Example 3.2: A servomechanism incorporating a hydraulic relay with displacement feedback through a dashpot and spring assembly is shown below. [Control System by Beard, 1984]

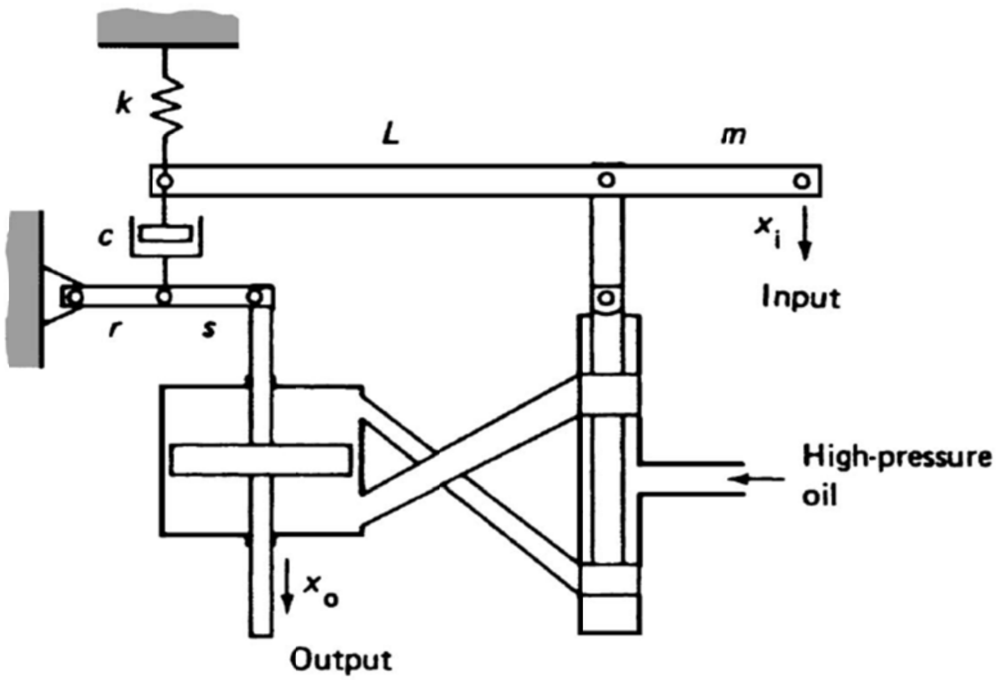
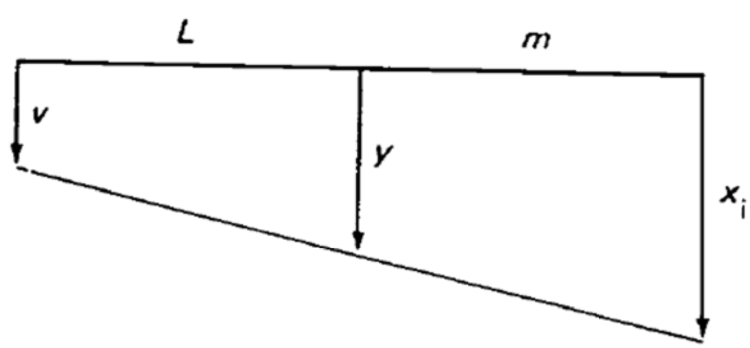


Fig. 3.7 Servomechanism with a hydraulic relay

The velocity of the output ram (Dx,) is equal to K times the movement of the pilot valve from its neutral position. Given that the inertia of all parts and the effect of friction (other than at the dashpot c) may be neglected, determine the equation of motion of the system.

For the control rod,



where v is the spring displacement and y is the movement of the spool valve. Thus;

$$\frac{y - v}{L} = \frac{x_i - v}{L + m}, \tag{3.19}$$

$$y = \left(\frac{L}{L + m}\right)x_i - \left(\frac{m}{L + m}\right)v \tag{3.20}$$

Force balance for the spring and dashpot gives

$$kv = cD\left(\left(\frac{r}{r + s}\right)x_o - v\right), \tag{3.21}$$

So that

$$(k + cD)v = \left(\frac{r}{r + s}\right)cDx_o. \tag{3.22}$$

The flow equation gives $Dx_o = Ky$. Substituting for y and v gives

$$Dx_o = K\left[\left(\frac{L}{L + m}\right)x_i - \left(\frac{m}{L + m}\right)\left(\frac{r}{r + s}\right)\left(\frac{cD}{k + cD}\right)x_o\right], \tag{3.23}$$

This can be rearranged to give the equation of motion as;

$$\left[D^2 + D\left(\frac{k}{c} + \frac{Kmr}{(L + m)(r + s)}\right)\right]x_o = \left(\frac{KL}{L + m}\right)\left(\frac{k + cD}{c}\right)x_i. \tag{3.24}$$

Problem 3.3:

A linear remote position control system with negative output feedback consists of a potentiometer giving **8** V/rad errors to an amplifier, and a motor which applies a torque of 3 N m/V to the load. The load has inertia of ***6*** kg m2 and viscous friction of 12 N m s/rad.

(i) Draw a block diagram for the system and derive its equation of motion.

(ii) Calculate the maximum overshoot in the output response to a step input of 2 rad, and

(iii) Given that a tacho-generator is employed to provide negative output velocity feedback, derive the new equation of motion and calculate the velocity feedback coefficient needed to give critical damping.

Sol:

(i) The block diagram for the system is

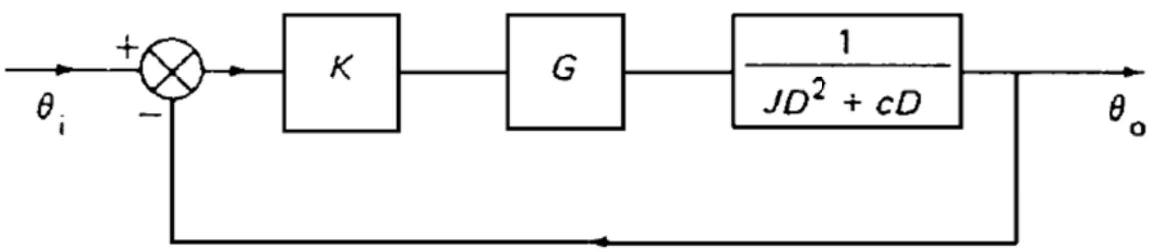


Fig. 3.8 Block Diagram

Equation is

$$(\theta_i - \theta_o)KG = (JD^2 + cD)\theta_o, \tag{3.25}$$

So the final equation of motion is

$$(JD^2 + cD + KG)\theta_o = KG\theta_i. \tag{3.26}$$

where

K = **8** V/rad error

G = 3 Nm/V,

J = 6 kg m^2, and

c = 12 N m s/rad,

ii) With a step input, overshoot is $\theta_i e^{-\zeta\pi/\sqrt{(1-\zeta^2)}}$

$$\zeta = \frac{c}{2\sqrt{(KGJ)}} = \frac{12}{2\sqrt{(8 \times 3 \times 6)}} = \frac{1}{2}. \tag{3.27}$$

$$e^{-1/2\pi/\sqrt{[1-(1/2)^2]}} = e^{-1.813} = 0.163 \text{ rad}$$

and overshoot = 2 x 0.163 rad = 0.326 rad, for a 2 rad input

(iii) With output velocity feedback, the block diagram is

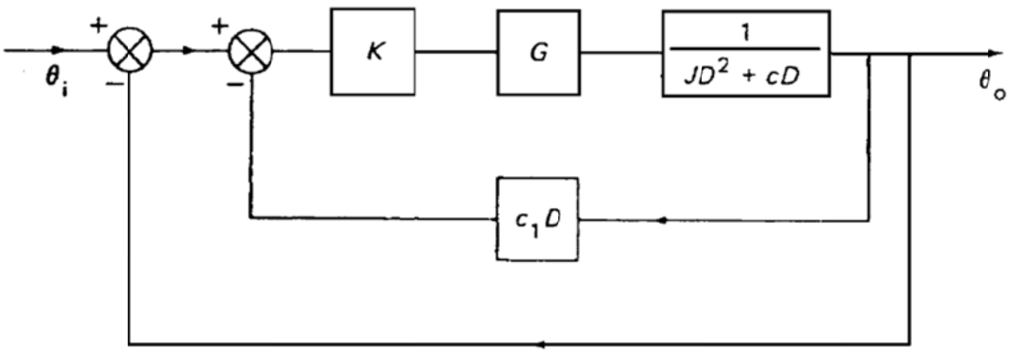


Fig. 3.9 Block diagram

So that

$$[\theta_i - \theta_o - c_1 D\theta_o]KG = [JD^2 + cD]\theta_o \qquad (3.28)$$

and the equation of motion is

$$[JD^2 + (c + c_1KG)D + KG]\theta_o = KG\theta_i. \qquad (3.29)$$

For critical damping

$$c + c_1KG = 2\sqrt{(KGJ)},$$

that is

$$12 + (c_1 \times 8 \times 3) = 2\sqrt{(8 \times 3 \times 6)}.$$

Hence,

$$c_1 = 0.5 \text{ (N m s/rad)/(Nm/rad)},$$

or

$$c_1 = 0.5 \text{ s.}$$

Ans.

Vibration Damping:

Damping is a phenomenon by which mechanical energy is dissipated (usually converted as thermal energy) in dynamic systems.

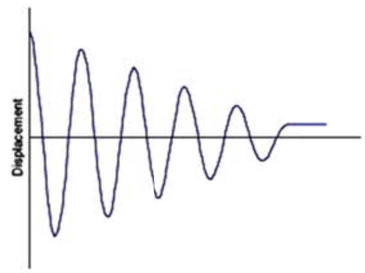


Fig. 3.10 Displacement time response of damped system

Three primary mechanisms involved in damping

- Internal damping – of material
- Structural damping – at joints and interface
- Fluid damping – through fluid -structure interactions

Two types of external dampers can be added to a mechanical system to improve its energy dissipation characteristics:

- Active dampers – require external source of power
- Passive dampers – Does not require

MATERIAL (Internal) Damping

Internal damping originates from energy dissipation associated with:

--microstructure defects (grain boundaries & impurities),

-thermo elastic effects (caused by local temperature gradients)

-eddy-current effects (ferromagnetic materials),

-dislocation motion in metals, etc

Types of Internal damping:

-Viscoelastic damping

-Hysteretic damping

Shunt Damping:

Structural damping is an important means of reducing vibration, noise and fatigue. Vibration can be suppressed by adding mass to a system, introducing a mechanical vibration absorber or a variety of other techniques. This paper explores the use of "smart materials" or piezoelectric polymers in conjunction with an electrical shunt circuit as a single mode damper. Piezoelectric shunt damping is a popular technique for vibration suppression in smart structures. These are characterized by the connection of electrical impedance to a structurally bonded piezoelectric transducer. Such methods do not require an external sensor, may guarantee stability of the shunted system and do not require parametric models for design purposes. The piezoelectric materials are used in conjunction with passive inductance-resistance-capacitance (RLC) circuits to dampen specific vibration modes. The piezoelectric materials convert mechanical energy to electrical energy, which is then dissipated in the RLC circuit through joule heating. Resonant shunt damping circuits, comprised of inductors, capacitors, and resistors, are simple to design and can significantly augment the damping of lightly-damped flexible structures.

Piezoelectric materials have the unique ability to convert mechanical energy into electrical energy and vice versa. When strained the piezoelectric materials produce a voltage difference across the poled terminals. This characteristic has been exploited in various configurations of mechanical sensors. Inversely, piezoelectric materials strain when a voltage is applied. This characteristic enables piezoelectric materials to be used as mechanical actuators. Active control systems require complex amplifiers and electronic sensors. Implementation of simple and robust passive control systems using piezoelectric materials decreases the risk of malfunction and deterioration. A passive control system is used to damp a single mode of a simple cantilever beam (1-DOF). Analysis was done to predict the optimal position of a piezoelectric tile on the beam. A resonant shunt circuit was created using an inductor and resistor in series and parallel. The electrical impedance frequency was tuned to equate the modal frequency that was to be damped. The efficiency of these shunts depends very much on the ability to (i) transfer strain from the vibrating structure to the transducer material, and (ii) transform the strain energy into electrical energy inside the active material. The latter is measured by the piezoelectric electromechanical coupling factor k.

Application of shunt damping for a cantilever beam:

An analytical model has been developed to determine point displacements, mode shapes and strain energy of a cantilever beam. A beam clamped at one end and free at the other was used.

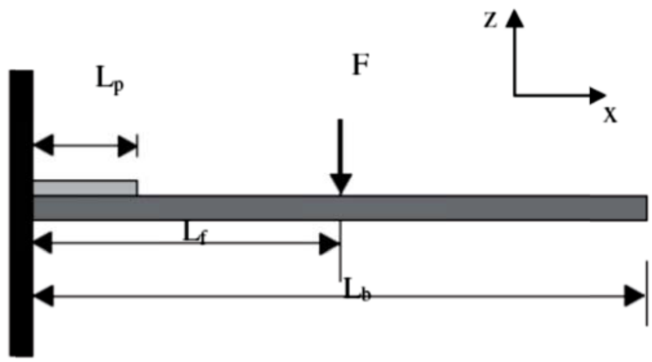


Fig. 3.11 Cantilever beam with PZT patch

The Euler-Bernoulli method is used to model the cantilever beam. The governing undamped equation of motion for the beam for forced motion under zero initial conditions can be written as:

$$\rho A \frac{\partial^2 w(x,t)}{\partial t^2} + E_b I_b \frac{\partial^4 w(x,t)}{\partial x^4} = F(t) \tag{3.30}$$

where w is the displacement of the beam, ρ is the density of the steel beam, A is the cross-sectional area, and F(t) is the external force applied to the beam. The boundary conditions are:

$$w(0,t) = 0,$$
$$w_x(0,t) = 0,$$
$$w_{xx}(L_b,t) = 0,$$
$$w_{xxx}(L_b,t) = 0$$

Considering a harmonic forcing function applied to a single point on the beam, according to Fig. 1, we can write:

$$\frac{\partial^2 w(x,t)}{\partial t^2} + c^2 \frac{\partial^4 w(x,t)}{\partial x^4} = \frac{F_0}{\rho A} sen(\omega t)\delta(x - L_f) \tag{3.31}$$

where ω is the frequency, L_f is the position of the applied force and c^2 can be written as:

$$c^2 = \frac{E_b I_b}{\rho A} \tag{3.32}$$

A piezoelectric material is a three-dimensional device poled along one axis. Most piezoelectric patches (PZT) are poled across the thickness with electrodes through the top and bottom planes. As a voltage difference is applied across the electrodes (x3 direction in Figure 2), a strain is produced in the other two directions (x_1 and x_2).

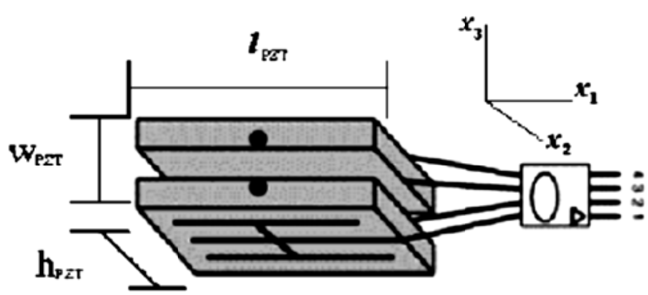


Fig. 3.12 PZT patch design on cantilever beam

For the given experiment, axis 1 is bonded along the horizontal neutral axis of the cantilever beam as shown in Figure 1. When deformed by the cantilever bending, the PZT produces a voltage across axis 3. The material transverse coupling constant (kij) is used to describe the relationship of energy transfer from the i-axis to the j-axis and is specific to the PZT patch design. A simplified description of the PZT damper is to convert mechanical energy into electrical energy and then dissipate the electrical energy in the form of joule heating through a resistor. The equation for power dissipated by the resistor is;

$$P = i^2 R = \frac{V^2}{R} \tag{3.33}$$

Maximizing the current through the resistor increases electrical damping. A simple resistor shunt circuit can be used as a broadband damper. The resistor will effectively dissipate energy from all modes of vibration.

Parallel RL Circuit

An inductor-resistor shunt circuit was implemented to create an electrical resonant frequency for single mode damping. Placing the inductor and resistor in parallel to the PZT allows for the simplest electrical frequency tuning because the inductor and resistor can be altered independently. Optimum resistor and inductor values for a parallel configuration were calculated using the method outlined by Wu and Bicos (1997) as;

1. Experimentally determine ωo and ωs for the cantilever beam. ωo and ωs are the PZT open-circuit and short-circuit modal frequencies for the cantilever beam.

2. Calculate the generalized electromechanical coupling coefficient for mode 31.

$$K_{31} = \sqrt{(\omega_o^2 - \omega_s^2)/\omega_s^2} \tag{3.34}$$

3. Determine the PZT capacitance at constant strain. CT is the pre-bonded PZT capacitance.

$$C^s = (1 - k_{31}^2)C^T \tag{3.35}$$

4. Calculate the normalized tuning frequency

$$\alpha = \sqrt{(1 - K_{31}^2 / 2)} \tag{3.36}$$

5. The optimum parallel inductance is

$$L_{opt-P} = 1/[C^s(\omega_s \alpha)^2] \tag{3.37}$$

6. The optimum parallel shunt resistance is

$$R_{opt-P} = 1/[\sqrt{2} * \omega_s C^s K_{31}] \tag{3.38}$$

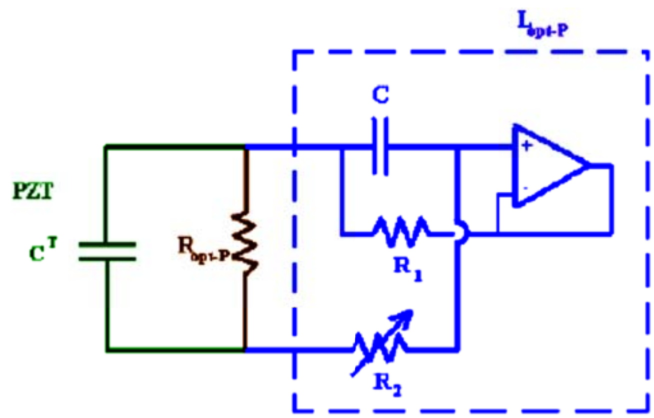


Fig. 3.13 RLC circuit for piezo-electric material

Series RL Circuit

By Kirchoff's law, current is constant through elements in series. Therefore, a resistor and inductor in series with the PZT can achieve maximum current through the resistor. The optimum resistor and inductor values for the series circuit were calculated following the procedures of Hagood and Von Flotow (1991). The dissipation tuning parameter was calculated by

$$r = RC^{s}\omega_{o} \tag{3.39}$$

The optimal circuit damping was determined by

$$r_{opt} = \sqrt{2} * K_{31} /(1 + K_{31}^{2}) \tag{3.40}$$

The optimal series shunt resistance is

$$R_{opt-S} = r_{opt} / C^{s}\omega_{o} \tag{3.41}$$

The electrical resonance frequency is

$$\omega_{e} = \frac{1}{\sqrt{LC^{s}}} \tag{3.42}$$

The optimal series configuration inductance value can be calculate by setting the electrical frequency equal to the short-circuit frequency ($\omega e = \omega o$).

$$L_{opt-S} = \frac{1}{C^{s}\omega_{s}^{2}} \tag{3.43}$$

Source:
http://nptel.ac.in/courses/112107088/9