives:

σ_rrdθh –$(\sigma_r + d\sigma_r)$(r+dr)dθh + 2σ_θ dr h sin$\frac{d\theta}{2} \mp$ 2mk r drdθ = 0

Dropping higher order terms, and applying $\sigma_\theta = \sigma_r$

$d\sigma_r$ = $\mp$ 2(m/h)k dr

We need to solve for σ_z the axial stress for upsetting the ring

We can apply Tresca yield criterion in order to replace σ_r in the above differential equation.

Let us assume that the two principal stresses acting on the ring are: σ_r and σ_z

Therefore, we have:

$\sigma_z - \sigma_r$ = Y

Replacing dσ_r with dσ_z, above we have:

$d\sigma_z$ = $\mp$ 2(m/h)k dr

Integrating once, we get:

σ_z = $\mp$ 2(m/h)k r +C

For solving the constant C we could apply the following boundary conditions:

iAt r = R_i, σ_r = 0 and σ_z = Y (From Tresca criterion)

ii At r = R_0, σ_r = 0 and σ_z = Y

We get the constant C as:

C = $Y - 2(m/h)k R_i$

Or

C = $Y + 2(m/h)k R_o$

Substituting for C in the general solution, we get:

σ_z = Y + 2(m/h)k(r-R_i) ---------- for the section inside the neutral section

and

σ_z = Y + 2(m/h)k(R_o-r) ---------- for outside neutral section

For continuity of the stresses, we can take the neutral section radius as:

$R_n = (\frac{R_i+R_o}{2})$

Also note that k = $\frac{Y}{\sqrt{3}}$ according to von Mises yield criterion

One can get the average upset force, F from the local stress as followed:

$$F = \int_{R_i}^{R_n} \sigma_z \, 2\pi r dr + \int_{R_n}^{R_o} \sigma_z \, 2\pi r dr$$

$$F = (1 + \frac{1}{2\sqrt{3}} \frac{m}{h} (R_o - R_i))\bar{\sigma} A$$

In the above equation the bracketed term represents the factor which accounts for friction effect during the forming. The limitation of uniform deformation assumption in slab method is overcome in another method of analysis called slipline field analysis, which is discussed in the next lecture.

The upset force is found to vary linearly with the friction factor m, as observed from the above equation. Further, we also note that the forming force required increases with reduction in height of the ring. Rings of smaller height require greater forming force as compared to rings of larger height. This is expected because the redundant deformation zone extends towards centre for rings of smaller height.

Ring compression test is a simple test for determination of friction factor or the coefficient of friction. It can also be used for studying the lubrication characteristics of different lubricants.

Source:
http://nptel.ac.in/courses/112106153/10