

# Module 2

# Measurement Systems

# Lesson 3

## Measurement Systems Specifications

## Instructional Objectives

At the end of this lesson, the student will be able to

1. Define the different terms used for characterizing the performance of an instrument/ measurement system.
2. Compare the performances of two similar type of instruments, looking at the specifications
3. Write down the performance specifications of a measurement system from its test data.

## Introduction

One of the most frequent tasks that an Engineer involved in the design, commissioning, testing, purchasing, operation or maintenance related to industrial processes, is to interpret manufacturer's specifications for their own purpose. It is therefore of paramount importance that one understands the basic form of an instrument specification and at least the generic elements in it that appear in almost all instrument specifications.

Different blocks of a measurement system have been discussed in lesson-2. The combined performance of all the blocks is described in the specifications. Specifications of an instrument are provided by different manufacturers in different wrap and quoting different terms, which sometimes may cause confusion. Moreover, there are several application specific issues. Still, broadly speaking, these specifications can be classified into three categories: (i) *static characteristics*, (b) *dynamic characteristics* and (iii) *random characteristics*.

### 1. Static Characteristics

Static characteristics refer to the characteristics of the system when the input is either held constant or varying very slowly. The items that can be classified under the heading static characteristics are mainly:

#### Range (or span)

It defines the maximum and minimum values of the inputs or the outputs for which the instrument is recommended to use. For example, for a temperature measuring instrument the input range may be 100-500 °C and the output range may be 4-20 mA.

#### Sensitivity

It can be defined as the ratio of the *incremental output* and the *incremental input*. While defining the sensitivity, we assume that the input-output characteristic of the instrument is approximately linear in that range. Thus if the sensitivity of a thermocouple is denoted as  $10\mu V/^{\circ}C$ , it indicates the sensitivity in the linear range of the thermocouple voltage vs. temperature characteristics. Similarly sensitivity of a spring balance can be expressed as 25 mm/kg (say), indicating additional load of 1 kg will cause additional displacement of the spring by 25mm.

Again sensitivity of an instrument may also vary with temperature or other external factors. This is known as *sensitivity drift*. Suppose the sensitivity of the spring balance mentioned above is 25 mm/kg at 20 °C and 27 mm/kg at 30°C. Then the sensitivity drift/°C is 0.2 (mm/kg)/°C. In order to avoid such sensitivity drift, sophisticated instruments are either kept at controlled temperature, or suitable in-built temperature compensation schemes are provided inside the instrument.

## Linearity

Linearity is actually a measure of nonlinearity of the instrument. When we talk about sensitivity, we assume that the input/output characteristic of the instrument to be approximately linear. But in practice, it is normally nonlinear, as shown in Fig.1. The *linearity* is defined as the maximum deviation from the linear characteristics as a percentage of the full scale output. Thus,

$$\text{Linearity} = \frac{\Delta O}{O_{\max} - O_{\min}} \quad (1)$$

where,  $\Delta O = \max(\Delta O_1, \Delta O_2)$ .

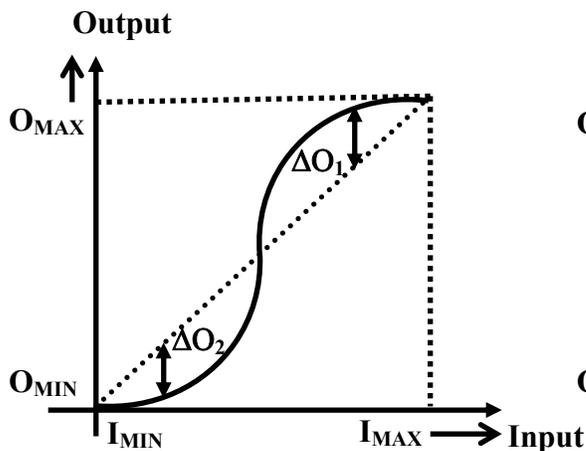


Fig. 1 Linearity

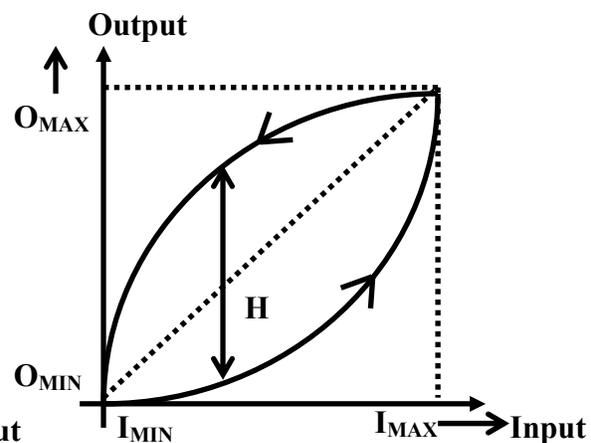


Fig. 2 Hysteresis

## Hysteresis

Hysteresis exists not only in magnetic circuits, but in instruments also. For example, the deflection of a diaphragm type pressure gage may be different for the same pressure, but one for increasing and other for decreasing, as shown in Fig.2. The *hysteresis* is expressed as the maximum hysteresis as a full scale reading, i.e., referring fig.2,

$$\text{Hysteresis} = \frac{H}{O_{\max} - O_{\min}} \times 100. \quad (2)$$

## Resolution

In some instruments, the output increases in discrete steps, for continuous increase in the input, as shown in Fig.3. It may be because of the finite graduations in the meter scale; or the

instrument has a digital display, as a result the output indication changes discretely. A  $3\frac{1}{2}$ -digit voltmeter, operating in 0-2V range, can have maximum reading of 1.999V, and it cannot measure any change in voltage below 0.001V. *Resolution* indicates the minimum change in input variable that is detectable. For example, an eight-bit A/D converter with +5V input can measure the minimum voltage of  $\frac{5}{2^8 - 1}$  or 19.6 *mv*. Referring to fig.3, *resolution* is also defined in terms of percentage as:

$$Resolution = \frac{\Delta I}{I_{max} - I_{min}} \times 100 \quad (3)$$

The quotient between the measuring range and resolution is often expressed as *dynamic range* and is defined as:

$$Dynamic\ range = \frac{measurement\ range}{resolution} \quad (4)$$

And is expressed in terms of dB. The dynamic range of an *n*-bit ADC, comes out to be approximately  $6n$  dB.

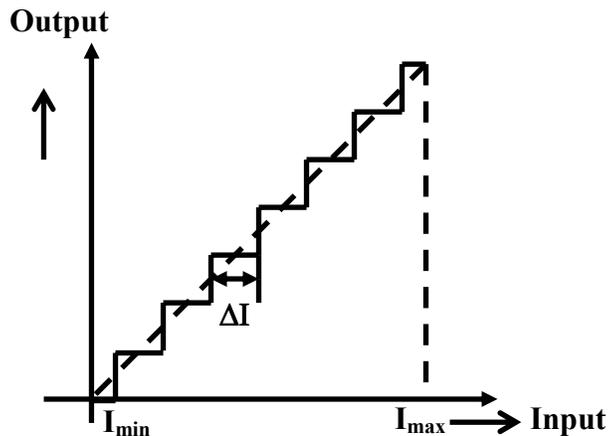


Fig. 3 Resolution

## Accuracy

Accuracy indicates the closeness of the measured value with the actual or true value, and is expressed in the form of the *maximum error* ( $= \text{measured value} - \text{true value}$ ) as a percentage of full scale reading. Thus, if the accuracy of a temperature indicator, with a full scale range of 0-500 °C is specified as  $\pm 0.5\%$ , it indicates that the measured value will always be within  $\pm 2.5$  °C of the true value, if measured through a standard instrument during the process of calibration. But if it indicates a reading of 250 °C, the error will also be  $\pm 2.5$  °C, i.e.  $\pm 1\%$  of the reading. Thus it is always better to choose a scale of measurement where the input is near full-scale value. But the true value is always difficult to get. We use standard calibrated instruments in the laboratory for measuring true value if the variable.

## Precision

Precision indicates the repeatability or reproducibility of an instrument (but does not indicate accuracy). If an instrument is used to measure the same input, but at different instants, spread

over the whole day, successive measurements may vary randomly. The random fluctuations of readings, (mostly with a Gaussian distribution) is often due to random variations of several other factors which have not been taken into account, while measuring the variable. A precision instrument indicates that the successive reading would be very close, or in other words, the standard deviation  $\sigma_e$  of the set of measurements would be very small. Quantitatively, the precision can be expressed as:

$$Precision = \frac{\text{measured range}}{\sigma_e} \quad (5)$$

The difference between precision and accuracy needs to be understood carefully. Precision means repetition of successive readings, but it does not guarantee accuracy; successive readings may be close to each other, but far from the true value. On the other hand, an accurate instrument has to be precise also, since successive readings must be close to the true value (that is unique).

## 2. Dynamic Characteristics

Dynamic characteristics refer to the performance of the instrument when the input variable is changing rapidly with time. For example, human eye cannot detect any event whose duration is more than one-tenth of a second; thus the dynamic performance of human eye cannot be said to be very satisfactory. The dynamic performance of an instrument is normally expressed by a differential equation relating the input and output quantities. It is always convenient to express the input-output dynamic characteristics in form of a linear differential equation. So, often a nonlinear mathematical model is linearised and expressed in the form:

$$a_n \frac{d^n x_0}{dt^n} + a_{n-1} \frac{d^{n-1} x_0}{dt^{n-1}} + \dots + a_1 \frac{dx_0}{dt} + a_0 x_0 = b_m \frac{d^m x_i}{dt^m} + b_{m-1} \frac{d^{m-1} x_i}{dt^{m-1}} + \dots + b_1 \frac{dx_i}{dt} + b_0 x_i \quad (6)$$

where  $x_i$  and  $x_0$  are the input and the output variables respectively. The above expression can also be expressed in terms of a transfer function, as:

$$G(s) = \frac{x_0(s)}{x_i(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} \dots + b_1 s + b_0}{a_n s^n + b_{n-1} s^{n-1} \dots + a_1 s + a_0} \quad (7)$$

Normally  $m < n$  an  $n$  is called the order of the system. Commonly available sensor characteristics can usually be approximated as either *zero-th order*, *first order* or *second order* dynamics. Here are few such examples:

### Potentiometer

Displacement sensors using potentiometric principle (Fig.4) have no energy storing elements. The output voltage  $e_o$  can be related with the input displacement  $x_i$  by an algebraic equation:

$$e_o(t)x_t = Ex_i(t); \quad \text{or,} \quad \frac{e_o(s)}{x_i(s)} = \frac{E}{x_t} = \text{constant} \quad (8)$$

where  $x_t$  is the total length of the potentiometer and  $E$  is the excitation voltage.. So, it can be termed as a *zeroth order system*.

## Thermocouple

A bare thermocouple (Fig.5) has a mass ( $m$ ) of the junction. If it is immersed in a fluid at a temperature  $T_f$ , then its dynamic performance relating the output voltage  $e_o$  and the input temperature  $T_f$ , can be expressed by the transfer function:

$$\frac{e_o(s)}{T_f(s)} = \frac{K_v}{1 + s\tau} \quad (9)$$

where,  $K_v$  = steady state voltage sensitivity of the thermocouple in V/°C.

$$\tau = \text{time constant of the thermocouple} = \frac{mC}{hA}$$

$m$  = mass of the junction

$C$  = specific heat

$h$  = heat transfer co-efficient

$A$  = surface area of the hot junction.

Hence, the bare thermocouple is a first order sensor. But if the bare thermocouple is put inside a metallic protective well (as it is normally done for industrial thermocouples) the order of the system increases due to the additional energy storing element (thermal mass of the well) and it becomes a second order system.

## Seismic Sensor

Seismic sensors (Fig.6.) are commonly used for vibration or acceleration measurement of foundations. The transfer function between the input displacement  $x_i$  and output displacement  $x_o$  can be expressed as:

$$\frac{x_o(s)}{x_i(s)} = \frac{Ms^2}{Ms^2 + Bs + K} \quad (10)$$

where:  $M$  = mass of the seismic body

$B$  = damping constant

$K$  = spring constant

From the above transfer function, it can be easily concluded that the seismic sensor is a *second order system*.

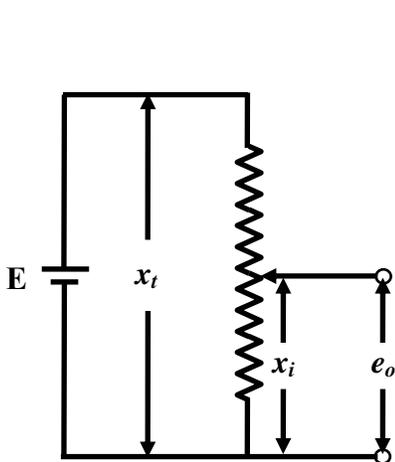


Fig. 4 Potentiometer

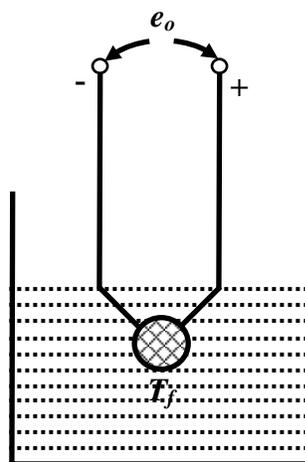


Fig. 5 Thermocouple

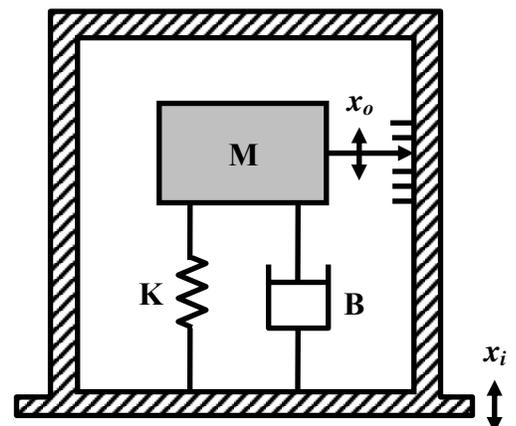


Fig. 6 Seismic sensor  
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Dynamic characteristics specifications are normally referred to the performance of the instrument with different test signals, e.g. impulse input, step input, ramp input and sinusoidal input. Few important specifications are:

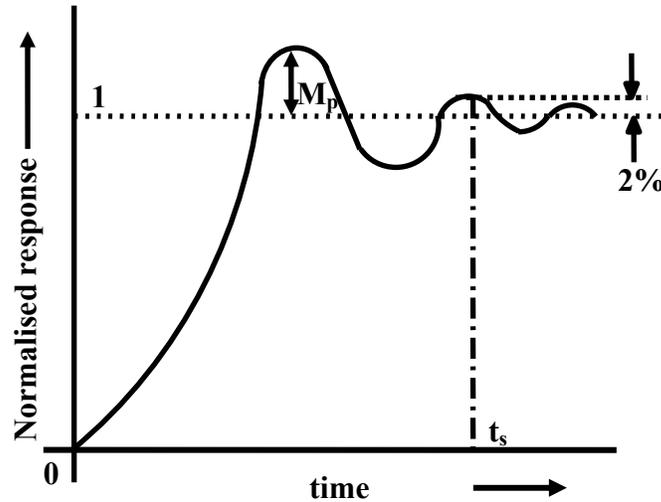


Fig. 7 Step response of a dynamic system

## Step response performance

The normalized step response of a measurement system normally encountered is shown in Fig. 7. Two important parameters for classifying the dynamic response are:

**Peak Overshoot ( $M_p$ ):** It is the maximum value minus the steady state value, normally expressed in terms of percentage.

**Settling Time ( $t_s$ ):** It is the time taken to attain the response within  $\pm 2\%$  of the steady state value.

**Rise time ( $t_r$ ):** It is the time required for the response to rise from 10% to 90% of its final value.

## Frequency Response Performance

The frequency response performance refers to the performance of the system subject to sinusoidal input of varying frequency.

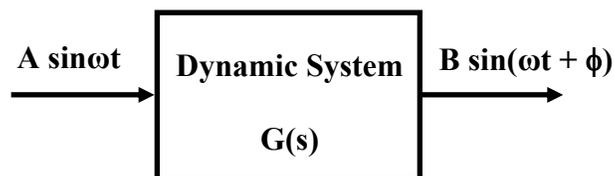
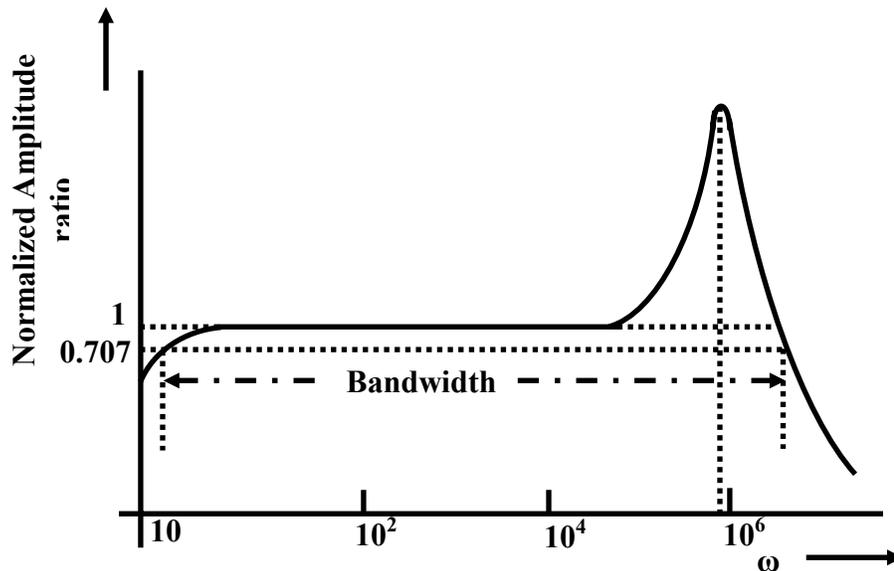


Fig. 8 Frequency Response

Suppose  $G(s)$  is the transfer function of the dynamic measurement system, represented by the general relation (7). If the input is a sinusoidal quantity of amplitude  $A$  and frequency  $\omega$ , then in the steady state, the output will also be of same frequency, but of different amplitude  $B$ , and there would be a phase difference between the input and output. It can be shown that the amplitude ratio and the phase difference can be obtained as:

$$\frac{B}{A} = |G(j\omega)| \quad \text{and} \quad \phi = \angle G(j\omega) \quad (11)$$



**Fig. 9 Amplitude vs. frequency characteristics of a piezoelectric accelerometer**

The plots showing variations of amplitude ratios and phase angle with frequency are called the magnitude and phase plots of the frequency response. Typical amplitude vs. frequency characteristics of a piezoelectric accelerometer is shown in Fig.9.

## Bandwidth and Natural Frequency

From fig. 9, it is apparent that the amplitude is fairly constant over a range of frequencies. This range is called the *bandwidth* of the measuring system (to be precise, it is the frequency range in which the normalized amplitude ratio does not fall below 0.707, or -3 dB limit). The instrument is suitable for use in this range. The lower and upper limits are called the *lower* and *upper* cut off frequencies. The frequency at which the amplitude ratio attains a peak is called the (damped) *natural frequency* of the system. For further details, the reader is requested to refer any standard book on control systems.

## 3. Random Characteristics

If repeated readings of the same quantity of the measurand are taken by the same instrument, under same ambient conditions, they are bound to differ from each other. This is often due to some inherent sources of errors of the instrument that vary randomly and at any point of time it is very difficult to exactly say, what would be its value. For example, the characteristics of resistance and diode elements of an electronic circuit are random, due to two sources of noises: *thermal noise* and *flicker noise*. To characterize these behaviors, statistical terminologies are

often used. Most common among them are *Mean* and *Standard deviation*. The mean of a set of readings is the most accurate estimation of the actual value, since, the positive and negative errors often cancelled out. On the other hand the standard deviation ( $\sigma$ ) is a measure of the spread of the readings. If successive measurements of the same parameter under same ambient condition are taken and the mean and standard deviations are calculated, then assuming normal distribution of the randomness in measurements, we can say that 68% of the readings would fall within the range of  $\text{mean} \pm \sigma$ . Naturally, smaller the value of  $\sigma$ , more would be the repeatability and higher would be the precision (refer equation (5)). This uncertainty limit is often extended to the  $3\sigma$  limit, that means, that with 99% confidence, we can say that the any reading taken, would give a value within the range of  $\text{mean} \pm 3\sigma$ . The interval of uncertainty is often called as the *confidence interval*. This part is discussed in detail in Lesson10.

## Exercise

1. Followings are the excerpts from the specifications of a laser displacement sensor:
  - (a) Measurement range:  $\pm 10\text{mm}$
  - (b) Measurement point:  $40\text{mm}$
  - (c) Resolution:  $3\ \mu\text{m}$
  - (d) Linearity: 1% Full Scale
  - (e) Response time:  $0.15\text{ms}$
  - (f) Linear output:  $4\text{-}20\text{mA}$

## Answer the following questions

- i. Explain the meaning of each term.
  - ii. Suppose, the distance between the sensor and the object is  $35\text{mm}$ . Then what would be output in mA?
  - iii. What is the error due to nonlinearity under the above condition?
  - iv. Find out the sensitivity of the sensor in mA/mm.
2. Find the resolution of a 10-bit ADC, if it is excited by a 10V source.
  3. The accuracy specified for a pressure gauge of range  $0\text{-}10\text{kPa}$  is 2%. Find the maximum error in measurement in Pa if it gives a reading of  $4.0\ \text{kPa}$ .
  4. Justify the following statements:
    - (a) A potentiometer is a zero-th order device.
    - (b) A bare thermocouple is a first order device.
    - (c) An accelerometer is a second order device.

## Answers

1. (ii)  $8\text{mA}$ , (iii)  $\pm 0.1\text{mm}$ , (iv)  $0.8\ \text{mA/mm}$ .
2.  $9.77\text{mv}$ .
3.  $\pm 0.2\text{kPa}$ .