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Performance Analysis of Edge Detection Methods on Hexagonal Sampling Grid

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Abstract

Hexagonal sampling grid offers less storage time, less computation time, increased coding efficiency, less quantization error, equidistant property and consistent connectivity, etc., Edge detection is one of the important preprocessing steps in many of the image processing applications. Various edge detection techniques are available for images on rectangular grids such as classical operators, CLAP algorithm based edge detection, wavelet based edge detection etc. Wavelet based edge detection is found to be a better technique for specific application such as iris recognition system, 3D vertebrae shape recognition, infrared target recognition etc. These techniques are not suitable for hexagonal domain. For these we need to choose proper addressing scheme. In this paper, (i) Edge detection using masks on hexagonal grid (using spiral addressing scheme) is performed (ii) CLAP algorithm is performed on rectangular domain and hexagonal domain (using alternate pixel suppressal and half-pixel shift method) (iii) Wavelet based edge detection was performed on rectangular domain. For hexagonal domain, we need to choose proper directional wavelet. Gabor wavelet based edge detection is proposed for the hexagonal grid and we were able to get good results. For performance evaluation we are considering Mean Square Error, Peak Signal to Noise Ratio and ratio of edge pixels to size of image. For all operations, edge detection on hexagonal domain gives better results and better visual appeal of images compared to edge detection on rectangular domain.

Keywords: Image resampling, Hexagonal sampled grid, spiral addressing, CLAP Algorithm, Gabor filters

Introduction

Edge detection is the most common approach for detecting meaningful discontinuities in gray level. An edge is a set of connected pixels that lie on the boundary between two regions. Edge is a 'local' concept [1]. In hexagonal domain the edge detection operations were performed on the hexagonally sampled image. Resampling is the process of transforming a discrete image which is defined at one set of coordinate locations to a new set of coordinate points [2] i.e, converting from rectangular to hexagonal grid.

Many classical edge detectors have been developed over time. They are based on the principle of matching local image segments with specific edge patterns. The edge detection is realized by the convolution with a set of directional derivative masks. Classical edge detection operators like Roberts, Sobel, Prewitt and Laplacian are defined on a 3 X 3 pattern grid, so they are efficient and easy to apply. In hexagonal domain we can use hexagonal operators for edge detection. These hexagonal masks are applied on the images which is represented using spiral addressing scheme.

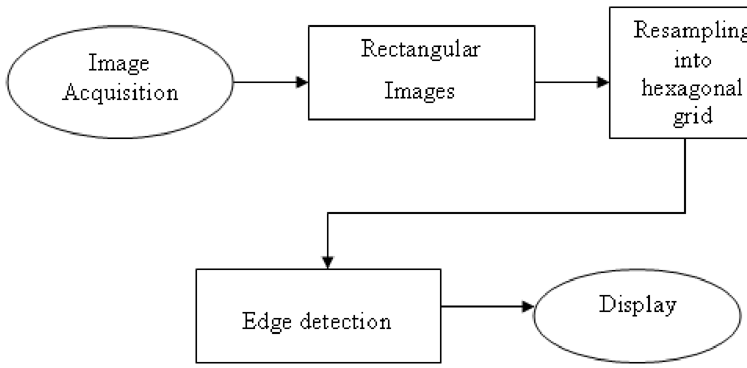


Figure 1: System diagram for performing edge detection on hexagonal sampled grid.

CLAP algorithm based edge detection was proposed by E.G.Rajan [3] for rectangular and hexagonal sampled grid. He used various Basis Structures to filter the values of interest from the scan window. The addressing scheme used was alternate pixel suppressal. The Basis structures would be convex polygons that will enclose the pixel in consideration. In this paper ,we have performed CLAP algorithm based edge detection on both rectangular grid and hexagonal grid .Two techniques as alternate pixel suppressal and half pixel shift addressing scheme were used in our work .Their performance was analysed and the results are discussed in section(4).

With the growth of wavelet theory, the wavelet transform have been found to be remarkable mathematical tool to analyze the singularities including the edges [4], and further to detect them effectively. The wavelet transform characterizes the local regularity of signals by decomposing signals into elementary building blocks that are well localized both in space and frequency. This not only explains the underlying mechanism of classical edge detectors, but also indicates a way of constructing optimal edge detectors under specific working conditions. Gabor filter [5] is the only filter with orientation selectivity that can be expressed as a sum of only two separable filters. It choose higher frequency information hence the edge is maximized. For rectangular domain we have to select four different orientations to obtain the edges, along 0, 90,270 and 360 degrees. But in Hexagonal domain 3 degree orientations give better edges, along 0, 60,120 degrees.

The paper is organized as follows .Section (2) deals with edge detection methods on hexagonal sampling grid and along with each method a comparative study is performed. Section (3) deals with performance evaluation and section (4) deals with results and discussion.

Edge detection methods on hexagonal sampling grid

To study the effect of using a hexagonal sampling regime for edge detection the HIP (Hexagonal Image Processing) framework was employed. One of the issue is choosing proper addressing scheme

Operator Based Edge detection

Various classical operators used for rectangular grid are Prewitt, Laplacian of Gaussian, Canny edge detector, etc., These operators are applicable exclusively for spiral addressing scheme representation of hexagonal grid. Sheridan [6] proposed a one-dimensional addressing system, as well as two operations as addition and multiplication based on this addressing system, for hexagonal structure. This system is called Spiral Architecture (Figure.2). It is inspired from anatomical consideration of the primate's vision system. Middleton and Sivaswamy [7] also proposed a single-index system for pixel addressing by modifying the Generalized Balanced Ternary system. Neighborhood operations are often used in image processing. Finding the neighbor in a hexagonal image makes use of the spiral addition operation. In a seven-pixel cluster, the neighborhood relation can be determined by spiral addition . One can find out the spiral address of any hexagonal pixel with centre on a certain hexagonal pixel whose spiral address is known.

The Prewitt edge operators are first derivative operators while the Laplacian of Gaussian (LOG) is a second derivative operator. In terms of directional sensitivity, the Prewitt operators essentially are maximally sensitive to edges in horizontal and vertical directions while the LOG operator is isotropic. The isotropic nature of the LOG operator is due to the Gaussian smoothing function employed to reduce the noise sensitivity of the second derivative operation. The Canny edge detector is a good example of a near optimal edge detector combining features of the Prewitt and LOG operators.

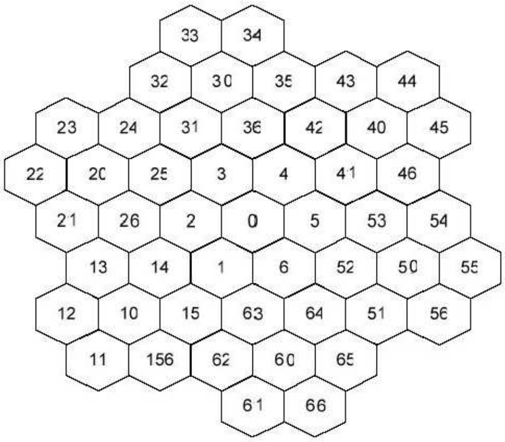


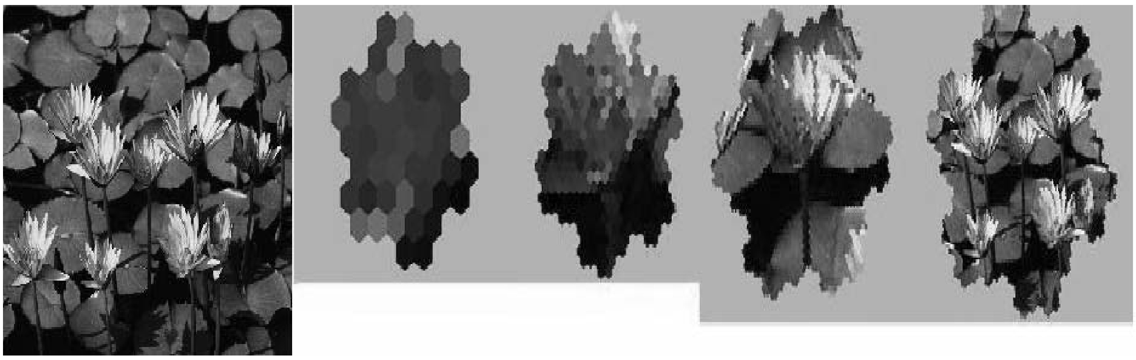
Figure 2: Spiral addressing.

The Prewitt edge detector is a gradient based edge detector. The detector is considered to be poor due to its bad approximation to the gradient operator. However, the ease of implementation and low computational cost overcome these disadvantages.Figure.3 shows Prewitt mask suitable for HIP framework using spiral addressing scheme.

$$h_1 = \begin{bmatrix} & 1 & & 1 & \\ 0 & & 0 & & 0 \\ & -1 & & -1 & \end{bmatrix} \; h_2 = \begin{bmatrix} & 0 & & 1 & \\ -1 & & 0 & & 1 \\ & -1 & & 0 & \end{bmatrix} \; h_3 = \begin{bmatrix} & -1 & & 0 & \\ -1 & & 0 & & 1 \\ & 0 & & 1 & \end{bmatrix}$$

Figure 3: Prewitt Hexagonal mask.

In this framework number of layers required to represent an image with M x N size is approximately $\frac{\log(M)+\log(N)}{\log 7}$.



(a) (b) (c) (d) (e)

Figure a) original image in rectangular grid (b) – (e) Image in HIP framework using layer λ as 2-layer, 3-layer, 4-layer and 5-layer respectively.

Figure shows original image is of size 128 x 128.So the number of layers required to display this image fully in HIP framework is given by

$$\lambda = \frac{\log\ 128 + \log 128}{\log 7} = \frac{4.214}{0.845} = 4.987 \approx 5.$$

Implementation of edge operator involved computing an equivalent mask for hexagonal case, this takes the advantage of the fact that the vertical direction gradient can be approximated by a combination of two masks at 60 degree and 120 degree to the horizontal. The horizontal mask for hexagonal case is equivalent to the square case. The masks are shown in figure 3. h1=h2-h3 .Then, only two of the masks are needed in the computation of gradient images.

CLAP Algorithm based edge detection

For CLAP algorithm based edge detection on hexagonal grid , basis structures for a hexagonal lattice can be defined as the possible convex polygons that enclose the central pixel in the 7- neighbourhood as shown in figure.4

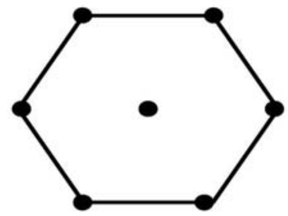


Figure 4: The pixel and its 7-neighborhood structure in a Hexagonal lattice.

The convex polygon connecting all the neighboring pixels of the central pixel is one such basis structure. Other polygons can be obtained by removing the neighborhood pixels in various combinations. Thus 18 convex polygons are obtained over the 7- neighborhood hexagonal lattice. The pixel wise representation of these polygons over the 7- neighborhood grid can be given as in Figure 5.

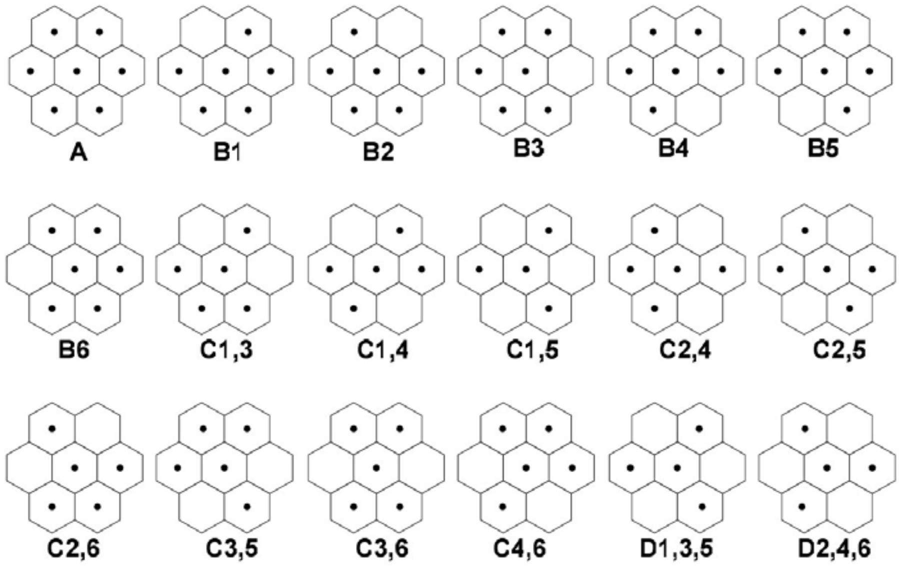


Figure 5: The Basis Structures over the 7-neighbourhood Hexagonal lattice.

Hexagonal resampling

We performed edge detection algorithm on two different types of representation of hexagonal lattice such as alternate pixel suppressal method and half-pixel shift method.

Hexagonal grid image based on alternate pixel suppressal method can be obtained from the conventional image by alternatively suppressing rows and columns of the existing rectangular grid and sub sampling it. All the other pixels of the rectangular grid which do not have any correspondence with the hexagonal counterparts are suppressed to zero. While processing this sub sampled image the suppressed pixels are not considered in computation. The sub sampled hexagonal grid is shown in Figure. 6.

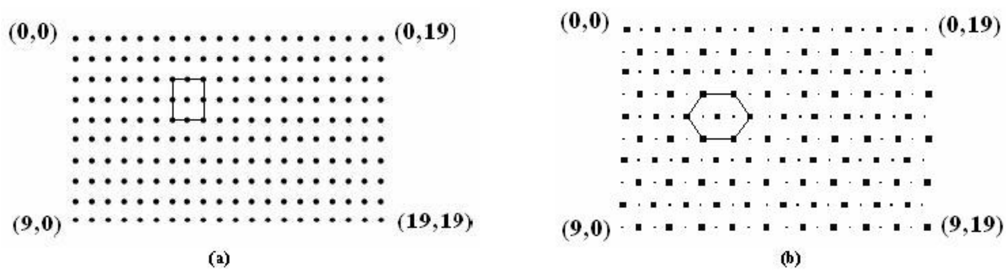


Figure 6: (a) Rectangular grid and (b) Simulated hexagonal grid obtained using alternate pixel suppression.

The hexagonal grid obtained above by sub sampling the rectangular grid image consists of fewer pixels compared to rectangular sampled image which will affect the quality of the image. Here, we consider hexagonal grid based on half-pixel shift method [8]. For each odd line , find the midpoint between two adjacent pixels by simple linear interpolation (i.e., mid = (left + right) / 2). Discard the left and right, keeping only the mid values. This gives us a hexagonal mapping from a regular square or rectangular grid.

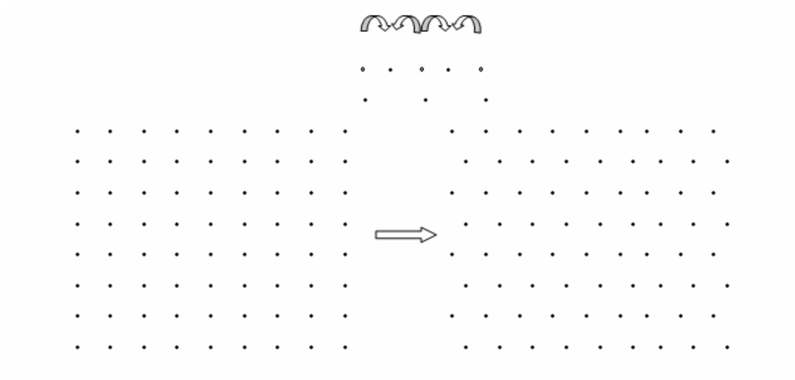


Figure 7: (a) Rectangular lattice (b) Hexagonal lattice using half-pixel shift method.

General procedure for edge detection

For hexagonally sub sampled images, we need to consider five basis structures. When we sub sample an image onto a hexagonal grid, the pixel in consideration being “13” the nearest neighbors are as circled in Figure.8.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Figure 8: Neighbourhood of the pixel in the hexagonally subsampled image.

We have five infima namely C1,4, C2,5, C3,6, D1,3,5, D2,4,6, (Figure.5) for the lattice of the convex polygons over Hexagonal Pixel Grid. If the sub image scanned by the 5 x 5 window is found to contain anyone of the five basis structures C1,4, C2,5, C3,6, D1,3,5, D2,4,6, then the central cell under consideration is assigned the 0 gray level, otherwise the original value is retained. This procedure is continued till the entire image is scanned. The overall effect is that the boundaries of various regions of an admissible gray range in the given image are retained and the interior parts are erased thus giving us the edge detected version of the original image. This procedure is repeated for whole of the image by sliding the window over it.

Performance wise, edge detection based on CLAP Algorithm gives better results on hexagonal grid representation based on half-pixel shift method rather than alternate pixel suppressal method. In the alternate pixel suppressal, due to the suppressal of pixels, most of the information will be lost. Half-pixel shift method provides better quality compared with alternate pixel suppressal method .We used half-pixel shift method for edge detection using CLAP Algorithm .For the comparative study we consider the case of hexagonal grid representation using alternate pixel suppression method and the results are discussed in the section (4).

Wavelet based edge detection

Different methods were proposed for edge detection on rectangular grid [9-13] in which wavelets can be directly used for edge detection. But for hexagonal domain, due to the three axis of symmetry, we have to consider the directionality of wavelets. We have to choose better wavelet which is having the property of directionality. To reflect the local and global direction of the image transform, the directional wavelet transform is applied using Daubachies 6 wavelet.

Let ((x , y) $\in L_2 (R^2)$ be a mother wavelet function, the wavelet transform of a function f (x, $y) \in L^2 (R)$ *at* scale s Corresponding to traditional wavelet function f *(X, Y)* , let $\theta_s(x, y,\theta_i) = (s (x \cos\theta_i , y \sin \theta_i)$. Assume the angle of the highest sensitivity of the wavelet function is θ_i, then we can obtain different directional images at various resolution by applying the above directional wavelet function with different θ_i, in wavelet transform. The directional wavelet transform of $f (x , y)$ is defined by

$$\boldsymbol{W}_S \boldsymbol{f}(x , y ,\theta_i) = f \ {}^{*} \phi_x (x , y ,\theta_i) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

In light of the analysis, we can obtain the image edge by computing the value of the directional wavelet transform instead of computing the modulus of gradient vector; moreover, we can select the proper angle θ_i, because it represents the direction of gradient. Then the edge of the original image can he obtained by the fusion of several edge images.

Gabor Wavelet Based Edge detection

Gabor is the only filter which is having the property of directionality. Due the different axis of symmetry of hexagonal lattice, we should give more importance for orientation selectivity. So we used the orientation selectivity filter like Gabor filter for the purpose of edge detection on hexagonal domain.Gabor expansion is a time-frequency analysis method which combine both the time/space and frequency information. Gabor expansion can be implemented as a multi-channel filter[15].

The Gabor filter is basically a Gaussian with variances along x and y-axes respectively, modulated by a complex sinusoid with centre frequencies U and V along x and y-axes respectively. Gabor expansion is a time-frequency analysis method, which was introduced in 1946 by Dennis Gabor [16]. This expansion introduces a time-localization Gaussian window function for extracting local information of signal with the form similar to the Fourier transformation. The Gabor filter tries to search and investigate the intermediate representations which combine the information of both time/space information ***f*** and frequency information ***F***. The goal is a simultaneous description of the temporal and spectral behavior of a function or signal ***f***. Such a representation is essentially two dimensional, measuring both behavior of the frequency ***w*** and time/space.

The classic Gabor expansion is computationally expensive, and since it combines all the space and frequency details of the original signal, it is difficult to take advantage of the gigantic amount of numbers. Fortunately, these problems can be partially overcome by using a multi-channel filtering technique, which uses specified filters to select the information at particular space/frequency points.

Multi-channel Gabor

Gabor expansion can be implemented as a multi-channel, wavelet-like filter and this multichannel filtering process mimics the characteristics of the human visual system (HVS). If an image containing more than two object regions is fed to a Gabor filter bank, then local spatial frequency differences between these objects hopefully will show variation in the filtered sub-images. This filtering approach is a natural way to manipulate variation between image objects.

Gabor filters have been used in detecting texture edge. However, it is based on traditional Gradient method. It does not specify the choice of frequency/scale range, and as a result, it cannot take advantage of the edge information for further analysis. The Gabor expansion is the only biologically plausible filters with orientation selectivity that can be exactly expressed as a sum of only two separable filters. This unique property has made Gabor filter an important transformation in image processing. There are many image segmentation algorithms and much research has successfully used Gabor filters for texture analysis.

When an image is processed by a Gabor filter, the output is the convolution of the image $I(x, y)$ and the Gabor function $g(x, y)$, i.e., $r(x, y) = g(x, y) * i(x, y)$ where * denotes the two dimensional convolution. This process can be used at different frequencies and different orientations, and the result is a multichannel filter bank.

Figure. 9 illustrates the multi-channel filtering system.

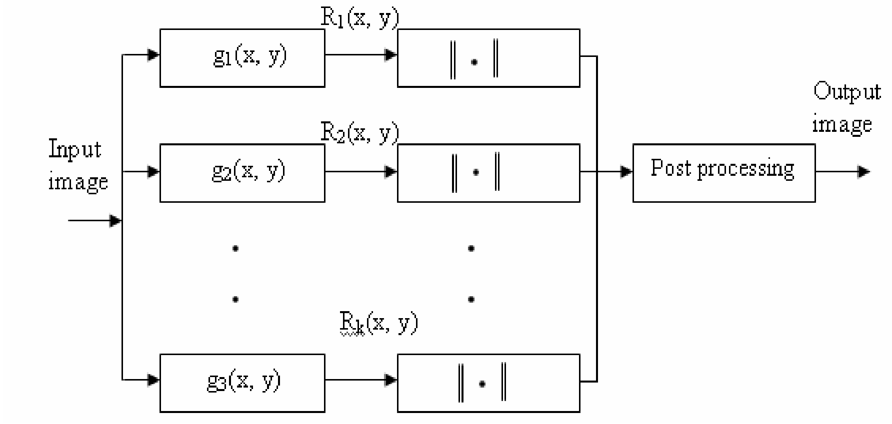


Figure 9: Multichannel Gabor system.

Gabor parameters and Edge detection

$$g(x, y; \omega, \theta, \sigma) = \frac{1}{2\prod\sigma^2}\exp(\frac{-(x^2+y^2)}{2\sigma^2})\exp(i(\omega_x\cos\theta+\omega_y\sin\theta)) \quad \text{……….(2)}$$

The Gabor filter can be mathematically represented as (3) where Gabor parameters like ω, θ and σ represents radial frequency, orientation and spatial extension respectively.

Due to the three axis of symmetry of hexagonal grid we should choose three different orientation along 0^0, 60^0 and 120^0. The central frequency is selected according to the image dimension. The radial frequencies are all 1 octave apart. Low frequency corresponds to smooth variations and constitutes the base of an image and high frequency presents the edge information which gives the detailed information in the image. Hence, this study neglects the very low radial frequencies. Using these frequencies and orientations, the Gabor filter Multi-channel system can present an image in various orientations and frequencies.

For performing edge detection operation on hexagonal domain using Gabor filters, the image is convolved with gabor filter bank to obtain the gabor filtered image along three different orientation . Then we find the edge detection of the resultant superimposed Gabor filtered image.

Performance Evaluation

In this work, the performances of edge detection on rectangular and hexagonal grids are computed using operators, CLAP algorithm and Wavelets. The performance was compared based on the parameters Mean Square Error (MSE), Peak Signal-to-Noise Ratio (PSNR), Computation time and mean square error.

Mean Square Error (MSE)

MSE indicates the average difference of the pixels throughout the image. A higher MSE indicates a greater difference between the original and processed image. Nevertheless, it is necessary to be very careful with the edges. The formula for the MSE calculation is given as:

$$MSE = \frac{1}{N}\sum_{i}\sum_{j}\left(X_{ij} - V_{ij}\right)^2 \qquad \text{...} (3)$$

Where N is the size of the image, X is the processed image, and v is the original image.

Peak Signal-to-Noise Ratio (PSNR)

$$PSNR = 10.\log_{10}\left(\frac{N * 255^2}{\sum_{i}\sum_{j}\left(X_{ij} - v_{ij}\right)^2}\right) \qquad \text{...} (4)$$

PSNR is used for quantitative comparison.

Results and discussion

Operator based edge detection

Classical edge detection operators are very easy to apply but they are sensitive to noise. Sobel , Roberts, Prewitt, Canny are the main classical edge detection operators.

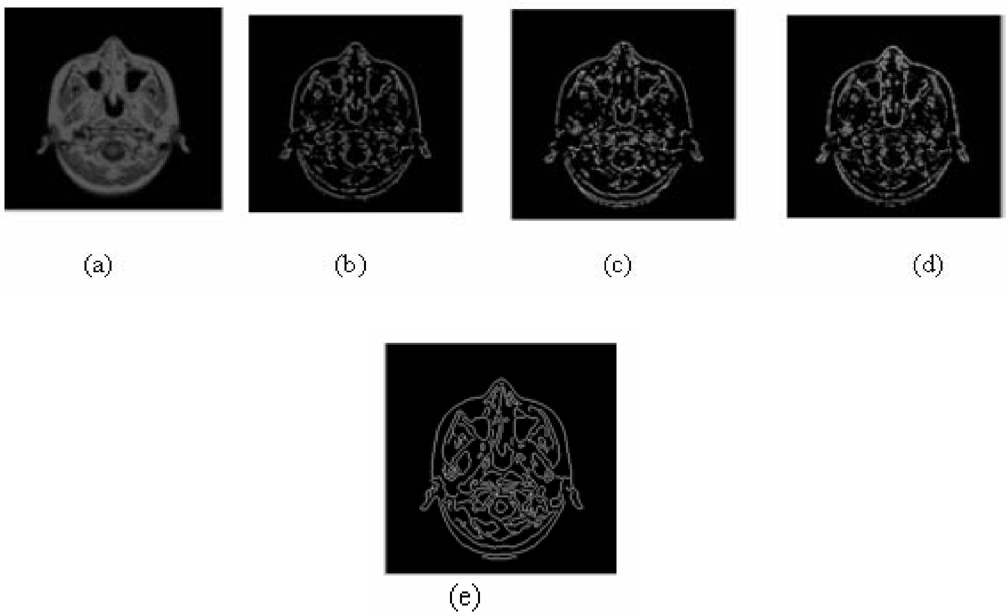


Figure 10: (a) Original Image on rectangular lattice (b) – (e) Edge detection using Sobel, Roberts, Prewitt and canny operators respectively.

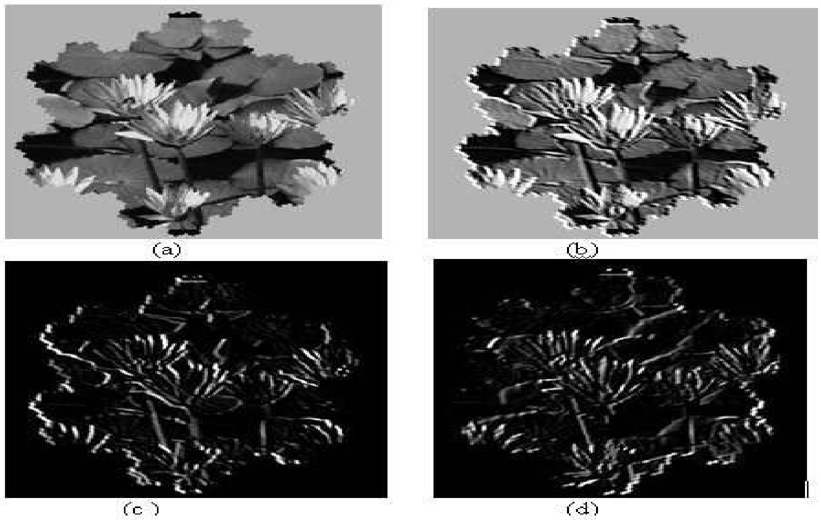


Figure 11: Edge detection using Prewitt hexagonal mask on spiral addressing scheme (a) original image (b) 0^oedges (c) 60^oedges (d) 120^oedges.

In rectangular domain, classical operators are very easy to apply .But the results shows discontinuous edges and contours. The image shows the three directional edges using prewitt hexagonal masks on spiral addressing scheme as mentioned in section II. Here the contours and curves have no discontinuities.For this implementation Python software is used.

CLAP Algorithm based edge detetcion

Using Cellular Logical Array Processing Algorithm (CLAP) , the edge detection in hexagonal grid (using half-pixel shift method) gives better results compared to rectangular domain.

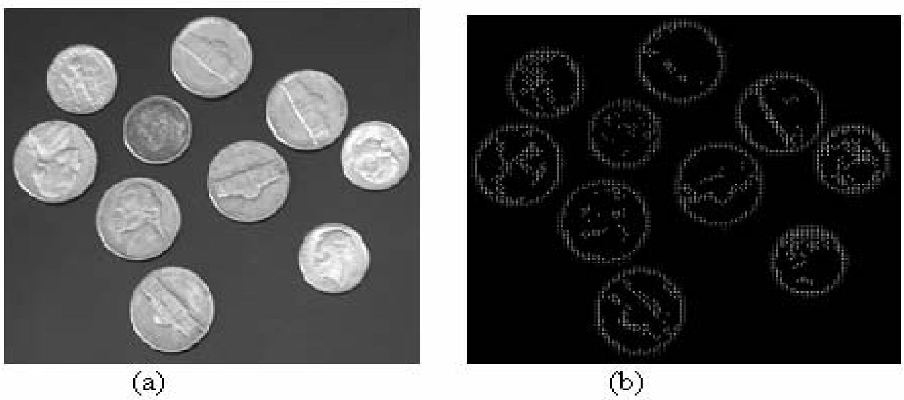


Figure 12: (a) Original Image (b) Edge Detection in Rectangular Grid.

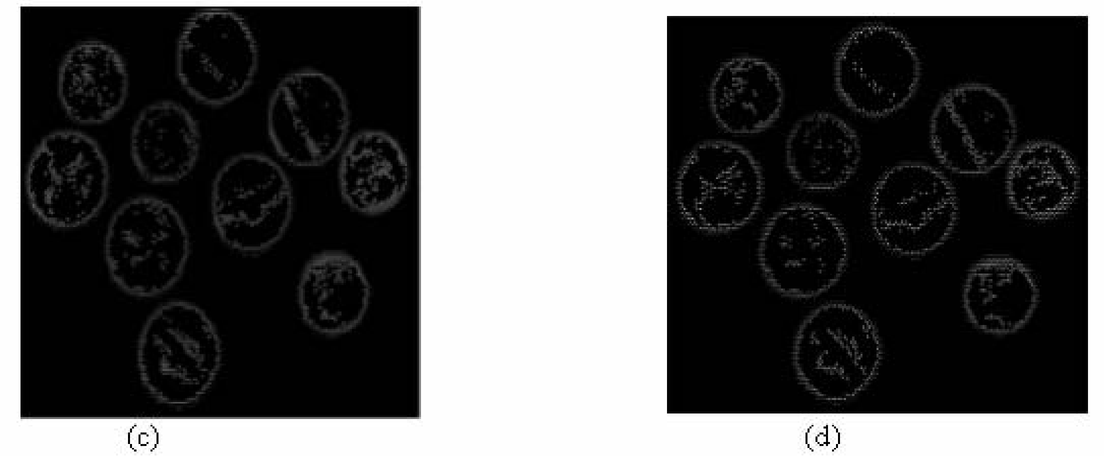


Figure 12: (c) Edge Detection in Hexagonal Grid (alternate pixel suppressal) (d) Edge Detection in Hexagonal Grid (half-pixel shift method)

Table 1: Performance measures of edge detection in rectangular and hexagonal lattice.

Test Data	Sampling	PSNR	MSE	Elapsed time(s)
Saturn.tif	Rectangular	30.77	7.8	9.8
Saturn.tif	Hexagonal(alternate pixel suppressal)	30.81	7.69	7.7
Saturn.tif	Hexagonal(half-pixel shift)	30.84	7.6	6.8
coins.png	Rectangular	23.24	14	9.96
coins.png	Hexagonal(alternate pixel suppressal)	23.36	12.8	8.3
coins.png	Hexagonal(half-pixel shift)	23.39	12	7.4
mri.tif	Rectangular	53.95	1.54	8.4
mri.tif	Hexagonal(alternate pixel suppressal)	54.202	1.519	7.2
mri.tif	Hexagonal(half-pixel shift)	54.204	1.518	6.4

Results show the CLAP Algorithm based edge detection on hexagonal domain gives good visual appeal and better performance ratio. Due to the three axis of symmetry of hexagonal domain the staircase effect is reduced in the case of hexagonal domain which leads better detection of curved edges.

Wavelet based edge detection

Application of wavelets on rectangular grid images results in the decomposition of image to various frequency components.

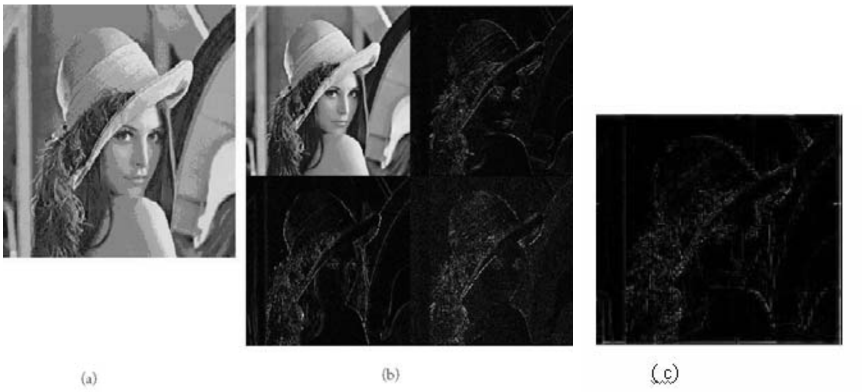


Figure 13: (a)Original Image (b)Wavelet decomposition (c)Edge detected image.

The figure shows the decomposition of lena image. In Figure. 13 upper left part shows low frequency (approximation) component and the other three gives to high frequency component which corresponds to edges. The superimposition of these three high frequency part gives the edge detected image.

Gabor filter based edge detection

Due the different axis of symmetry of hexagonal lattice we used the orientation selectivity filter like Gabor filter for the purpose of edge detection on hexagonal domain. For the comparative study gabor based edge detection on rectangular domain is also considered.

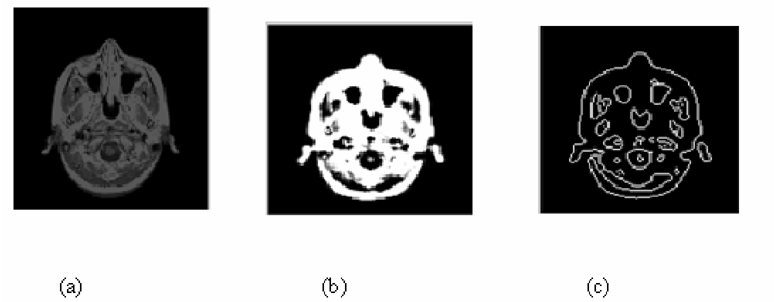


Figure 14: (a) Original Image (b) Gabor filtered image(c)Edge detected image on rectangular domain.

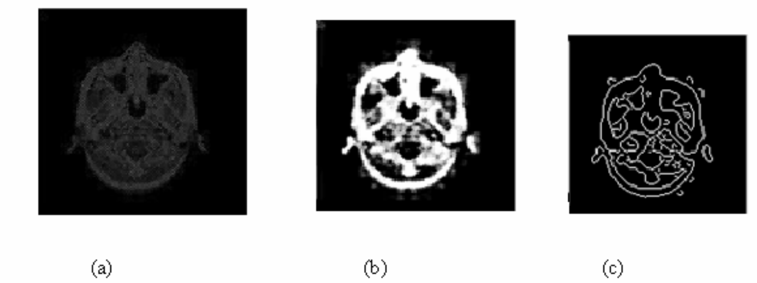


Figure 15: (a) Original Image (b)Gabor filtered image (c)Edge detected image on hexagonal domain.

Table 2: Performance measures of edge detection in rectangular and hexagonal lattice using Gabor filter.

Test Data	Sampling	PSNR	MSE	Elapsed time(s)	Ratio of edge pixels to size of image(%)
mri.tif	Rectangular	54.77	1.8	9.8	5.4
mri.tif	Hexagonal	66.14	0.6	6.8	7.4
bacteria.tif	Rectangular	42.24	7.4	9.96	5.2
bacteria.tif	Hexagonal	58.36	1.3	8.3	6
saturn.tif	Rectangular	53.95	1.54	8.4	5.5
saturn.tif	Hexagonal	66.0992	0.87	1.7	7.2

The results shows that using gabor filter the edge detection on hexagonal domain detects more edges compared to rectangular domain. Performance wise, hexagonal grid gives better PSNR, and ratio of edge pixels to size of image also elapsed time and mean square error is reduced for hexagonal grid.

Conclusions

In this work various edge detection techniques like operator based edge detection and CLAP algorithm based edge detection were implemented on both rectangular lattice and hexagonal lattice. Performance comparison shows improved results for edge detection on hexagonal lattice based on both subjective analysis and objective analysis. Gabor wavelet is found to be suitable wavelet for hexagonal domain because of its directional selectivity. Due to non-availability of hexagonal displays, we used software based approach by converting rectangular lattice into hexagonal lattice which is a limitation of our work.

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