

ROLE OF VORONOI DIAGRAM APPROACH IN PATH PLANNING

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Abstract:

Graph theoretic approaches play an important role in the path planning problems. The usefulness of Graph Theoretic approaches mainly the Voronoi diagram is provided for the path planning in static and dynamic environment in this paper. Voronoi Diagram plays an important role in computing the optimal path between the source & destination in static as well as dynamic environment. So, through the literature survey and a detailed description about the methods, we provide the significance of this graph theoretic approach of Voronoi diagram in the robot motion planning.

Keywords: Graph Theoretic approach, Voronoi Diagram, static & dynamic path planning

1. INTRODUCTION

Path planning is an important issue in the field of robot motion planning as it allows a robot to get from source point to target point. Path planning algorithms are measured by their computational complexity. The accuracy of the map (or floor plan), robot localization and on the number of obstacles decides the feasibility of real time motion planning. The problem of path planning is related to the shortest path problem of finding a route between two nodes in a graph. When the autonomous robot decides its action, it is necessary to plan an optimal path depending on their tasks. Moreover, it is necessary to plan a collision free path minimizing a cost in terms of time, energy and distance. When an autonomous robot moves from a source to a target point in its given environment (static or dynamic), it is necessary to plan a feasible path avoiding obstacles in its way. An important aspect of the capability of a robot in Robot motion planning is to have robots with the ability of planning their paths while moving in their surrounding environment, preventing collision with other objects (whether stationary or in motion) and meeting the targeted task(s). The rigorous study of motion planning results in a rich and comprehensive list of approaches and algorithms. But here, the main focus of our study is on graph-based approaches. Graph representation of the robot's working environment is considered as one of the earliest attempts on constructing a well-organized and well-structured map of the agent's world for the purpose of planning a safe navigation.

The basic idea behind the graph-representation approach is to connect all the available free spaces of the field (i.e. places that are obstacle-free) via connected networks of lines so as to provide the robot with the opportunity of performing a safe, target-oriented, collision-free motion, where the robot's trajectories are planned. The available free-spaces are generally represented as vertices of a graph whose edges are in fact a network of connected lines. On formation of one such network, an algorithm such as Dijkstra's shortest path algorithm is used to find the shortest, obstacle-free path from the current robot's location to the target point.

Voronoi Diagram (VD) is a type of versatile geometric data structure. It has been widely used in physics, astronomy, geographical information systems, computer graphics, image processing, robotics, etc. Basically, Voronoi diagram is the graph theoretic approach in Robot motion planning. Some of the limitations that might be attributed to this approach are:

- Time complexity of constructing such a connected network (graph) as the robot's field of operation expands.
- Need of a global map of the environment for which the graph is constructed.
- Uncertainty introduced by the application of moving/movable objects.

2. LITERATURE REVIEW

The topic of robot motion planning (RMP) using Voronoi diagram has been the subject of in-depth studies and analysis during the past years. There are a number of valuable sources available in the literature. In this article, our main focus is on the roadmap-based path planning and utilizes a powerful computational geometry data structure, the Voronoi diagram, to obtain the shortest path. The merit of using a Voronoi diagram as a roadmap

over alternative methods, among which the visibility graph prevails, is its efficiency in finding path. The Voronoi diagram can be constructed in just $O(n \log n)$ time, even the fastest known algorithm for constructing visibility graph [1] can take $O(n^2)$ time in the worst case when the visibility graph has $O(n^2)$ edges.

A general method for refining a path obtained from a roadmap based on classical numerical optimization techniques can be found in [2]. The authors apply costs to each edge and use an augmented Dijkstra's algorithm to determine an optimal path. The edges which are closer to obstacles are assigned higher costs. However, there is not possibility that the method will generate an optimal path, as the path is constrained to the edges in the roadmap. To improve the smoothness of the path obtained from a roadmap, a B-Spline approximation has been used in [3].

In [4], the authors merges the Voronoi diagram, visibility graph, and potential field method for path planning into a single algorithm to obtain the optimal safe and the shortest paths. The algorithm is fairly complicated, and although the path length is shorter than those obtained from the potential field method or the Voronoi diagram. The obtained path contains bumps and rudimentary turns and is not smooth.

In [5], the authors create a new diagram called the VV(c) diagram (the Visibility-Voronoi diagram for clearance c). Method is applied to obtain an optimal path for a specified clearance value. The diagram evolves from the visibility graph to the Voronoi diagram as the value of c increases.

Another recent work on reducing the length of the path obtained from a Voronoi diagram [6] involves constructing polygons at the vertices in the roadmap where more than two Voronoi edges meet at a point. The path is sharper (smoother and shorter) than that obtained directly from the Voronoi diagram, but still path is not optimal.

3. INTRODUCTION TO VORONOI DIAGRAM

Voronoi diagrams are one of the main structures in the computational geometry area, commonly used as visibility graphs and for finding networks collision-free paths, they are also one of the most common techniques for building trajectory maps.

3.1. Definition:

Let $P = \{p_1, p_2, \dots, p_n\}$ be a set of n given point sites in the plane. We define the Voronoi diagram of set of points P as the subdivision of the plane into n cells, one for each site in P, with the property that a point q lies in the cell corresponding to a cell p_i iff $\text{dist}(q, p_i) < \text{dist}(q, p_j)$ for each p_j in P with $j \neq i$. We denote the Voronoi diagram of P by $\text{Vor}(P)$. The cell that corresponds to a site p_i is denoted $V(p_i)$.

The Voronoi diagram is a planar subdivision whose edges are straight line segments. Some edges are line segments and others are half-lines. Unless all sites are collinear there will be no edges that are full lines.

3.2. Properties of The Voronoi Diagram:

For the Voronoi diagram $\text{Vor}(S)$ of a set of points S the following holds:

- A point q is a vertex of $\text{Vor}(S)$ iff its largest empty circle $C(q)$ contains three or more sites on its boundary.
- The bisector between sites p_i and p_j defines an edge of $\text{Vor}(S)$ iff there is a point q in R^2 such that $C(q)$ contains both p_i and p_j on its boundary but no other site.

3.3. Computation Of Voronoi Diagram (Fortune's Algorithm):

3.3.1 Method: Sweep a horizontal line - the sweep line - from top to bottom over the plane.

3.3.2 Difficulty in method: The part of $\text{Vor}(P)$ above the sweep line l depends not only on the sites above l but also on sites below l. Here one observation is that, For any point q above the sweep line l, the nearest site of q cannot lie below l if q is at least as near to some site above l as q is to l.

3.3.3 Beach line: The locus of the points that are closer to some site above the line l than to l itself is bounded by parabolic arcs. We call the sequence of parabolic arcs as the beach line. This is shown by the thick piecewise smooth curve in the figure below.

3.3.4 Inductive Assumption: By the observation above, we inductively assume that $\text{Vor}(P)$ is completely determined above the beach line.

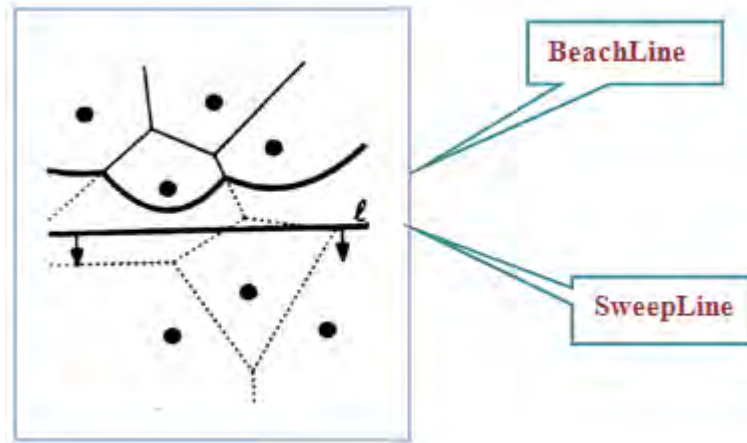


Fig.1. Voronoi Diagram construction using Fortune's Sweepline algorithm

It is easy to see that several disjoint portions of the same parabola can be part of the beach line. Notice that the breakpoints between the different parabolic arcs forming the beach line lie on edges of the Voronoi diagram. The breakpoints observed trace out the Voronoi diagram as the sweep line moves from top to bottom. The algorithm for construction of voronoi Diagram is given below:

Algorithm: VoronoiDiagram(P)

Input: A set $P = \{p_1, p_2, \dots, p_n\}$ of point sites in the plane.

Output: The Voronoi diagram $Vor(P)$ given inside a bounding box in a doubly-connected edge list structure.

1. Initialize the event queue Q with all site events.
2. While Q is not empty Do
3. Consider the event with largest y -coordinate in Q.
4. If the event is a site event, occurring at site p_i
5. Then HandleSiteEvent(p_i)
6. Else HandleCircleEvent(p_i), where p_i is the lowest point of the circle causing the event.
7. Remove the event from Q.
8. The internal nodes still present in T correspond to the half-infinite edges of the Voronoi diagram. Compute a bounding box that contains all vertices of the Voronoi diagram in its interior, and attach the half-infinite edges to the bounding box by updating the doubly connected edge list appropriately.
9. Traverse the half-edges of the doubly connected edge list to add the cell records and the pointers to and from them.

4. DISCUSSION ON ROADMAP BASED METHODS

Now, we will discuss some of the methods which uses the concept of roadmap for path planning using voronoi diagram. Following are some of the methods that uses the voronoi diagram for the construction of roadmaps and then find the path for robot motion planning.

4.1 Path Planning for Unmanned Vehicles using Ant Colony Optimization on a Dynamic Voronoi Diagram :

It is a biologically inspired path planning algorithm using the Ant Colony Optimization (ACO) on obstacle geometry described by the Voronoi diagram. [7]The safe paths from source to destination between obstacles are chosen as the boundaries of the Voronoi cells centered the obstacles. Based on the Voronoi diagram, Ant Colony Optimization is then applied to produce quasi-optimal paths. The combined Voronoi and ACO approach is expected to provide quasi-optimal paths adaptively to a dynamically changing environment. Following diagram shows the experimental results obtained.

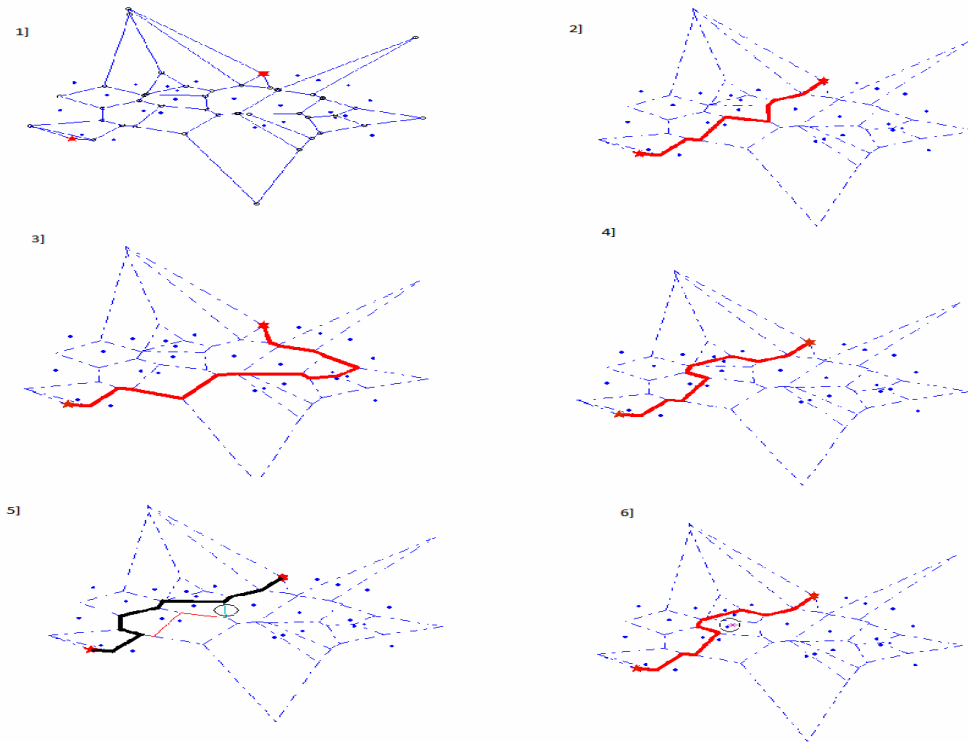
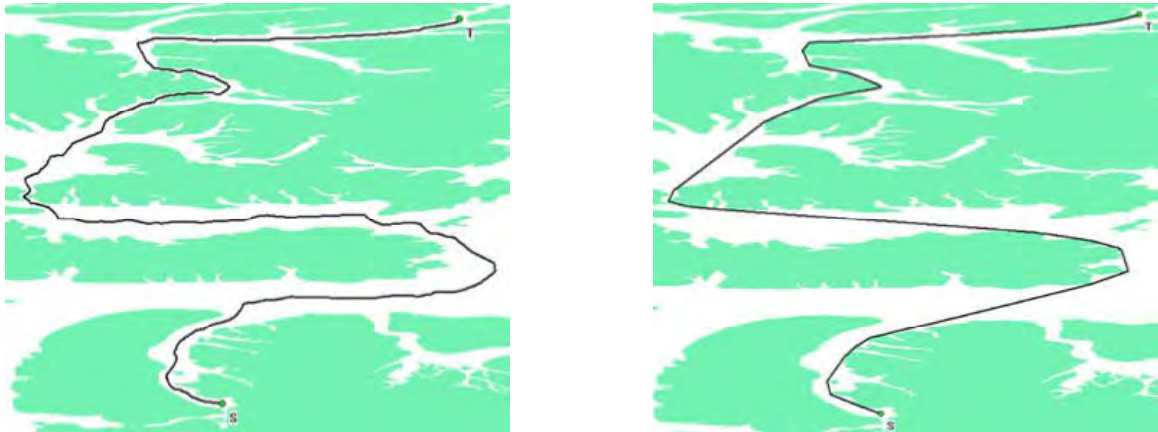


Fig. 2. Results obtained by ACO algorithm (1. Voronoi diagram based on 27 obstacle points 2. Optimal Path Generated using the A-Star Algorithm (length: 229.8, time: 0.515s) 3. Shortest Path Found by 10 Ant Agents (length: 341.5, time: 0.172s) 4. Shortest Path Found by 100 Ant Agents (length: 239.8, time: 1.172s) 5. The ACO algorithm generates an alternative path when one of the Voronoi edge (marked by a circle) is blocked. 6. The ACO algorithm produces a new optimal path when a slight shift of an obstacle point causes the topology change of the Voronoi diagram

In their simulation experiments, they first generate random obstacle points in a 2D plane. Afterward, a Voronoi diagram is generated based on these obstacle points. In Figure 2 first figure shows the Voronoi diagram generated using MATLAB based on 27 random obstacle points. The highlighted vertices represent the starting point and the target. The lines in the graph are the boundaries of the Voronoi cells, which constitute various possible paths from the starting vertex to the ending vertex. The path highlighted in figure 2.2 is the optimal path (length = 229.8) from the starting point to the destination obtained by the well-known A-star algorithm. The computation time is 0.515 second. As the number of ant agents increases, the shortest path converges to the optimal solution. Figures 2.3 and 2.4 shows the shortest paths in the same Voronoi diagram found by 10 and 100 ant agents, respectively. Figure 2.5 shows the situation that an edge is blocked, e.g. the unmanned vehicle is not allowed to cross the region between two obstacle points. Based on the previous pheromone distribution in the Voronoi diagram, the ACO then produces an alternative path to the destination. In Figure 2.6, a slight shift of an obstacle point from location $[x=74, y=93]$ to $[x=71, y=90]$ leads to topology changes in the Voronoi diagram.

4.2 Roadmap-Based Path Planning Using the Voronoi Diagram for a Clearance-Based Shortest Path:

It is a technique based on Voronoi diagram to compute an optimal path between source and destination in the presence of simple disjoint polygonal obstacles in the plane. This method has a number of unique features, the iterative refinement technique based on Steiner points for path optimization, and the possibility of performing dynamic updates on the structure during the path computation process[8][9].



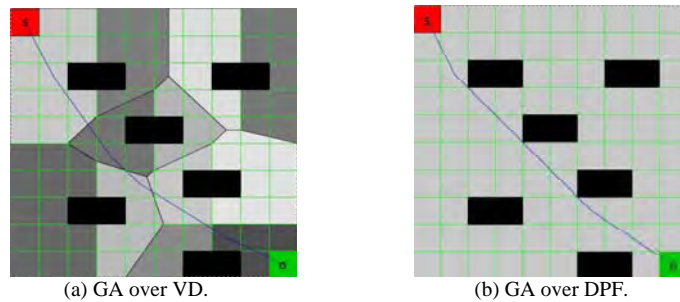
(a) Shortest path obtained from Voronoi diagram based Roadmap

(b) Optimal path after iterative refinement (Cmin = 0)

Fig 3: Shortest path using above proposed algorithm (Number of vertices in dataset = 11,797).

4.3 Real Path Planning based on Genetic Algorithm and Voronoi Diagrams:

In this method, the path planning is based on Voronoi diagrams, where obstacles in the environment are considered as the generating points of the diagram, and a genetic algorithm is used to find a path without collisions from the robot initial to target position. And they have used a genetic algorithm for computing paths on a regular grid based environment, considering certain quality attributes[10].



(a) GA over VD.

(b) GA over DPF.

Fig. 4. Visual comparison.

In this method, they have compared the previous method for path planning over GA i.e. Digital Potential Field method (DPF) with GA over voronoi diagram (VD). Their simulation results are shown in above figure. The GA based on DPF approach requires a 25.05% more time in the initial phase of building the environment. Also, when considering the quality of the solutions obtained, the GA based on VD achieves better solutions, finding safer paths with similar values for the length and smoothness.

4.4 Motion Planning for Mobile Robot Navigation using combine Quad-Tree Decomposition and Voronoi Diagrams:

The method uses a path selection mechanism that creates innovative paths through the workspace & This approach is implemented on motion robots which verified the shortest path via Quad-tree Decomposition (QD) and then used Voronoi Diagrams (VD(S)) algorithm called (Q&V) algorithm.

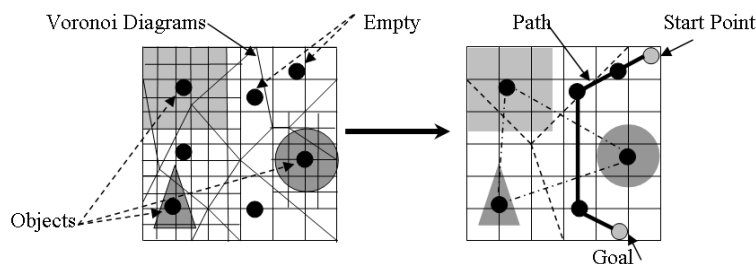


Fig. 5. Q&V Algorithm

5. CONCLUSION

In this paper, we have found that how the voronoi diagram mapping is efficient over the other methods in path planning. Path in static as well as dynamic environment can be planned using graph theoretic approach i.e. voronoi diagram more efficiently with better optimized path than the path obtained by other approaches. Our study shows the role of voronoi diagram in path planning.

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