Analysis of cold rolling – a more accurate method

1.1 Rolling of strip – more accurate slab analysis

The previous lecture considered an approximate analysis of the strip rolling. However, the deformation zone in rolling process is very complex and is curved. Therefore, we have to consider the various states of stresses acting, considering the curvature of the deformation zone. In cold rolling the work material is likely to undergo strain hardening as it comes out of the rolls. In the present lecture we consider the analysis considering various stresses acting on an elemental strip. Slab method of analysis is applied in order to obtain the rolling load in terms of the geometry of the deformation zone and roll diameter. We assume that rolls are not undergoing any elastic deformation.

Consider an elemental strip within the deformation zone, as shown below:

We assume that the rolling is plane strain process, as there is little spread of material along the width of the strip. Further, the friction coefficient remains constant through the rolling process.

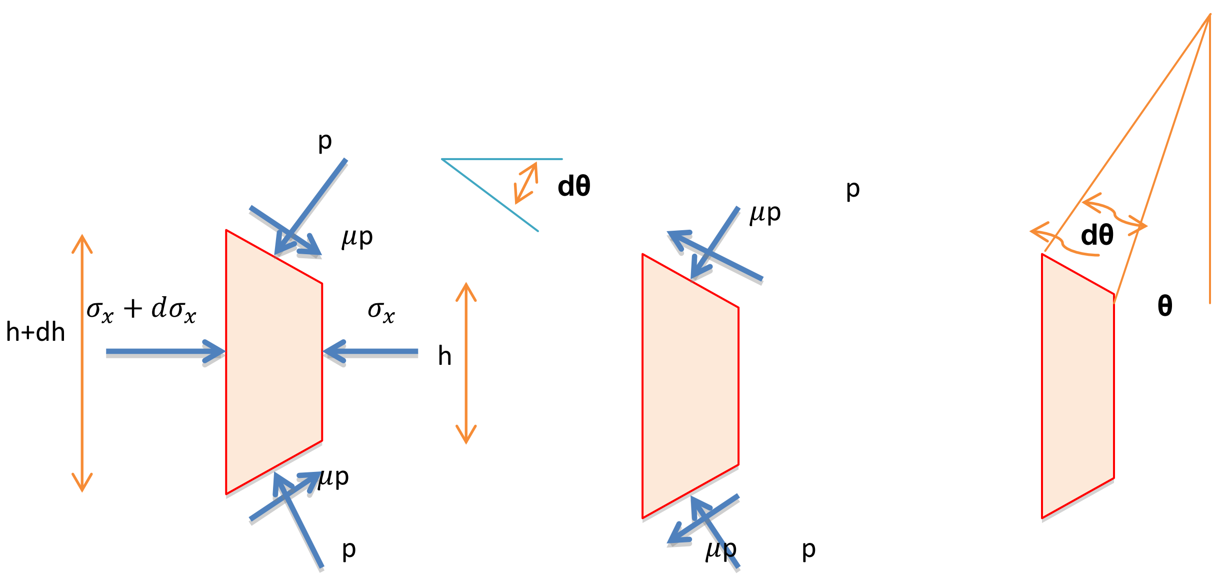

Fig. 1.1.1: Elemental strip taken from the rolling deformation zone

The element makes an angle of $d\theta$ with the roll centre.

Consider the element at an angle of θ from the line joining centres of the rolls

The following forces act on the element:

Normal roll pressure force: pR $d\theta$

Tangential friction force: μpR $d\theta$

The compressive forces: $\sigma_x h$ and $[\sigma_x + d\sigma_x]$[h+dh]

The normal and tangential forces can be resolved along the direction of rolling – x axis:

pR $d\theta sin\theta$ and μpR $d\theta cos\theta$

Making a force balance on the element shown above:

$[\sigma_x + d\sigma_x]$[h+dh] - $\sigma_x h$ - 2pR $d\theta sin\theta \pm$ 2μpR $d\theta cos\theta$ = 0 --------------23

Ignoring the products of small quantities, dividing by $d\theta$ and simplifying, we get:

$\frac{d(\sigma_x h)}{d\theta} = 2pR(sin\theta \pm \mu cos\theta)$ ----------------24

This equation is called von Karman equation.

In cold rolling, under low friction conditions angle θ is small [6 degrees]. We can approximately take; sinθ = θ and cosθ = 1. These approximates were proposed by Bland and Ford.

Now the above equation becomes:

$\frac{d(\sigma_x h)}{d\theta} = 2pR(\theta \pm \mu)$ -------------------25

From von Mises yield criterion applied to plane strain we have:

$\sigma_1 - \sigma_3 = \frac{2}{\sqrt{3}}Y$ --------26

In rolling, for small angle, the two principal stresses are: the roll pressure p and σ_x

Therefore, we have:

p - σ_x= $\frac{2}{\sqrt{3}}Y$ = Y' --------27

Substituting this in the above equation,

$\frac{d(p-Y\prime)h)}{d\theta} = 2pR(\theta \pm \mu)$ ----------------28

Or Y'h$\frac{d(\frac{p}{Y'})}{d\theta} - (p/Y' - 1)\frac{dY'h}{d\theta}$ = $2pR(\theta \pm \mu)$ --------------29

The second term on left hand side can be ignored because, Y'h is constant. That is, when h increases, Y' decreases and vice versa.

Now we have:$Y'h\,\frac{d(\frac{p}{Y'})}{d\theta} = 2pR(\theta \pm \mu)$ ----------30

$\frac{\frac{d}{d\theta}\left(\frac{p}{Y'}\right)}{\frac{p}{Y\prime}} = \frac{2R}{h}(\theta \pm \mu)$ ---------31

we can approximately write: h = h_f +Rθ^2

Substituting this in 31 and integrating we get the general solution to the above differential equation as:

$p = AY'\frac{h}{R}e^{\pm\mu H}$----------32

where H = 2$\sqrt{\frac{R}{h_f}}$tan^{-1}[$\sqrt{\frac{R}{h_f}}\theta$] ---------32A

Applying the boundary conditions: At entry, $\theta = \alpha$ and H = H_o

AT exit, $\theta = 0$ and H = 0

We get the roll pressure as:

p = $Y'\frac{h}{h_o}e^{\mu(Ho-H)}$ at the entry ------33

p = $Y'\frac{h}{h_f}e^{\mu H}$ at exit -----------43

From the above expressions we note that the local rolling pressure depends on the angular position of the section and the height of the work, h. It is also dependent on R/h_fR/$h_f$is equivalent of a/h in forging. As this ratio increases, the rolling pressure also increases.

The total rolling force P can be evaluated by integrating the local rolling force over the arc of contact.

P = Rb$\int_0^{\alpha} pd\theta$, where b is width of the strip ----44

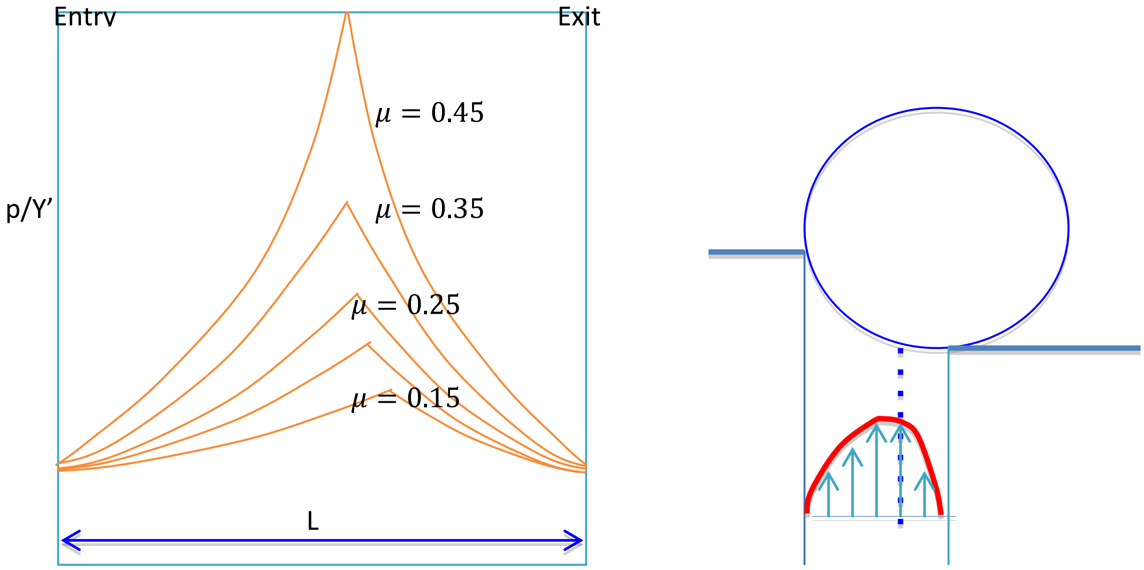

Fig. 1.1.2: Actual variation of roll pressure

The above figure shows the variation of the non-dimensional roll pressure with respect to the coefficient of friction – the friction hill. We observe that the roll pressure increases with increase in coefficient of friction. The area under the curves gives the total roll force. Further, we also observe that the neutral point also shifts towards the exit as the coefficient of friction reduces. As the friction gets reduced, there is slipping between the rolls and the work. Hence the relative velocity between roll and strip is in the same direction.

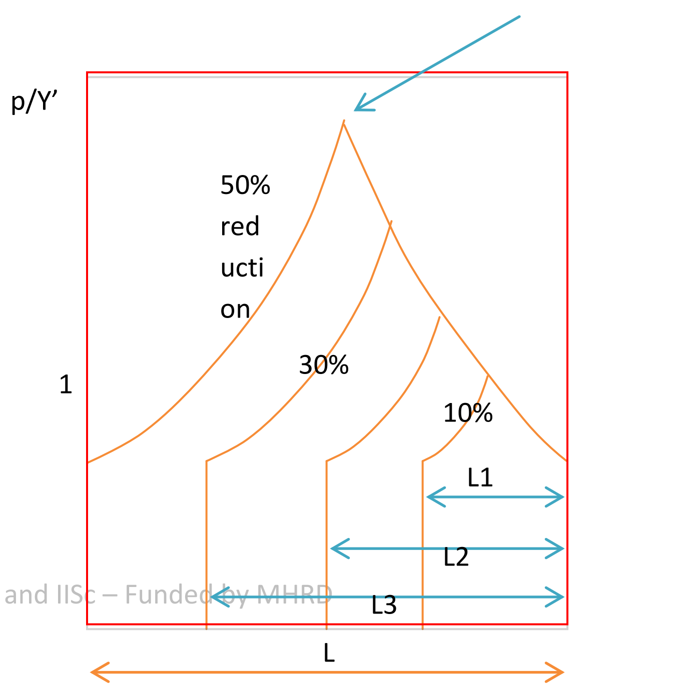

Fig.4.1.3: Roll Entry **pressure versus reduction in thickness**

The above figure represents the variation of roll pressure with respect to thickness reduction of the strip. As the reduction increases the roll pressure also increases. This is because, for larger reductions, the length of contact between roll and strip increases.

1.2 Determination of neutral point:

The neutral point can be determined by equating the roll pressure before neutral point to that after neutral point. Equating the equations 33 and 34 and solving for Hn,

$H_n = \frac{1}{2}(H_o - \frac{1}{\mu} ln\frac{h_o}{h_f})$ ------45

Substituting this in equation 32A and solving for θ,

$\theta_n = \sqrt{\frac{h_f}{R}}\tan[\sqrt{\frac{h_f}{R}}\frac{H_n}{2}]$ ---------------46

Example:

Determine the rolling power required to roll low carbon steel strip, 250 mm wide, 12 mm thick, if the final thickness is 9 mm. Assume sliding friction between the rolls and work, with a coefficient of friction 0.12. The 250 mm radius rolls rotate at a speed of 300 rpm. Take k= 550 MPa, n = 0.26 for steel.

Solution:

We can take the average roll force for sliding friction condition as:

$F = Lw\bar{Y}'(1+\frac{\mu L}{2h_{av}})$

True strain = ε = ln(ho/hf) = 0.287

The average flow stress of the material = $k\varepsilon^n/1+n$ = 315.73 MPa

Plane strain flow stress Y' = $\frac{2}{\sqrt{3}}$Average flow stress = 364.58 MPa

hav = (12+9)/2 = 10.5 mm

L = $\sqrt{R\Delta h}$ = 27.39 mm

Rolling load F = 2.9MN

Roll torque = FXL/2

Power = $\pi NTorque$ = 1,25 MW

Source:
http://nptel.ac.in/courses/112106153/21