TRIANGULAR FUZZY MULTINOMIAL CONTROL CHART WITH VARIABLE SAMPLE SIZE USING α – CUTS

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Abstract:

Statistical Process Control concepts and methods have been very important in the manufacturing and process industries. The main objective is to monitor the performance of a process over time in order for the process to achieve a state of statistical control. So many of the quality characteristics found in these process industries are not easily measured on a numerical scale. Quality characteristics of this type are called attribute data. Many basic attribute control charts like, p-chart, c-chart and u-chart are readily available in this process industry. This paper designs control chart for multiple-attribute quality characteristics. Aggregate sample quality is estimated by using interactive weighted addition of fuzzy values assigned to each quality characteristics. Triangular Fuzzy multinomial Control charts are drawn using Multinomial distribution using **α –** cuts for variable sample sizes. The proposed method is compared and numerical example is the evidence for improvement in the process.

Keyword: TFM chart, Multi-attribute chart, **α –** cuts chart, Multinomial chart, Fuzzy control chart

1. Introduction:

Statistical Process control (SPC) represents a set of tools for managing process, and also for determining and monitoring the quality of final product within an organization. The SPC can be viewed as a strategy for reducing variation in production process, where the variation represents an unwanted thing for any company producing goods or providing service. D.C. Montgomerry [9] illustrated as “Statistical Process control is a powerful collection of problem - solving tools useful in achieving process stability and improving capability through the reduction of variability. In 1924, Walter Shewhart designed the first control charts as follows: Let w be a number of samples used to measure some quality characteristic of interest and suppose that the mean of w is μ_w and the standard deviation of w is σ_W. Then the upper control limit, central limit and lower control limit are respectively given by

$$UCL = \mu_w + k\sigma_w , \quad CL = \mu_w , \quad LCL = \mu_w \tilde{\ } k\sigma_w$$

Where k is the “distance” of the control limits from the centre line and it is expressed in unit’s standard deviation.

A single measureable quality characteristic such as dimension, weight or volume is called a variable. In such cases, control charts for variables are used. These include $\bar{X}$ - chart for controlling the process average and R-chart for controlling the process variability. If the quality related characteristics such as characteristics for appearance, softness, colour, taste, etc., attribute control chart such as p-chart , c-chart are used to monitor the production process. Sometimes classified as either “conforming or nonconforming”, depending upon whether or not they must meet the specifications. The p-chart

2. Fuzzy logic and Linguistic Variable:

The fuzzy set theory and fuzzy logics are playing very important role in Statistical Process control. In many industrial situations, we may come across situation where quality has to be defined using linguistic variables using subjective measures like rating on a scale. These are variables whose states are fuzzy numbers. Bradshaw [2] used fuzzy set theory as the basis for interpreting the representation of a grade degree of product conformance with a quality standard. Raz and Wang [12] explained two approaches for constructing variable control chart based on linguistic data when the product quality is classified ‘perfect’, ‘good’, ‘poor’ etc. The representative fuzzy measures are obtained by using any of the four commonly used methods, namely, Fuzzy average, fuzzy mode, and fuzzy median and α- level fuzzy-midrange, to construct the control chart. The membership functions defined for the linguistic variables in the above method are chosen arbitrarily and hence decision for process control may change as per the user's choice of values of decision parameter. Amirzadeh et al. [1] have developed Fuzzy Multinomial control chart for fixed Sample Size and Pandurnagan et al. [11] illustrated fuzzy multinomial control chart based on linguistic variable which is classified into more than two categories with variable sample size.

In this paper, a Triangular Fuzzy Multinomial control chart (TFM chart) with VSS for linguistics variables using fuzzy number with α –cut fuzzy midrange transform techniques is proposed. The proposed method is compared with regular p-chart and FM-chat with VSS and which is more effective.

3. Methodology:

Based up on fuzzy set theory, a linguistics variable $\tilde{\boldsymbol{L}}$ which is classified by the set of k mutually, exclusive members $\{l_1, l_2, \dots\dots l_k\}$. We estimate the weight w_i to each term $\boldsymbol{l_i}$ and the fuzzy set is defined as

$$\tilde{L} = \{(l_1, m_1), (l_2, m_2), \dots\dots (l_k, m_k)\} \qquad (1)$$

To monitor the out of control signal in the production process we are taking independent samples of different size and categorized as 'perfect', 'good', 'poor' etc form the {$\boldsymbol{n_1, n_2, n_3, \dots\dots n_s}$}.

4. Fuzzy Multinomial control chart:

Pandurnagan et al. [11], defined $\tilde{\boldsymbol{L}}$ is a linguistic variable which can take k mutually exclusive members $\{l_1, l_2, \dots\dots l_k\}$ and $\boldsymbol{p_i}$ is the probability that an item l_i is produced. Then $\{X_1, X_2, \dots\dots X_k\}$ has multinomial distribution with parameters $\boldsymbol{n_r}$ and $\boldsymbol{p_1, p_2, \dots.p_k}$. It is known that each $\boldsymbol{X_i}$ Marginally has Binomial distribution with parameters $\boldsymbol{n_r p_i}$ and $\boldsymbol{n_r p_i}$ (1- $\boldsymbol{p_i}$) respectively. Then the FM Control Chart with VSS is given by

$$UCL = E[\overline{\tilde{L}}] + d\sqrt{Var(\overline{\tilde{L}})} \qquad CL = E[\overline{\tilde{L}}] \qquad LCL = E[\overline{\tilde{L}}] - d\sqrt{Var(\overline{\tilde{L}})}$$

5. Triangular Fuzzy Multinomial control Chart:

$\tilde{\boldsymbol{L}}$ is a linguistic variable which can categorize k mutually exclusive members $\{l_1, l_2, \dots\dots l_k\}$ and each members are more skewed for each variable sample sizes. The weights of the membership degree are also assumed as 1, 0.75, 0.5, 0.25 and 0 in the Fuzzy Multinomial distribution control chart. To overcome these drawbacks, we propose Triangular Fuzzy multinomial Control chart with variable sample size based on α –cut fuzzy midrange transform techniques.

6. Fuzzy number construction:

A method of constructing fuzzy numbers is given in the following steps.

Step1: Let the observation for quality characteristics from samples of different sizes are assigned on a rank of 1 to k. a relative distance matrix D = $[d_{ij}]_{kxk}$ where $d_{ij} = |R_i - R_j|$ is evaluated.

Step 2: The average of relative distance for each $\boldsymbol{l_i}$ is calculate by $\bar{d}_i = \sum_{j=1}^{k} \frac{d_{ij}}{k-1}$. This distance average is used to measure the centre of all the ranking for each quality characteristics.

Step3: Find a pair-wise comparison matrix P = $[p_{ij}]_{kxk}$, where $p_{ij} = \frac{\bar{d}_j}{\bar{d}_i}$.

Step4: Evaluates weights by weight determination method of Saaty (1980) as $w_j = \frac{1}{\sum_{i=1}^{k} p_{ij}}; j = 1, 2, \dots k$ where $\sum_{i=1}^{k} w_i = 1$

Step5: The importance of degree $\boldsymbol{w_i}$ represents the weight to be associated with $\boldsymbol{l_i}$ when estimating the mode of the fuzzy number. The fuzzy mode is given by $m = \sum_{i=1}^{k} l_i w_i$

Step6: Separate the sample quality characteristics $\boldsymbol{l_i}$ and find as fuzzy subset A and C, which is by obtained by $m < l_i$ and $m > l_i$. The fuzzy subset A and C which is represented as follows

$$A = \left\{ \begin{matrix} (f_1, w_i)(f_2, w_2) \dots.(f_r, w_r) \\ m < l_i \end{matrix} \right\} \text{ And } C = \left\{ \begin{matrix} (m_1, w_1), (m_2, w_2), \dots.(m_t, w_t) \\ m > l_i \end{matrix} \right\}, r + t = n_i$$

Step7: Apply fuzzy multinomial distribution separately for the fuzzy subset A , M and C and find

$$E[\bar{\tilde{L}}_{iA}] = \sum_{i=1}^{r} p_i w_i \quad E[\bar{\tilde{L}}_{iM}] = \sum_{i=1}^{K} p_i w_i \qquad E[\bar{\tilde{L}}_{iC}] = \sum_{i=1}^{s} p_i w_i$$

Step8: Apply an α – cut to the fuzzy sets, the values are obtained as follows

$$E\left[\bar{\tilde{L}}_{iA}{}^{\alpha}\right] = E[\bar{\tilde{L}}_{iA}] + \alpha\{E[\bar{\tilde{L}}_{iM}] - E[\bar{\tilde{L}}_{iA}]\} \text{ and}$$

$$E\left[\bar{\bar{L}}_{iC}{}^{\alpha}\right] = E\left[\bar{\bar{L}}_{iC}\right] - \alpha\left\{E\left[\bar{\bar{L}}_{iC}\right] - E\left[\bar{\bar{L}}_{iM}\right]\right\}$$

Step9: Construct α-cut Triangular FM control chart based on Multinomial distribution

$$UCL^{\alpha}{}_{\bar{\bar{L}}_i} = \left\{\begin{array}{c} E\left[\bar{\bar{L}}_{iA}{}^{\alpha}\right] + 3\sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iA}\right)}, \\ E\left[\bar{\bar{L}}_{iM}\right] + 3\sqrt{var\left(\left(\bar{\bar{L}}_{iM}\right)\right)}, \\ E\left[\bar{\bar{L}}_{iC}{}^{\alpha}\right] + 3\sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iC}\right)} \end{array}\right\}$$

$$CL^{\alpha}{}_{\bar{\bar{L}}_i} = \left\{E\left[\bar{\bar{L}}_{iA}{}^{\alpha}\right], E\left[\bar{\bar{L}}_{iM}\right], E\left[\bar{\bar{L}}_{iC}{}^{\alpha}\right]\right\}$$

$$LCL^{\alpha}{}_{\bar{\bar{L}}_i} = \left\{\begin{array}{c} E\left[\bar{\bar{L}}_{iA}{}^{\alpha}\right] - 3\sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iA}\right)}, \\ E\left[\bar{\bar{L}}_{iM}\right] - 3\sqrt{var\left(\left(\bar{\bar{L}}_{iM}\right)\right)}, \\ E\left[\bar{\bar{L}}_{iC}{}^{\alpha}\right] - 3\sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iC}\right)} \end{array}\right\}$$

Where $$var^{\alpha}\left(\bar{\bar{L}}_{iA}\right) = var\left(\bar{\bar{L}}_{iA}\right) + \alpha\left\{var\left(\bar{\bar{L}}_{iM}\right) - var\left(\bar{\bar{L}}_{iA}\right)\right\}$$

$$var^{\alpha}\left(\bar{\bar{L}}_{iC}\right) = var\left(\bar{\bar{L}}_{ic}\right) - \alpha\left\{var\left(\bar{\bar{L}}_{iC}\right) - var\left(\bar{\bar{L}}_{iM}\right)\right\}$$

Step10: α – level fuzzy midrange is one of four transformation techniques used to determine the fuzzy control limits. These control limits are used to give a decision such as in-control or out-of-control for a process. In this study α – level fuzzy midrange is used as the fuzzy transformation method while calculating the control limits.

$$UCL^{\alpha}{}_{\bar{\bar{L}}_i - mr} = \left\{CL^{\alpha}{}_{\bar{\bar{L}}_i - mr} + 3\left(\frac{\sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iA}\right)} + \sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iC}\right)}}{2}\right)\right\}$$

$$CL^{\alpha}{}_{\bar{\bar{L}}_i - mr} = f^{\alpha}{}_{mr-\bar{\bar{L}}}(CL) = \left\{\frac{E\left[\bar{\bar{L}}_{iA}{}^{\alpha}\right] + E\left[\bar{\bar{L}}_{iC}{}^{\alpha}\right]}{2}\right\}$$

$$LCL^{\alpha}{}_{\bar{\bar{L}}_i - mr} = \left\{CL^{\alpha}{}_{\bar{\bar{L}}_i - mr} - 3\left(\frac{\sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iA}\right)} + \sqrt{var^{\alpha}\left(\bar{\bar{L}}_{iC}\right)}}{2}\right)\right\}$$

Step11: The definition of α- level fuzzy midrange for sample ni for Triangular FM control chart is defined

$$S^{\alpha}{}_{mr-\bar{\bar{L}}_i} = \left\{\frac{\left(E\left[\bar{\bar{L}}_{iA}\right] + E\left[\bar{\bar{L}}_{iC}\right]\right) + \alpha\left[\left\{E\left[\bar{\bar{L}}_{iM}\right] - E\left[\bar{\bar{L}}_{iA}\right]\right\} - \left\{E\left[\bar{\bar{L}}_{iC}\right] - E\left[\bar{\bar{L}}_{iM}\right]\right\}\right]}{2}\right\}$$

Step12: Then the condition of process control for each sample can be defined by as

$$\textbf{process control} = \begin{cases} \textbf{in-control for} & LCL^{\alpha}{}_{\bar{\bar{L}}_i - mr} \le S^{\alpha}{}_{mr-\bar{\bar{L}}_i} \le UCL^{\alpha}{}_{\bar{\bar{L}}_i - mr} \\ \textbf{out-of control} & otherwise \end{cases}$$

7. Numerical Example:

The numerical example given by Pandurnagan et al. [11], are taken for constructing TFM control chart. On a production line, a visual control of the aluminum die-cast of a lighting component might have the following assessment possibilities

1.”reject” if the aluminum die-cast does not works;

2. ”poor quality” if the aluminum die-cast works but has some defects;

3. ”medium quality” if the aluminum die-cast works and has no defects, but has some aesthetic flaws;

4. “good quality” if the aluminum die-cast works and has no defects, but has few aesthetic flaws;

5. ”excellent quality” if the aluminum die-cast works and has neither defects nor aesthetic flaws of any kind. The data with $\bar{\bar{L}}_i$ and $\hat{P}_i$ are given table – 1.

Table – 1.

Sample So.	Sample Size	Reject	Poor Quality (PQ)	Medium Quality (MQ)	Good Quality (GQ)	Excellent Quality (EQ)	$\bar{\bar{L}}_{iM}$	$\hat{P}$
1	100	12	10	12	54	12	0.2450	0.120
2	80	8	7	9	48	8	0.1845	0.100
3	80	6	11	12	43	8	0.1695	0.075
4	100	9	7	13	53	18	0.2168	0.090
5	110	10	16	18	54	12	0.2365	0.091
6	110	12	5	17	60	16	0.2373	0.109
7	100	11	12	13	50	14	0.2175	0.110
8	100	10	22	18	45	5	0.2080	0.100
9	90	10	8	13	50	9	0.1985	0.111
10	90	6	5	14	51	14	0.1989	0.067
11	110	20	13	23	47	7	0.2280	0.182
12	120	15	13	20	58	14	0.2595	0.125
13	120	9	12	22	64	13	0.2615	0.075
14	120	8	9	20	61	22	0.2568	0.067
15	110	6	10	19	61	14	0.2393	0.055
16	80	8	5	12	47	8	0.1821	0.100
17	80	10	8	12	40	10	0.1774	0.125
18	80	7	13	10	42	8	0.1743	0.088
19	90	5	7	14	54	10	0.1993	0.056
20	100	8	11	14	50	17	0.2150	0.080
21	100	5	8	16	58	13	0.2190	0.050
22	100	8	9	15	51	17	0.2182	0.080
23	100	10	12	14	50	14	0.2205	0.100
24	90	6	13	17	45	9	0.1925	0.067
25	90	9	10	14	46	11	0.1960	0.100

7.1. Construction of control limits for p-chart:

The value of $\bar{\bar{L}}_i$ can be calculated to the following ways

$$\bar{\bar{L}}_i = \frac{\sum_{i=1}^{k} X_i m_i}{\sum_{i=1}^{k} X_i} = \frac{\sum_{i=1}^{k} X_i m_i}{n_r}, n_r \in \{n_1, n_2, \dots n_s\}$$

$$\bar{\bar{L}}_1 = \frac{\{(12X1) + (10X0.75) + (12X0.5) + (54X0.25) + (12X0)\}}{100} = 0.390$$

$$\bar{\bar{L}}_2 = \frac{\{(8X1) + (7X0.75) + (9X0.5) + (48X0.25) + (8X0)\}}{80} = 0.372$$

And so on, and the value of $\hat{P}_i$, the control limits for P-charts can be calculated as follows

$$\hat{P}_i = \frac{D_i}{n_r}, n_r \in \{n_1, n_2, \dots n_s\}$$

$\hat{P}_1 = \frac{D_1}{n_1} = \frac{12}{100} = 0.120$, $\hat{P}_2 = \frac{D_2}{n_2} = \frac{8}{80} = 0.100$, $\hat{P}_3 = \frac{D_3}{n_3} = \frac{6}{80} = 0.075$; and so on.

The control limits for P – chart is obtained as follows

$$\bar{P} = \frac{\sum_{i=1}^{k} D_i}{\sum_{r=1}^{s} n_r} = \frac{234}{2450} = 0.096$$

For sample 1: $UCL_1 = \bar{P} + d\sqrt{\frac{\bar{P}(1-\bar{P})}{n_r}} = 0.096 + 3\sqrt{\frac{0.096(1-0.096)}{100}} = 0.184$

$CL_1 = \bar{P} = 0.096$

$$LCL_1 = \bar{P} - d\sqrt{\frac{\bar{P}(1-\bar{P})}{n_r}} = 0.096 - 3\sqrt{\frac{0.096(1-0.096)}{100}} = 0.008$$

For sample 2: $UCL_2 = \bar{P} + d\sqrt{\frac{\bar{P}(1-\bar{P})}{n_r}} = 0.096 + 3\sqrt{\frac{0.096(1-0.096)}{80}} = 0.195$

$$CL_2 = \bar{P} = 0.096$$

$$LCL_2 = \bar{P} - d\sqrt{\frac{\bar{P}(1-\bar{P})}{n_r}} = 0.096 - 3\sqrt{\frac{0.096(1-0.096)}{80}} = -0.003$$ and so on.

7.2. Construction of control limits for FM-chart:

To construct the FM-chart, the control limits are computed for each sample as follows.

For sample 1: $UCL_1 = E[\ \bar{\bar{L}}_i\ ] + 3\sqrt{var\ (\bar{\bar{L}}_i)} = 0.375 + 3\sqrt{0.0008214} = 0.4609$

$$CL_1 = E[\ \bar{\bar{L}}_i\ ] = \sum_{i=1}^{K} p_i m_i = 0.3750$$

$$LCL_1 = E[\ \bar{\bar{L}}_i\ ] - 3\sqrt{var\ (\bar{\bar{L}}_i)} = 0.375 - 3\sqrt{0.0008214} = 0.2891$$

For sample 2: $UCL_2 = E[\ \bar{\bar{L}}_i\ ] + 3\sqrt{var\ (\bar{\bar{L}}_i)} = 0.3863 + 3\sqrt{0.001234} = 0.4917$

$$CL_2 = E[\ \bar{\bar{L}}_i\ ] = \sum_{i=1}^{K} p_i m_i = 0.3863$$

$$LCL_2 = E[\ \bar{\bar{L}}_i\ ] - 3\sqrt{var\ (\bar{\bar{L}}_i)} = 0.3863 - 3\sqrt{0.001234} = 0.2809$$ and so on.

Where $E[\ \bar{\bar{L}}_i\ ] = \sum_{i=1}^{K} p_i m_i$ and $var\ (\bar{\bar{L}}_i) = \frac{1}{n_r}\left[\sum_{i=1}^{k} m\,_i^{\,2} p_i(1-p_i) - 2\sum_{i=1<j}^{k}\sum_{j=1}^{k} m_i m_j p_i p_j\right]$

7.3. Construction of control limits for TFM-chart (Proposed Method):

To construct TFM control chart with VSS for each sample, first we must estimate $E[\ \bar{\bar{L}}_{iA}],\ E[\ \bar{\bar{L}}_{iM}]\ and\ E[\ \bar{\bar{L}}_{iC}]$ for each Fuzzy subset as follows

For sample 1 $E[\ \bar{\bar{L}}_{1A}] = \sum_{i=1}^{r} p_i w_i\ 0.3063$

$$E[\ \bar{\bar{L}}_{1M}\ ] = \sum_{i=1}^{K} p_i w_i = 0.2450$$

$$E[\ \bar{\bar{L}}_{1C}\ ] = \sum_{i=1}^{s} p_i w_i = 0.1688$$

For Sample 2 $E[\ \bar{\bar{L}}_{2A}] = \sum_{i=1}^{r} p_i w_i = 0.3030$

$$E[\ \bar{\bar{L}}_{2M}\ ] = \sum_{i=1}^{K} p_i w_i = 0.1855$$

$$E[\ \bar{\bar{L}}_{2C}\ ] = \sum_{i=1}^{s} p_i w_i = 0.1579$$ and so on.

Apply an α – cut to the fuzzy sets by taking α = 0.65, the values are obtained as follows

$$E\left[\bar{\bar{L}}_{iA}{}^{\alpha}\right] = E[\ \bar{\bar{L}}_{iA}] + \alpha\{E[\ \bar{\bar{L}}_{iM}\ ] - E[\ \bar{\bar{L}}_{iA}]\}$$ and

$$E\left[\bar{\bar{L}}_{iC}{}^{\alpha}\right] = E[\ \bar{\bar{L}}_{iC}] - \alpha\{E[\ \bar{\bar{L}}_{iC}\ ] - E[\ \bar{\bar{L}}_{iM}]\}$$

For sample 1

$$E\left[\bar{\bar{L}}_{1A}{}^{0.65}\right] = E[\ \bar{\bar{L}}_{1A}] + 0.65\{E[\ \bar{\bar{L}}_{1M}\ ] - E[\ \bar{\bar{L}}_{1A}]\}$$

$$E\left[\bar{\bar{L}}_{1A}{}^{0.65}\right] = 0.3069 + 0.65\ (0.2450 - 0.3063) = 0.2664$$

$$E\left[\bar{\bar{L}}_{1C}{}^{0.65}\right] = E[\ \bar{\bar{L}}_{1C}] - 0.65\{E[\ \bar{\bar{L}}_{1C}\ ] - E[\ \bar{\bar{L}}_{1M}]\}$$

$$E\left[\bar{\bar{L}}_{1C}{}^{0.65}\right] = 0.1688 - 0.65\ (0.1688 - 0.2450) = 0.2183$$

For sample 2 $E\left[\bar{\bar{L}}_{2A}{}^{0.65}\right] = E[\ \bar{\bar{L}}_{2A}] + 0.65\{E[\ \bar{\bar{L}}_{2M}\ ] - E[\ \bar{\bar{L}}_{2A}]\}$

$$E\left[\bar{\bar{L}}_{2A}{}^{0.65}\right] = 0.3030 + 0.65\ (0.1855 - 0.3030) = 0.2259$$

$$E\left[\bar{\bar{L}}_{2C}{}^{0.65}\right] = E[\ \bar{\bar{L}}_{2C}] - 0.65\{E[\ \bar{\bar{L}}_{2C}\ ] - E[\ \bar{\bar{L}}_{2M}]\}$$

$$E\left[\bar{\bar{L}}_{2C}{}^{0.65}\right] = 0.1579 - 0.65\ (0.1579 - 0.1855) = 0.1752$$ and so on.

α – level fuzzy midrange is used as the fuzzy transformation method while calculating the control limits.

For Sample 1

$$UCL^{0.65}{}_{1-mr} = \left\{ CL^{0.65}{}_{1-mr} + 3\left(\frac{\sqrt{var^{0.65}\,(\bar{\tilde{L}}_{1A})} + \sqrt{var^{0.65}\,(\bar{\tilde{L}}_{1C})}}{2}\right)\right\}$$

$$UCL^{0.65}{}_{1-mr} = \left\{0.2424 + 3\left(\frac{\sqrt{0.00012552+0.00008908}}{2}\right)\right\} = 0.2707$$

$$CL^{0.65}{}_{\bar{\tilde{L}}_1-mr} = f^{0.65}{}_{mr-\bar{\tilde{L}}}(CL) = \left\{\frac{E\left[\bar{\tilde{L}}_{1A}{}^{0.65}\right] + E\left[\bar{\tilde{L}}_{1C}{}^{0.65}\right]}{2}\right\} = \frac{0.2664+0.2183}{2} = 0.2424$$

$$LCL^{0.65}{}_{1-mr} = \left\{0.2424 - 3\left(\frac{\sqrt{0.00012552+0.00008908}}{2}\right)\right\} = 0.2141$$

For Sample 2

$$UCL^{0.65}{}_{2-mr} = \left\{ CL^{0.65}{}_{2-mr} + 3\left(\frac{\sqrt{var^{0.65}\,(\bar{\tilde{L}}_{2A})} + \sqrt{var^{0.65}\,(\bar{\tilde{L}}_{2C})}}{2}\right)\right\} = 0.2190$$

$$CL^{0.65}{}_{\bar{\tilde{L}}_2-mr} = f^{0.65}{}_{mr-\bar{\tilde{L}}}(CL) = \left\{\frac{E\left[\bar{\tilde{L}}_{2A}{}^{0.65}\right] + E\left[\bar{\tilde{L}}_{2C}{}^{0.65}\right]}{2}\right\} = \frac{0.2259+0.1752}{2} = 0.2006$$

$$LCL^{0.65}{}_{2-mr} = \left\{0.2006 - 3\left(\frac{\sqrt{0.00011592+0.00001987}}{2}\right)\right\} = 0.1758 \text{ and so on}$$

The Triangular Fuzzy Multinomial control chart for VSS with α – level fuzzy midrange for each samples are calculated and given in the table 2.

Table -2

Sample No	$\bar{\tilde{L}}_{iA}{}^{\alpha}$	$\bar{\tilde{L}}_{iM}$	$\bar{\tilde{L}}_{iC}{}^{\alpha}$	$Var(\bar{\tilde{L}}_{iA}{}^{\alpha})$	$Var(\bar{\tilde{L}}_{iM})$	$Var(\bar{\tilde{L}}_{iC}{}^{\alpha})$
1	0.2664	0.2450	0.2183	0.00012552	0.00008	0.00005265
2	0.2259	0.1845	0.1752	0.00011592	0.00003	0.00001987
3	0.2269	0.1695	0.1539	0.00009359	0.00001	0.00000703
4	0.2836	0.2168	0.1873	0.00007422	0.00002	0.00001114
5	0.2788	0.2365	0.1967	0.00006538	0.00002	0.00001042
6	0.2777	0.2373	0.2019	0.00006567	0.00002	0.00001040
7	0.2754	0.2175	0.1851	0.00007553	0.00002	0.00001109
8	0.2693	0.2080	0.1746	0.00006192	0.00002	0.00001193
9	0.2477	0.1985	0.1776	0.00008933	0.00002	0.00001169
10	0.2517	0.1989	0.1815	0.00008859	0.00003	0.00002171
11	0.2876	0.2280	0.1856	0.00005528	0.00002	0.00001083
12	0.2970	0.2595	0.2110	0.00006092	0.00001	0.00000923
13	0.2843	0.2615	0.2166	0.00006297	0.00001	0.00000893
14	0.3115	0.2568	0.2114	0.00005899	0.00002	0.00000986
15	0.2735	0.2393	0.2040	0.00006809	0.00002	0.00001004
16	0.2245	0.1821	0.1725	0.00010790	0.00003	0.00002013
17	0.2349	0.1774	0.1613	0.00010200	0.00003	0.00002212
18	0.2337	0.1743	0.1592	0.00009504	0.00002	0.00001374
19	0.2401	0.1993	0.1820	0.00008958	0.00002	0.00001159
20	0.2816	0.2150	0.1835	0.00007175	0.00002	0.00001138
21	0.2594	0.2190	0.1931	0.00007701	0.00002	0.00001098
22	0.2727	0.2182	0.1888	0.00007859	0.00003	0.00001965
23	0.2670	0.2205	0.1894	0.00008123	0.00003	0.00001857
24	0.2464	0.1925	0.1689	0.00007779	0.00002	0.00001275
25	0.2478	0.1960	0.1721	0.00008428	0.00002	0.00001216

Evaluated weights by weight determination method of Saaty (1980) as w_j where $\sum_{i=1}^{k} w_i = 1$ are presented in table 3 through column from 7 to 11 and various probabilities corresponding to each sample are presented in the table 3 through column from 12 to 16.

Table 3

Sample No.	Rej.	PQ	MQ	GQ	EQ	w_1	w_2	w_3	w_4	w_5	p_1	p_2	p_3	p_4	p_5
1	12	10	12	54	12	0.125	0.313	0.125	0.313	0.125	0.120	0.100	0.120	0.540	0.120
2	8	7	9	48	8	0.145	0.263	0.184	0.263	0.145	0.100	0.088	0.113	0.600	0.100
3	6	11	12	43	8	0.233	0.209	0.163	0.233	0.163	0.075	0.138	0.150	0.538	0.100
4	9	7	13	53	18	0.175	0.250	0.150	0.250	0.175	0.090	0.070	0.130	0.530	0.180
5	10	16	18	54	12	0.250	0.150	0.175	0.250	0.175	0.091	0.145	0.164	0.491	0.109
6	12	5	17	60	16	0.175	0.250	0.175	0.250	0.150	0.109	0.045	0.155	0.545	0.145
7	11	12	13	50	14	0.250	0.175	0.150	0.250	0.175	0.110	0.120	0.130	0.500	0.140
8	10	22	18	45	5	0.175	0.175	0.150	0.250	0.250	0.100	0.220	0.180	0.450	0.050
9	10	8	13	50	9	0.150	0.250	0.175	0.250	0.175	0.111	0.089	0.144	0.556	0.100
10	6	5	14	51	14	0.184	0.263	0.145	0.263	0.145	0.067	0.056	0.156	0.567	0.156
11	20	13	23	47	7	0.150	0.175	0.175	0.250	0.250	0.182	0.118	0.209	0.427	0.064
12	15	13	20	58	14	0.150	0.250	0.175	0.250	0.175	0.125	0.108	0.167	0.483	0.117
13	9	12	22	64	13	0.250	0.175	0.175	0.250	0.150	0.075	0.100	0.183	0.533	0.108
14	8	9	20	61	22	0.250	0.175	0.150	0.250	0.175	0.067	0.075	0.167	0.508	0.183
15	6	10	19	61	14	0.250	0.175	0.175	0.250	0.150	0.055	0.091	0.173	0.555	0.127
16	8	5	12	47	8	0.145	0.263	0.184	0.263	0.145	0.100	0.063	0.150	0.588	0.100
17	10	8	12	40	10	0.145	0.263	0.184	0.263	0.145	0.125	0.100	0.150	0.500	0.125
18	7	13	10	42	8	0.250	0.175	0.150	0.250	0.175	0.088	0.163	0.125	0.525	0.100
19	5	7	14	54	10	0.250	0.175	0.175	0.250	0.150	0.056	0.078	0.156	0.600	0.111
20	8	11	14	50	17	0.250	0.175	0.150	0.250	0.175	0.080	0.110	0.140	0.500	0.170
21	5	8	16	58	13	0.250	0.175	0.175	0.250	0.150	0.050	0.080	0.160	0.580	0.130
22	8	9	15	51	17	0.263	0.184	0.145	0.263	0.145	0.080	0.090	0.150	0.510	0.170
23	10	12	14	50	14	0.263	0.184	0.145	0.263	0.145	0.100	0.120	0.140	0.500	0.140
24	6	13	17	45	9	0.250	0.150	0.175	0.250	0.175	0.067	0.144	0.189	0.500	0.100
25	9	10	14	46	11	0.250	0.175	0.175	0.250	0.150	0.100	0.111	0.156	0.511	0.122

The control limits for Triangular Fuzzy Multinomial control chart for VSS with α – level fuzzy midrange for each sample are calculated and given in the table 4. The processes control $\boldsymbol{S^{\alpha}}_{\boldsymbol{mr-\tilde{\bar{L}}_i}}$ is also calculated and presented in table 4 through column 4 and the process control is also present in the table 4 through column 5.

Table 4

Sample No	$UCL^{\alpha}_{\tilde{\tilde{L}}_i-mr}$	$CL^{\alpha}_{\tilde{\tilde{L}}_i-mr}$	$LCL^{\alpha}_{\tilde{\tilde{L}}_i-mr}$	$S^{\alpha}_{mr-\tilde{\tilde{L}}_i}$	$LCL^{\alpha}_{\tilde{\tilde{L}}_i-mr} \leq S^{\alpha}_{mr-\tilde{\tilde{L}}_i} \leq UCL^{\alpha}_{\tilde{\tilde{L}}_i-mr}$ Process Control
1	0.2707	0.2424	0.2141	0.2176	In Control
2	0.2253	0.2006	0.1758	0.1919	In Control
3	0.2117	0.1904	0.1691	0.1759	In Control
4	0.2550	0.2354	0.2158	0.2081	Out-of-Control
5	0.2562	0.2377	0.2193	0.2007	Out-of-Control
6	0.2583	0.2398	0.2213	0.2071	Out-of-Control
7	0.2500	0.2303	0.2105	0.2002	Out-of-Control
8	0.2401	0.2219	0.2037	0.1909	Out-of-Control
9	0.2340	0.2127	0.1914	0.1933	In Control
10	0.2389	0.2166	0.1943	0.2004	In Control
11	0.2538	0.2366	0.2193	0.1972	Out-of-Control
12	0.2718	0.2540	0.2362	0.2089	Out-of-Control
13	0.2685	0.2505	0.2325	0.2088	Out-of-Control
14	0.2790	0.2614	0.2438	0.2193	Out-of-Control
15	0.2575	0.2388	0.2200	0.2061	Out-of-Control
16	0.2225	0.1985	0.1745	0.1896	In Control
17	0.2218	0.1981	0.1745	0.1832	In Control
18	0.2186	0.1964	0.1743	0.1825	In Control
19	0.2324	0.2110	0.1897	0.1950	In Control
20	0.2519	0.2325	0.2132	0.2033	Out-of-Control
21	0.2462	0.2263	0.2064	0.2022	Out-of-Control
22	0.2518	0.2308	0.2097	0.2035	Out-of-Control
23	0.2494	0.2282	0.2070	0.1993	Out-of-Control
24	0.2278	0.2076	0.1874	0.1857	Out-of-Control
25	0.2308	0.2099	0.1891	0.1878	Out-of-Control

From the above table, the process is out of control at samples at sample 4, the corresponding sample sizes are 100. The corresponding control limits are given by

For sample 4:

UCL = 0.2550 CL = 0.2354 LCL = 0.2158 $S^{\alpha}_{mr-\tilde{\tilde{L}}_i} = 0.2081$ Sample Size 100

$P_R = 0.090$ $P_{PQ} = 0.070$ $P_{MQ} = 0.130$ $P_{GQ} = 0.530$ $P_{EQ} = 0.180$

For Sample 5:

UCL = 0.2562 CL = 0.2377 LCL = 0.2193 $S^{\alpha}_{mr-\tilde{\tilde{L}}_i} = 0.2007$ Sample Size 110

$P_R = 0.091$ $P_{PQ} = 0.145$ $P_{MQ} = 0.164$ $P_{GQ} = 0.491$ $P_{EQ} = 0.109$ and so on.

It is clear that the TFM chart for VSS with α – level fuzzy midrange gives the first signal of special causes corresponding to 4th sample. The FM chart for VSS show the existence of assignable cause at 8th sample and in p-chart the first signal for out of control is identified at the 11th sample. Only 360 samples are inspected to get the first out of control signal, but the 780 and 1070 samples are to be inspected to get the alarm with the help of FM chart with VSS and p-chart respectively. TFM chart for VSS with α – level fuzzy midrange is more economical and more sensitive to identify the shift in the multi-attribute quality data.

8. Conclusion:

TFM control chart with VSS using α – level fuzzy midrange has been proposed for linguistic data. To draw the chart, samples of varying sizes are chosen from free defined set. α – level fuzzy midrange techniques are also applied to construct TFM chart with VSS. The proposed method is compared with regular p-chart with VSS and FM chart with VSS. The TFM control chart with VSS using α – level fuzzy midrange is more economical and more sensitive in identifying the shift in the process for multi-attribute quality data in linguistic terms. In future the work can be extended using Trapezoidal fuzzy number and Markov dependent sample sizes.

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