# ANALYSIS OF STRAINS

CONCEPT OF STRAIN

Concept of strain : if a bar is subjected to a direct load, and hence a stress the bar will change in length. If the bar has an original length L and changes by an amount δL, the strain produce is defined as follows:

$$strain(\epsilon) = \frac{change\ in\ length}{orginal\ length} = \frac{\delta L}{L}$$

Strain is thus, a measure of the deformation of the material and is a nondimensional Quantity i.e. it has no units. It is simply a ratio of two quantities with the same unit.
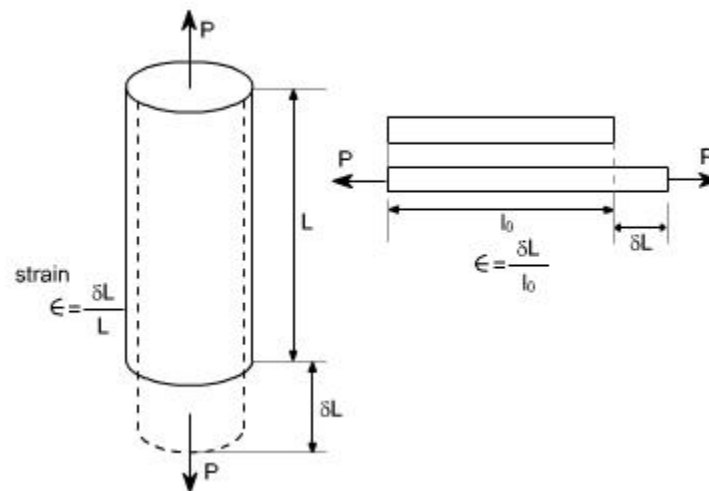

Since in practice, the extensions of materials under load are very very small, it is often convenient to measure the strain in the form of strain x 10^{-6} i.e. micro strain, when the symbol used becomes $\mu \epsilon$.

Sign convention for strain:

Tensile strains are positive whereas compressive strains are negative. The strain defined earlier was known as linear strain or normal strain or the longitudinal strain now let us define the shear strain.

Definition: An element which is subjected to a shear stress experiences a deformation as shown in the figure below. The tangent of the angle through which two adjacent sides rotate relative to their initial position is termed shear strain. In many cases the angle is very small and the angle it self is used, (in radians), instead of tangent, so that $\gamma = \angle AOB - \angle A'OB' = \phi$

Shear strain: As we know that the shear stresses acts along the surface. The action of the stresses is to produce or being about the deformation in the body consider the distortion produced b shear sheer stress on an element or rectangular block

$$\gamma = \angle AOB - \angle A'OB'$$
$$= \phi$$
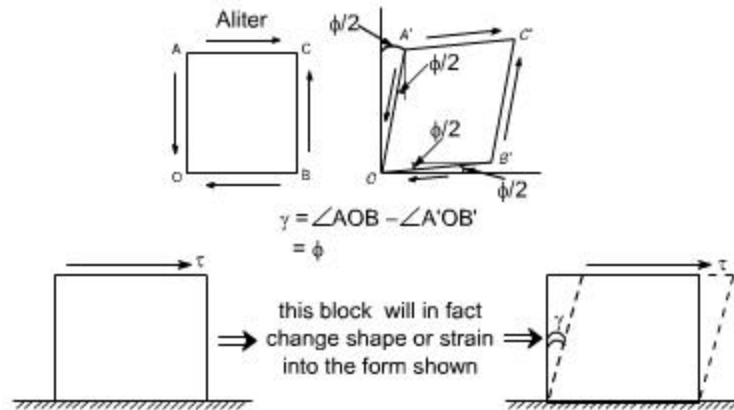
This shear strain or slide is ϕ and can be defined as the change in right angle. or The angle of deformation γ is then termed as the shear strain. Shear strain is measured in radians & hence is non – dimensional i.e. it has no unit. So we have two types of strain i.e. normal stress & shear stresses.

Hook's Law :

A material is said to be elastic if it returns to its original, unloaded dimensions when load is removed.

Hook's law therefore states that

Stress (σ) α strain($\in$)

$$\boxed{\frac{stress}{strain} = constant}$$

Modulus of elasticity : Within the elastic limits of materials i.e. within the limits in which Hook's law applies, it has been shown that

Stress / strain = constant

This constant is given by the symbol E and is termed as the modulus of elasticity or Young's modulus of elasticity

$$E = \frac{strain}{stress} = \frac{\sigma}{\in}$$
$$= \frac{P/A}{\delta L/L}$$

Thus $\boxed{E = \frac{PL}{A\delta L}}$

The value of Young's modulus E is generally assumed to be the same in tension or compression and for most engineering material has high, numerical value of the order of 200 GPa

Poisson's ratio: If a bar is subjected to a longitudinal stress there will be a strain in this direction equal to σ / E . There will also be a strain in all directions at right angles to σ . The final shape being shown by the dotted lines.

It has been observed that for an elastic materials, the lateral strain is proportional to the longitudinal strain. The ratio of the lateral strain to longitudinal strain is known as the poison's ratio .

Poison's ratio (μ) = – lateral strain / longitudinal strain

For most engineering materials the value of μ his between 0.25 and 0.33.

Three – dimensional state of strain : Consider an element subjected to three mutually perpendicular tensile stresses σ_x , $\sigma_y$and σ_z as shown in the figure below.
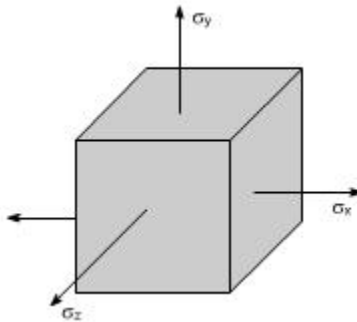
If σ_y and σ_z were not present the strain in the x direction from the basic definition of Young's modulus of Elasticity E would be equal to

$$\epsilon_x = \sigma_x / E$$

The effects of σ_y and σ_z in x direction are given by the definition of Poisson's ratio ' μ ' to be equal as $-\mu \, \sigma_y / E$ and $-\mu \, \sigma_z / E$

The negative sign indicating that if $\sigma_y$and σ_z are positive i.e. tensile, these they tend to reduce the strain in x direction thus the total linear strain is x direction is given by

$$\epsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E} - \mu\frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_x}{E}$$

Principal strains in terms of stress:

In the absence of shear stresses on the faces of the elements let us say that σ_x , σ_y , σ_z are in fact the principal stress. The resulting strain in the three directions would be the principal strains.

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1 - \mu\sigma_3\right]$$

$$\epsilon_3 = \frac{1}{E}\left[\sigma_3 - \mu\sigma_1 - \mu\sigma_2\right]$$

i.e. We will have the following relation.

For Two dimensional strain: system, the stress in the third direction becomes zero i.e $\sigma_z = 0$ or $\sigma_3 = 0$
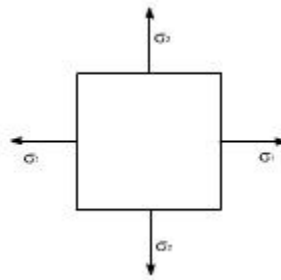
Although we will have a strain in this direction owing to stresses σ_1 & σ_2 .

$$\epsilon_1 = \frac{1}{E}\left[\sigma_1 - \mu\sigma_2\right]$$

$$\epsilon_2 = \frac{1}{E}\left[\sigma_2 - \mu\sigma_1\right]$$

$$\epsilon_3 = \frac{1}{E}\left[-\mu\sigma_1 - \mu\sigma_2\right]$$

Hence the set of equation as described earlier reduces to

Hence a strain can exist without a stress in that direction

i.e if $\sigma_3 = 0$; $\epsilon_3 = \frac{1}{E}\left[-\mu\sigma_1 - \mu\sigma_2\right]$

Also

$\epsilon_1 .E = \sigma_1 - \mu\sigma_2$

$\epsilon_2 .E = \sigma_2 - \mu\sigma_1$

so the solution of above two equations yields

$$\sigma_1 = \frac{E}{(1 - \mu^2)}\left[\epsilon_1 + \mu\epsilon_2\right]$$

$$\sigma_2 = \frac{E}{(1 - \mu^2)}\left[\epsilon_2 + \mu\epsilon_1\right]$$

Hydrostatic stress : The term Hydrostatic stress is used to describe a state of tensile or compressive stress equal in all directions within or external to a body. Hydrostatic stress causes a change in volume of a material, which if expressed per unit of original volume gives a volumetric strain denoted by☐ϵ_v. So let us determine the expression for the volumetric strain.

Volumetric Strain:
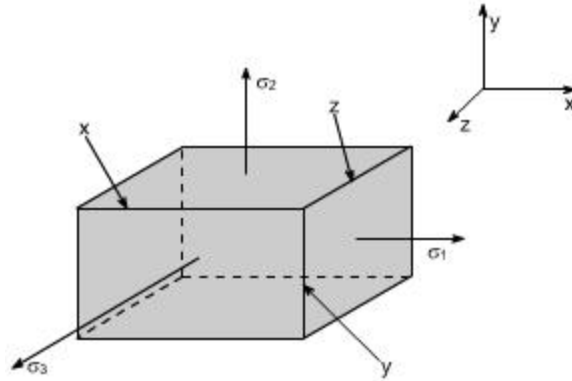
Consider a rectangle solid of sides x, y and z under the action of principal stresses σ_1, σ_2, σ_3 respectively.

Then ϵ_1, ϵ_2, and ϵ_3 are the corresponding linear strains, than the dimensions of the rectangle becomes

$(x + \epsilon_1 . x)$; $(y + \epsilon_2 . y)$; $(z + \epsilon_3 . z)$

$$\text{Volumetric strain} = \frac{\text{Increase in volume}}{\text{Original volume}}$$

$$= \frac{x(1+\epsilon_1)y(1+\epsilon_2)(1+\epsilon_3)z - xyz}{xyz}$$

$$= (1+\epsilon_1)y(1+\epsilon_2)(1+\epsilon_3) - 1 \cong \epsilon_1 + \epsilon_2 + \epsilon_3 \left[\text{Neglecting the products of } \epsilon's\right]$$

hence the

ALITER : Let a cuboid of material having initial sides of Length x, y and z. If under some load system, the sides changes in length by dx, dy, and dz then the new volume $(x + dx)(y + dy)(z + dz)$

New volume = xyz + yzdx + xzdy + xydz

Original volume = xyz

Change in volume = yzdx + xzdy + xydz

Volumetric strain = $(yzdx + xzdy + xydz) / xyz = \epsilon_x + \epsilon_y + \epsilon_z$

Neglecting the products of epsilon's since the strains are sufficiently small.

Volumetric strains in terms of principal stresses:

As we know that

$$\epsilon_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_3}{E}$$

$$\epsilon_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Futher Volumetric strain $= \epsilon_1 + \epsilon_2 + \epsilon_3$

$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{E} - \frac{2\mu(\sigma_1 + \sigma_2 + \sigma_3)}{E}$$
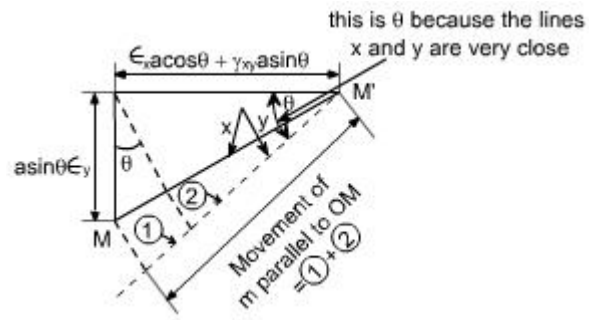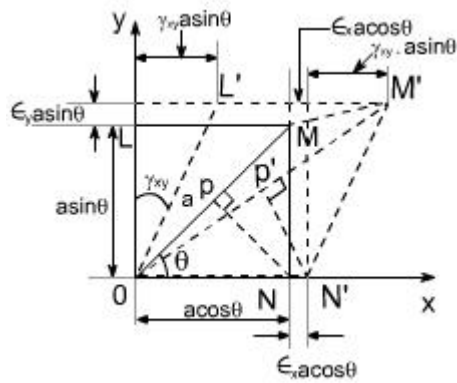
$$= \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}$$

hence the

$$\boxed{\text{Volumetric strain} = \frac{(\sigma_1 + \sigma_2 + \sigma_3)(1 - 2\mu)}{E}}$$

Strains on an oblique plane

(a) Linear strain

Consider a rectangular block of material OLMN as shown in the xy plane. The strains along ox and oy are ϵ_x and ϵ_y , and γ_{xy} is the shearing strain.

Then it is required to find an expression for ϵ_θ, i.e the linear strain in a direction inclined at θ to OX, in terms of ϵ_x, ϵ_y , γ_{xy} and θ.

Let the diagonal OM be of length 'a' then ON = a cos θ and OL = a sin θ , and the increase in length of those under strains are $\epsilon_x$acos θ and ϵ_ya sin θ (i.e. strain x original length) respectively.

If M moves to M', then the movement of M parallel to x axis is $\epsilon_x$acos θ + γ_{xy} sin θ and the movement parallel to the y axis is $\epsilon_y$asin θ

Thus the movement of M parallel to OM , which since the strains are small is practically coincident with MM'. and this would be the summation of portions (1) and (2) respectively and is equal to

$$= (\epsilon_y \, a \sin\theta)\sin\theta + (\epsilon_x \, a \cos\theta + \gamma_{xy} a \sin\theta)\cos\theta$$

$$= a\left[\epsilon_y \sin\theta.\sin\theta + \epsilon_x \cos\theta.\cos\theta + \gamma_{xy}\sin\theta.\cos\theta\right]$$

hence the strain along OM

$$= \frac{extension}{original\,length}$$

$$\epsilon_\theta = \epsilon_x \cos^2\theta + \gamma_{xy}\sin\theta.\cos\theta + \epsilon_y \sin^2\theta$$

$$\epsilon_\theta = \epsilon_x \cos^2\theta + \epsilon_y \sin^2\theta + \gamma_{xy}\sin\theta.\cos\theta$$

$$\text{Recalling } \cos^2\theta - \sin^2\theta = \cos2\theta$$

$$\text{or } 2\cos^2\theta - 1 = \cos2\theta$$

$$\cos^2\theta = \left[\frac{1 + \cos2\theta}{2}\right]$$

$$\sin^2\theta = \left[\frac{1 - \sin2\theta}{2}\right]$$

hence

$$\epsilon_\theta = \epsilon_x\left[\frac{1 + \cos2\theta}{2}\right] + \epsilon_y\left[\frac{1 - \sin2\theta}{2}\right] + \gamma_{xy}a\sin\theta.\cos\theta$$

$$= \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2}\cos2\theta + \frac{1}{2}\gamma_{xy}\sin2\theta$$

$$\epsilon_\theta = \left\{\frac{\epsilon_x + \epsilon_y}{2}\right\} + \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos2\theta + \frac{1}{2}\gamma_{xy}\sin2\theta$$

This expression is identical in form with the equation defining the direct stress on any inclined plane θ with ϵ_x and ϵ_y replacing σ_x and σ_y and ½ γ_{xy} replacing τ_{xy} i.e. the shear stress is replaced by half the shear strain

Shear strain: To determine the shear stain in the direction OM consider the displacement of point P at the foot of the perpendicular from N to OM and the following expression can be derived as

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin2\theta - \frac{1}{2}\gamma_{xy}\cos2\theta\right]$$

In the above expression ½ is there so as to keep the consistency with the stress relations.

Futher -ve sign in the expression occurs so as to keep the consistency of sign convention, because OM' moves clockwise with respect to OM it is considered to be negative strain.

The other relevant expressions are the following :

Principal planes :

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x - \epsilon_y}$$

Principal strains :

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
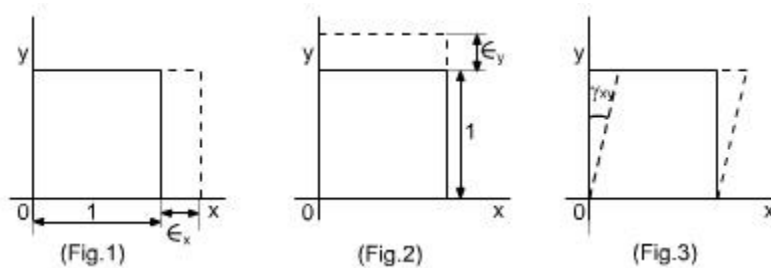
Maximum shear strains :

$$\frac{\gamma_{max}}{2} = \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Let us now define the plane strain condition

Plane Strain :

In xy plane three strain components may exist as can be seen from the following figures:

(Fig.1)   (Fig.2)   (Fig.3)

Therefore, a strain at any point in body can be characterized by two axial strains i.e ϵ_x in x direction, ϵ_y in y - direction and γ_{xy} the shear strain.

In the case of normal strains subscripts have been used to indicate the direction of the strain, and ϵ_x , ϵ_y are defined as the relative changes in length in the co-ordinate directions.

With shear strains, the single subscript notation is not practical, because such strains involves displacements and length which are not in same direction.The symbol and subscript γ_{xy} used for the shear strain referred to the x and y planes. The order of the subscript is unimportant. γ_{xy} and γ_{yx} refer to the same physical quantity. However, the sign convention is important.The shear strain γ_{xy} is considered to be positive if it represents a decrease the angle between the sides of an element of material lying parallel the positive x and y axes. Alternatively we can think of positive shear strains produced by the positive shear stresses and viceversa.

Plane strain :

An element of material subjected only to the strains as shown in Fig. 1, 2, and 3 respectively is termed as the plane strain state.

Thus, the plane strain condition is defined only by the components ϵ_x , ϵ_y , γ_{xy} : $\epsilon_z = 0$; $\gamma_{xz}= 0$; $\gamma_{yz}= 0$

It should be noted that the plane stress is not the stress system associated with plane strain. The plane strain condition is associated with three dimensional stress system and plane stress is associated with three dimensional strain system.

PRINCIPAL STRAIN

For the strains on an oblique plane we have an oblique we have two equations which are identical in form with the equation defining the direct stress on any inclined plane θ .

$$\epsilon_\theta = \left\{\frac{\epsilon_x + \epsilon_y}{2}\right\} + \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin 2\theta - \frac{1}{2}\gamma_{xy}\cos 2\theta\right]$$

Since the equations for stress and strains on oblique planes are identical in form, so it is evident that Mohr's stress circle construction can be used equally well to represent strain conditions using the horizontal axis for linear strains and the vertical axis for half the shear strain.

It should be noted, however that the angles given by Mohr's stress circle refer to the directions of the planes on which the stress act and not the direction of the stresses themselves.

The direction of the stresses and therefore associated strains are therefore normal (i.e. at 90^0) to the directions of the planes. Since angles are doubled in Mohr's stress circle construction it follows therefore that for a true similarity of working a relative rotation of axes of 2 x 90^0 = 180^0 must be introduced. This is achieved by plotting positive sheer strains vertically downwards on the strain circle construction.

The sign convention adopted for the strains is as follows:

Linear Strains : extension - positive

  compression - negative

{ Shear of strains are taken positive, when they increase the original right angle of an unstrained element. }

Shear strains : for Mohr's strains circle sheer strain γ_{xy} - is +ve referred to x - direction the convention for the shear strains are bit difficult. The first subscript in the symbol γ_{xy} usually denotes the shear strains associated with direction. e.g. in γ_{xy}– represents the shear strain in x - direction and for γ_{yx}– represents the shear strain in y - direction. If under strain the line associated with first subscript moves counter clockwise with respect to the other line, the shearing strain is said to be positive, and if it moves clockwise it is said to be negative.

N.B: The positive shear strain is always to be drown on the top of$\square\epsilon_x$ .If the shear stain γ_{xy} is given]

Moh's strain circle

For the plane strain conditions can we derivate the following relations

$$\epsilon_\theta = \left\{\frac{\epsilon_x + \epsilon_y}{2}\right\} + \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos2\theta + \frac{1}{2}\gamma_{xy}\sin2\theta \qquad (1)$$

$$\frac{1}{2}\gamma_\theta = -\left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin2\theta - \frac{1}{2}\gamma_{xy}\cos2\theta\right] \qquad (2)$$

Re writing the equation (1) as below :

$$\left[\epsilon_\theta - \left(\frac{\epsilon_x + \epsilon_y}{2}\right)\right] = \left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos2\theta + \frac{1}{2}\gamma_{xy}\sin2\theta \qquad (3)$$

squaring and adding equations (2) and (3)

$$\left[\epsilon_\theta - \left(\frac{\epsilon_x + \epsilon_y}{2}\right)\right]^2 + \left\{\frac{1}{2}\gamma_\theta\right\}^2 = \left[\left\{\frac{\epsilon_x - \epsilon_y}{2}\right\}\cos2\theta + \frac{1}{2}\gamma_{xy}\sin2\theta\right]^2$$

$$+ \left[\frac{1}{2}(\epsilon_x - \epsilon_y)\sin2\theta - \frac{1}{2}\gamma_{xy}\cos2\theta\right]^2$$

$$\left[\epsilon_\theta - \left(\frac{\epsilon_x + \epsilon_y}{2}\right)\right]^2 + \left\{\frac{1}{2}\gamma_\theta\right\}^2 = \left(\frac{\epsilon_x + \epsilon_y}{2}\right)^2 + \frac{\gamma_{xy}^2}{4}$$

Now as we know that

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\epsilon_1 + \epsilon_2 = \epsilon_x + \epsilon_y$$

$$\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 = \left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \frac{\gamma_{xy}^2}{4}$$

Therefore the equation gets transformed to

$$\left[\epsilon_\theta - \left(\frac{\epsilon_1 + \epsilon_2}{2}\right)\right]^2 + \left[\frac{\gamma_\theta}{2}\right]^2 = \left(\frac{\epsilon_1 - \epsilon_2}{2}\right)^2 \qquad (4)$$

If we plot equation (4) we obtain a circle of radius $\left(\frac{\epsilon_1 - \epsilon_2}{2}\right)$ with center at $\left(\frac{\epsilon_1 + \epsilon_2}{2}, 0\right)$

A typical point P on the circle given the normal strain and half the sheer strain $1/2\gamma_{xy}$ associated with a particular plane. We note again that an angle subtended at the centre of Mohr's circle by an arc connecting two points on the circle is twice the physical angle in the material.

Mohr strain circle :

Since the transformation equations for plane strain are similar to those for plane stress, we can employ a similar form of pictorial representation. This is known as Mohr's strain circle.

The main difference between Mohr's stress circle and stress circle is that a factor of half is attached to the shear strains.
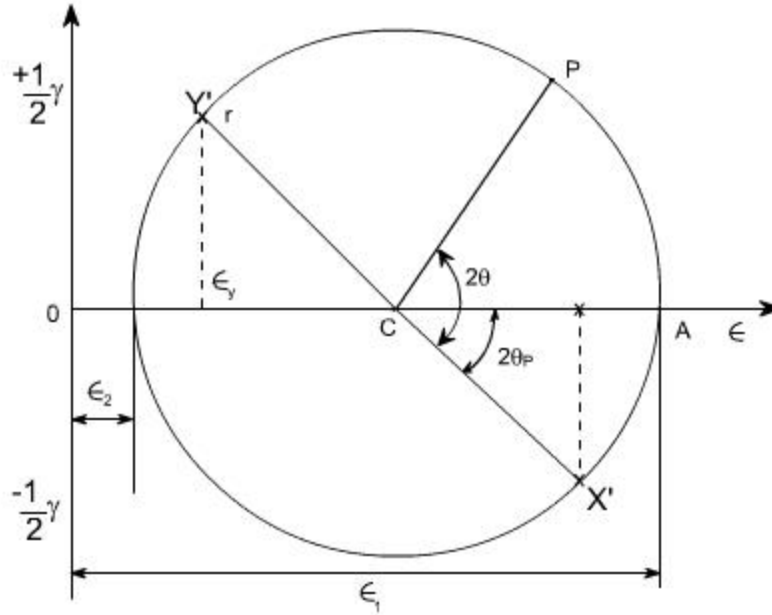
Points X' and Y' represents the strains associated with x and y directions with $\in$ and $\gamma_{xy}/2$ as co-ordiantes

Co-ordinates of X' and Y' points are located as follows :

$$X' = \left(\in_{x} , - \frac{\gamma_{xy}}{2} \right)$$

$$Y' = \left(\in_{y} , + \frac{\gamma_{xy}}{2} \right)$$

In x – direction, the strains produced, the strains produced by σ_x, and $-\tau\square_{xy}$ are $\in_x$ and $-\gamma_{xy}/2$

where as in the Y - direction, the strains are produced by $\in_y$ and $+\gamma_{xy}$ are produced by σ_y and $+\tau_{xy}$

These co-ordinated are consistent with our sign notation (i.e. + ve shear stresses produces produce +ve shear strain & vice versa)

on the face AB is τ_{xy}+ve i.e strains are ($\in_y$, $+\gamma_{xy}/2$) where as on the face BC, $\square\tau_{xy}$ is negative hence the strains are ($\square\in_x$, $-\gamma_{xy}/2$)
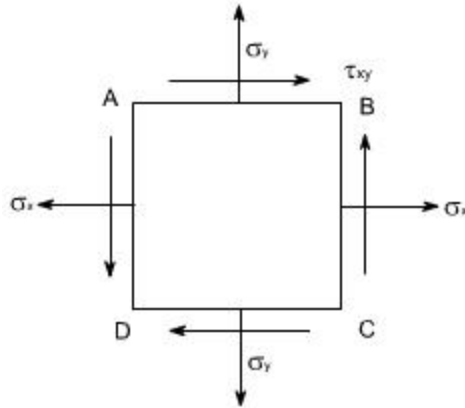
A typical point P on the circle gives the normal strains and half the shear strain, associated with a particular plane we must measure the angle from x – axis (taken as reference) as the required formulas for $\in_\theta$,$-1/2\ \gamma_\theta$ have been derived with reference to x-axis with angle measuring in the c.c.W direction
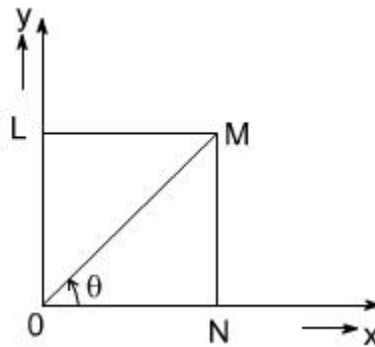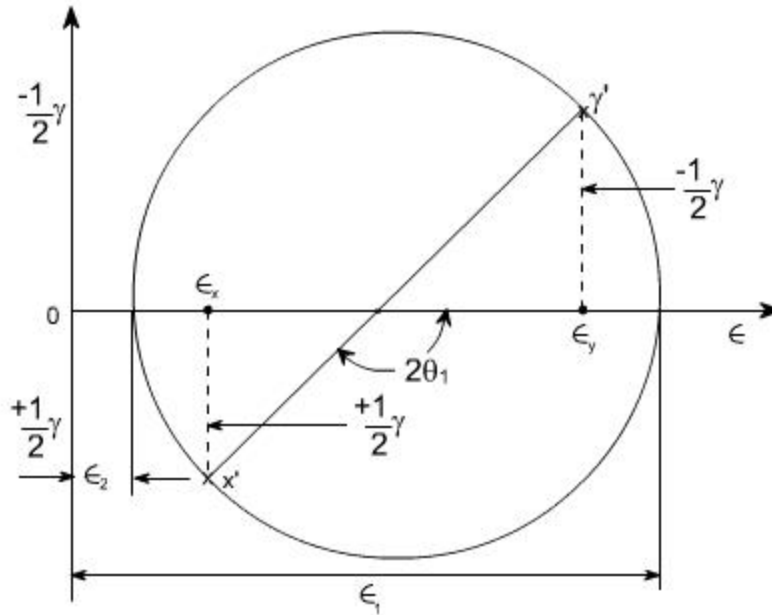

CONSTRUCTION :

In this we would like to locate the points x' & y' instead of AB and BC as we have done in the case of Mohr's stress circle.

steps

1. Take normal or linear strains on x-axis, whereas half of shear strains are plotted on y-axis.

2. Locate the points x' and y'

3. Join x' and y' and draw the Mohr's strain circle

4. Measure the required parameter from this construction.

Note: positive shear strains are associated with planes carrying positive shear stresses and negative strains with planes carrying negative shear stresses.

ILLUSTRATIVE EXAMPLES :

1. At a certain point, a material is subjected to the following state of strains:

$\epsilon_x = 400 \times 10^{-6}$ units

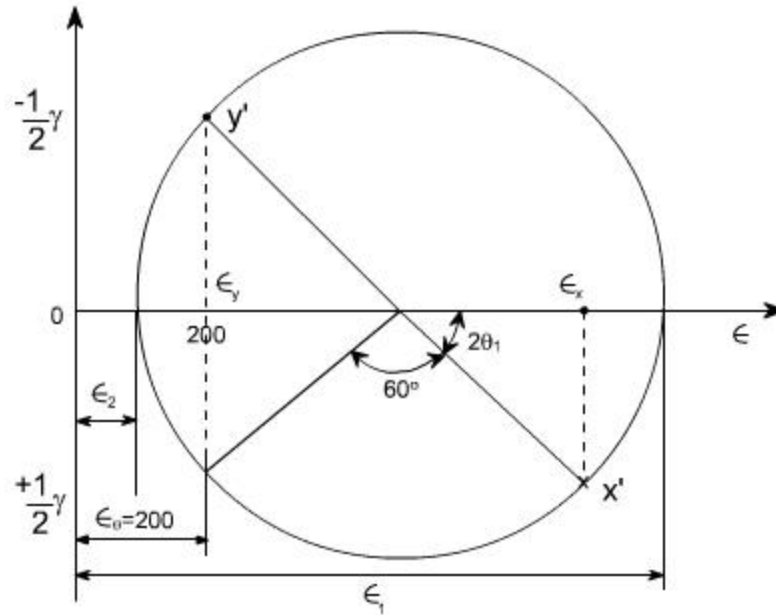$\epsilon_y = 200 \times 10^{-6}$ units

$\gamma_{xy} = 350 \times 10^{-6}$ radians

Determine the magnitudes of the principal strains, the direction of the principal strains axes and the strain on an axis inclined at 30^0 clockwise to the x – axis.

Solution:

Draw the Mohr's strain circle by locating the points x' and y'

By Measurement the following values may be computed

$\in_1 = 500 \times 10^{-6}$ units

$\in_2 = 100 \times 10^{-6}$ units

$\theta_1 = 60^0 /2 = 30^0$

$\theta_2 = 90 + 30 = 12^0$

$\in_{30} = 200 \times 10^{-6}$ units

The angles being measured c.c.w. from the direction of $\in_x$.

PROB 2.

A material is subjected to two mutually perpendicular strains $\in_x = 350 \times 10^{-6}$ units and $_{\in y} = 50 \times 10^{-6}$ units together with an unknown sheer strain γ_{xy} if the principal strain in the material is 420×10^{-6} units Determine the following.

(a) Magnitude of the shear strain

(b) The other principal strain

(c) The direction of principal strains axes

(d) The magnitude of the principal stresses

If E = 200 GN / m^2;$\square\gamma$ = 0.3

Solution :

The Mohr's strain circle can be drawn as per the procedure described earlier. from the graphical construction, the following results may bre obtained :

(i) Shear strain γ_{xy} = 324 x 10^{-6} radians

(ii) other principal strain = -20 x 10^{-6}

(iii) direction of principal strain = 47^0 / 2 = 23^0 30'

(iv) direction of other principal strain = 90^0 +23^0 30' = 113^0 30'

In order to determine the magnitude of principle stresses, the computed values of $\in_1$ and $\in_2$ from the graphical construction may be substituted in the following expressions

$$\sigma_1 = \frac{(\in_1 + \gamma\in_2)}{(1 - \gamma^2)}.E = 91 \ \frac{MN}{m^2}$$

$$\sigma_2 = \frac{(\in_2 + \gamma\in_1)}{(1 - \gamma^2)}.E = 23 \ \frac{MN}{m^2}$$

Use of strain Gauges :

Although we can not measure stresses within a structural member, we can measure strains, and from them the stresses can be computed, Even so, we can only measure strains on the surface. For example, we can mark points and lines on the surface and measure changes in their spacing angles. In doing this we are of course only measuring average strains over the region concerned. Also in view of the very small changes in dimensions, it is difficult to archive accuracy in the measurements

In practice, electrical strain gage provide a more accurate and convenient method of measuring strains.

A typical strain gage is shown below.
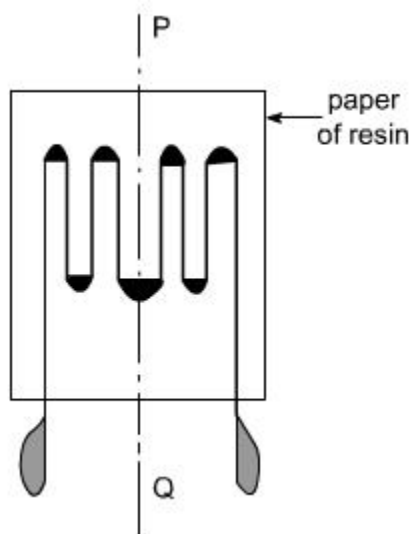
The gage shown above can measure normal strain in the local plane of the surface in the direction of line PQ, which is parallel to the folds of paper. This strain is an average value of for the region covered by the gage, rather than a value at any particular point.

The strain gage is not sensitive to normal strain in the direction perpendicular to PQ, nor does it respond to shear strain. therefore, in order to determine the state of strain at a particular small region of the surface, we usually need more than one strain gage.
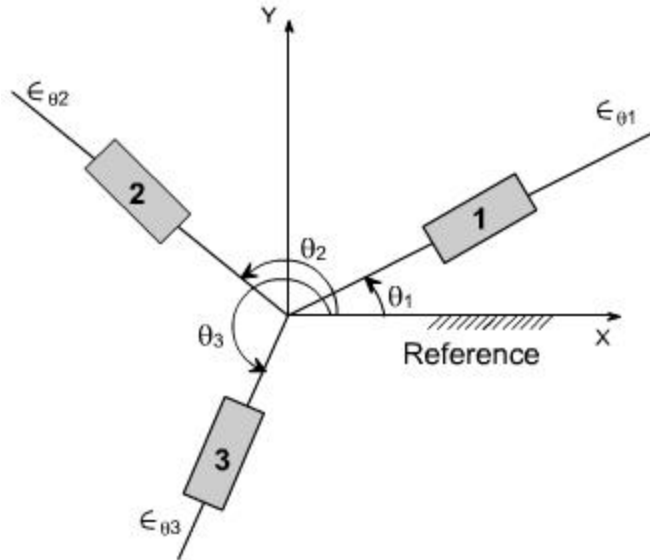
To define a general two dimensional state of strain, we need to have three pieces of information, such as ϵ_x, ϵ_y and γ_{xy} referred to any convenient orthogonal co-ordinates x and y in the plane of the surface. We therefore need to obtain measurements from three strain gages. These three gages must be arranged at different orientations on the surface to from a strain rossett. Typical examples have been shown, where the gages are arranged at either 45^0 or 60^0 to each other as shown below :

(i)   (ii)

(iii)
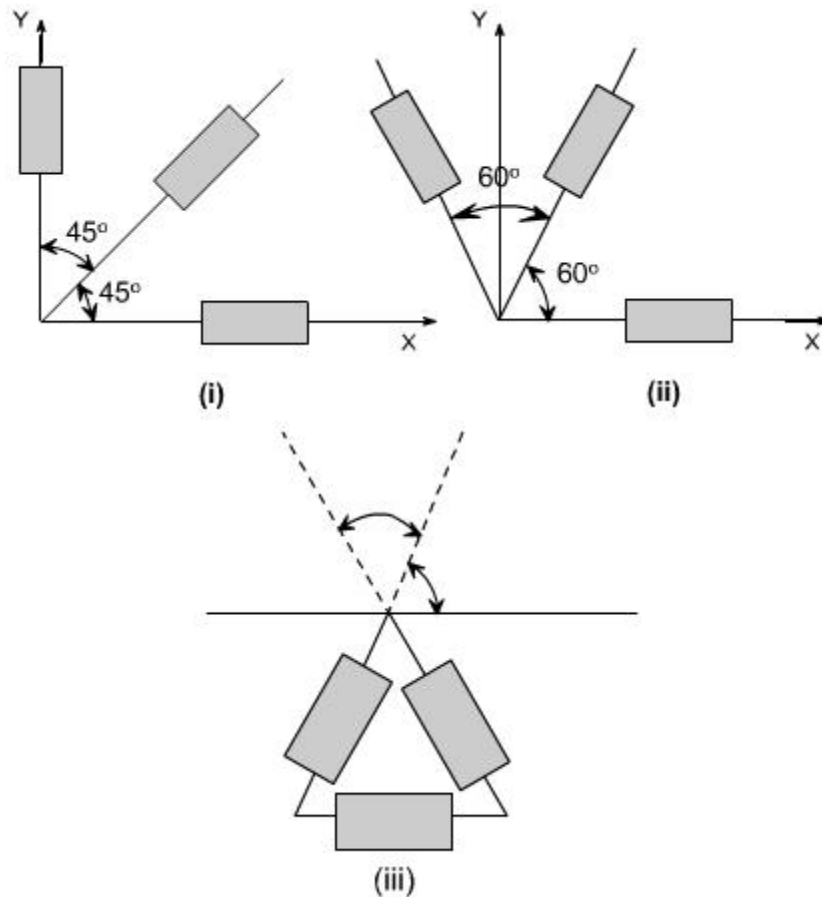
A group of three gages arranged in a particular fashion is called a strain rosette. Because the rosette is mounted on the surface of the body, where the material is in plane stress, therefore, the transformation equations for plane strain to calculate the strains in various directions.

Knowing the orientation of the three gages forming a rosette, together with the in – plane normal strains they record, the state of strain at the region of the surface concerned can be found. Let us consider the general case shown in the figure below, where three strain gages numbered 1, 2, 3, where three strain gages numbered 1, 2, 3 are arranged at an angles of θ_1, θ_2, θ_3 measured c.c.w from reference direction, which we take as x – axis.

Now, although the conditions at a surface, on which there are no shear or normal stress components. Are these of plane stress rather than the plane strain, we can still use strain transformation equations to express the three measured normal strains in terms of strain components ϵ_x, ϵ_y, ϵ_z and γ_{xy} referred to x and y co-ordiantes as

$$\epsilon_{\theta_1} = \epsilon_x \cos^2 \theta_1 + \epsilon_y \sin^2 \theta_1 + \gamma_{xy} \sin \theta_1 . \cos \theta_1$$

$$\epsilon_{\theta_2} = \epsilon_x \cos^2 \theta_2 + \epsilon_y \sin^2 \theta_2 + \gamma_{xy} \sin \theta_2 . \cos \theta_2$$

$$\epsilon_{\theta_3} = \epsilon_x \cos^2 \theta_3 + \epsilon_y \sin^2 \theta_3 + \gamma_{xy} \sin \theta_3 . \cos \theta_3$$

This is a set of three simultaneous linear algebraic equations for the three unknows ϵ_x, ϵ_y , γ_{xy} to solve these equation is a laborious one as far as manually is concerned, but with computer it can be readily done.Using these later on, the state of strain can be determined at any point.

Let us consider a 45^0 degree stain rosette consisting of three electrical – resistance strain gages arranged as shown in the figure below :
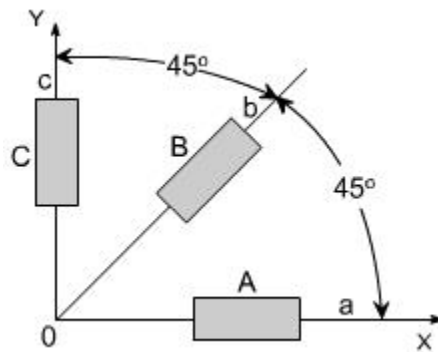
The gages A, B,C measure the normal strains ϵ_a , ϵ_b , ϵ_c in the direction of lines OA, OB and OC.

Thus

$$\epsilon_{\theta_1} = \epsilon_x \cos^2\theta_1 + \epsilon_y \sin^2\theta_1 + \gamma_{xy}\sin\theta_1 . \cos\theta_1$$

for $\theta_1 = 0; \epsilon_{\theta_1} = \epsilon_a$

$$\boxed{\epsilon_{\theta_1} = \epsilon_a} \qquad\qquad (1)$$

again

$$\epsilon_{\theta_2} = \epsilon_x \cos^2\theta_2 + \epsilon_y \sin^2\theta_2 + \gamma_{xy}\sin\theta_2 . \cos\theta_2$$

for gage B; $\theta_2 = 45^0$

$$\epsilon_{\theta_2} = \epsilon_b$$

$$\epsilon_b = \epsilon_x \cos^2 45^0 + \epsilon_y \sin^2 45^0 + \gamma_{xy}\sin 45^0 . \cos 45^0$$

$$\epsilon_b = \epsilon_x \frac{1}{2} + \epsilon_y \frac{1}{2} + \frac{1}{2}\gamma_{xy} = \frac{\epsilon_x + \epsilon_y + \gamma_{xy}}{2}$$

$$\gamma_{xy} = 2\epsilon_b - (\epsilon_x + \epsilon_y)$$

since $\epsilon_x = \epsilon_a$

$$\boxed{\gamma_{xy} = 2\epsilon_b - (\epsilon_a + \epsilon_y)} \qquad\qquad (2)$$

for the gage C

$$\epsilon_{\theta_3} = \epsilon_x \cos^2\theta_3 + \epsilon_y \sin^2\theta_3 + \gamma_{xy}\sin\theta_3 . \cos\theta_3$$

for $\theta_3 = 90^0; \epsilon_{\theta_3} = \epsilon_c$

or $\qquad \boxed{\epsilon_c = \epsilon_y} \qquad\qquad (3)$

Thus, substituting the relation (3) in the equation (2) we get

$\gamma_{xy} = 2\epsilon_b - \Box(\epsilon_a + \epsilon_c)$ and other equation becomes $\epsilon_x = \epsilon_a$; $\epsilon_y = \epsilon_c$

Since the gages A and C are aligned with the x and y axes, they give the strains ϵ_x and ϵ_y directly

Thus, ϵ_x , ϵ_y and γ_{xy} can easily be determined from the strain gage readings. Knowing these strains, we can calculate the strains in any other directions by means of Mohr's circle or from the transformation equations.

The 60^0 Rossett:

For the 60^0 strain rosette, using the same procedure we can obtain following relation.

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \frac{1}{3}(2.\epsilon_b + 2.\epsilon_c - \epsilon_a)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_c - \epsilon_b)$$