Abstract—Orthogonal Legendre moments are used in several pattern recognition and image processing applications. Translation and scale Legendre moment invariants are achieved directly by using Legendre polynomials. Unfortunately, this method can not convert into a rotational invariant form. This is a big disadvantage and limits the use of Legendre moment invariants in many vital applications. In this paper, an indirect method is used in computing invariant Legendre moments as a linear combination of translation, scaling and rotation geometric moment invariants. Geometric moment invariants are computed exactly for symmetric as well as non-symmetric binary and gray-scale images. According to the tremendous reduction of the computational complexity, the proposed set of Legendre descriptors is suitable for large size images. The performance of these descriptors is evaluated by using a set of standard images.

Keywords—Legendre moment invariants, geometric moments, symmetric images.

I. INTRODUCTION

ORTHOGONAL Legendre moments were introduced by Teague [1]. These moments are used in a wide range of applications [2]-[7]. Teh and Chain [8] reported that, orthogonal Legendre moments can be used to represent an image with minimum amount of information redundancy. Computer vision and pattern recognition applications usually required the computation of invariant features. These features can be obtained through Legendre moment invariants.

The invariance of Legendre moments could be achieved mainly through two methods. These methods are direct and indirect. In the direct method, Legendre moment invariants are computed directly by using Legendre polynomials [9]. The first drawback of this method is the high computational demands especially with big size images and the higher order moments. A refined method is proposed [10] to overcome this problem. The other drawback is the impossibility of deriving rotational invariance property.

On the other side, the indirect method easily achieves all kinds of invariance including the rotation invariance and is highly effective when the computational complexity is tremendously reduced.

In this paper, we propose an indirect method that efficiently produce Legendre moments descriptors. The computed Legendre moments are invariants to translation, scaling and rotation. These moments works well for symmetric as well non-symmetric images. Exact Legendre moments are computed as linear combination of exact geometric moments. A fast algorithm is applied to accelerate the computation process, where the computational speed is greatly reduced. A recurrence relation is derived to avoid the direct computation of the factorial terms. The coefficient matrix is pre-computed and stored for any future use.

The rest of the paper is organized as follows: In section II, an overview of Legendre moments is given. The proposed method is described in section III. Section IV is devoted to give detailed analysis of computational complexity and some experimental results. Conclusion and concluding remarks are presented in section V.

II. LEGENDRE MOMENTS

The two-dimensional Legendre moments of order \( p + q \) for image intensity function \( f(x, y) \) are defined as:

\[
L_{pq} = \frac{(2p+1)(2q+1)}{4} \int_{-1}^{1} \int_{-1}^{1} P_p(x) P_q(y) f(x, y) dx dy,
\]

where \( P_p(x) \) is the \( p \) th-order Legendre polynomial defined as [11]:

\[
P_q(y) = \sum_{k=0}^{q} B_{k,q} y^{q-2k},
\]

where

\[
B_{k,q} = \frac{(-1)^k (2q-2k)!}{2^q k!(q-k)!(q-2k)!},
\]

Legendre polynomial coefficients are represented by \( B_{k,q} \). The operator \( \lfloor n \rfloor \) represents an integer less than or equal to \( n \). Legendre polynomial \( P_p(x) \) obeys the following recursive relation:

\[
P_{p+1}(x) = \frac{(2p+1)}{(p+1)} x P_p(x) - \frac{p}{(p+1)} P_{p-1}(x),
\]

with \( P_0(x) = 1, P_1(x) = x \) and \( p > 1 \). The set of Legendre polynomials \( \{P_p(x)\} \) forms a complete orthogonal basis set on the interval \([-1,1]\]. The orthogonality property is defined as:
A digital image of size $M \times N$ is an array of pixels. Centers of these pixels are the points $(x_i, y_j)$, where the image intensity function is defined only for this discrete set of points $(x_i, y_j)$. Since Legendre polynomials are defined only in the square $[-1,1] \times [-1,1]$, therefore, the digital image must be mapped into this square. The mapping transformations are defined as follows:

$$x_i = -1 + (i - \frac{1}{2}) \Delta x, \quad y_j = -1 + (j - \frac{1}{2}) \Delta y,$$

with $i = 1, 2, 3, \ldots \ldots M$, and $j = 1, 2, 3, \ldots \ldots N$. $\Delta x = x_i+1 - x_i$, $\Delta y = y_j+1 - y_j$ are sampling intervals in the $x$- and $y$- directions respectively. In the literature of digital image processing, the intervals $\Delta x$ and $\Delta y$ are fixed at $\frac{M}{3}$ and $\frac{N}{3}$, respectively. In the digital image, the intervals $\Delta x_i$ and $\Delta y_j$ are fixed at constant values $\Delta x_i = 2/M$, and $\Delta y_j = 2/N$ respectively. For this discrete-space version of the image, "(1)," is usually approximated by using zeroth-order approximation (ZOA) as follows:

$$\tilde{I}_{pq} = \frac{(2p+1)(2q+1)}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} P_p(x_i)P_q(y_j)f(x_i, y_j)$$

### III. INDIRECT LEGENDRE MOMENT INVARIANTS

Indirect method to get Legendre moment invariants represent the 2D Legendre moments as a linear combination of geometric moment invariants or affine moment invariants. Geometric moments are invariant to translation, non-uniform scaling and rotation.

#### A. Exact Geometric Moment Invariants

Translation invariance is achieved by shifting the image so that the image centroid $(\bar{x}, \bar{y})$ is coincides with the origin of the coordinate system. The centroid of 2D image is:

$$\bar{x} = \frac{m_{10}}{m_{00}}, \quad \bar{y} = \frac{m_{01}}{m_{00}}$$

The central moments

$$\mu_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \bar{x})^p (y - \bar{y})^q f(x, y) \, dx \, dy$$

where the $(p + q)$-order geometric moments are defined as:

$$M_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) \, dx \, dy$$

By using the binomial theorem, central geometric moments simply represented as:

$$\mu_{pq} = \sum_{k=0}^{p} \sum_{j=0}^{q} \left( \begin{array}{c} p \\end{array} \right) \left( \begin{array}{c} q \\end{array} \right) (-\bar{x})^{p-k} (-\bar{y})^{q-j} m_{kj}$$

Scaling invariance could be achieved through the cancellation of the scaling factors. By setting $\mu_{p0} = 1$, the scale-normalized moments are:

$$\mu'_{pq} = \mu_{pq} / \mu_{00}^{\frac{p+q}{2}}$$

Rotation through an angle $\theta$ about the coordinate origin is represented by the following form:

$$\eta_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x \cos \theta - y \sin \theta)^p (y \cos \theta + x \sin \theta)^q \times f(x, y) \, dx \, dy$$

By using the binomial theorem, the moments of the normalized image with respect to rotation could be written as follows:

$$\eta_{pq} = \sum_{k=0}^{p} \sum_{j=0}^{q} \left( \begin{array}{c} p \\end{array} \right) \left( \begin{array}{c} q \\end{array} \right) (-\bar{x})^{p-k} (-\bar{y})^{q-j} \times \left( \cos \theta \right)^{p-j-k} \mu'_{p+q-k-j,k+j}$$

Rotation normalization can be achieved by the major principal axis method [13] or complex moment’s method [14]. The principal axis moments are obtained by rotating the axis of the central moments until $\eta_{11}$ is zero. This method gives accurate results only in case of non-symmetric images and shapes while it fails with the N-fold symmetrical objects with N > 2. Complex moments of order $(p + q)$ for image intensity function $f(x, y)$ are:

$$c_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x + i y)^p (x - i y)^q f(x, y) \, dx \, dy$$

where $i = \sqrt{-1}$. By using the binomial theorem, each complex moment can be expressed as a combination of geometric moments of the same order or less as follows:

$$c_{pq} = \sum_{k=0}^{p} \sum_{j=0}^{q} \left( \begin{array}{c} p \\end{array} \right) \left( \begin{array}{c} q \\end{array} \right) (-1)^j i^{k+j} m_{p+q-k-j,k+j}$$

In the complex moment’s approach, both original and rotating
images have the same magnitude values of these moments, while the phase is shifted with the rotation angle as follows:
\[ c_{pq}' = e^{i(p-q)\theta} c_{pq} \]  
(18)

Rotation normalization via complex moments requires \( c_{pq}' \) being real and positive. For \( Ic_{pq} \) and \( Rc_{pq} \) are the imaginary and real parts of the complex moments \( c_{pq} \), the rotation angle \( \theta \) evaluated as follows:
\[ \theta = -\frac{1}{p-q} tan^{-1} \left( \frac{Ic_{pq}}{Rc_{pq}} \right) \]  
(19)

Any non-zero complex moment could be used for the rotation normalization process. It is preferable to keep the moment order as low as possible. An algorithm is proposed for automatic selection of the normalization non-zero complex moment, where \( \text{Max} \) is the maximum order of the moment [15].

B. Relation with Geometric Moments

The \((p + q)\)th-order Legendre moments are expressed as a combination of geometric moments of the same order or less as follows:
\[ L_{p,q} = \frac{(2p+1)(2q+1)}{4} \sum_{i=0}^{\frac{p}{2}} \sum_{j=0}^{\frac{q}{2}} B_{p,i} B_{q,j} \eta_{p-2i, q-2j} \]  
(20)

The geometric moment invariants \( \eta_{p-2i, q-2j} \) are defined by using "(15)." According to the "(3)," the expensive factorial computations are avoided by implementing the following recurrence relations:
\[ B_{0,q} = \frac{(2q-1)}{q} B_{0,q-1} , \]  
(21-1)
\[ B_{k,q} = \frac{(2q-2k-1)}{(q-2k)} B_{k,q-1} , \]  
(21-2)
\[ B_{k,q} = \frac{(q-2k-1)(q-2k-2)}{2k(2q-2k+1)} B_{k-1,q} , \]  
(21-3)

with \( B_{0,0} = 1 \) and \( k \geq 1 \).

To reduce the computational complexity, "(20)," could be rewritten in a separable form as follows:
\[ L_{p,q} = \frac{(2p+1)(2q+1)}{4} \left| \frac{\frac{p}{2}}{\frac{q}{2}} \right| \sum_{i=0}^{\frac{p}{2}} B_{p,i} Y_{q,i} \]  
(21-1)
\[ Y_{p,i} = \sum_{j=0}^{\frac{q}{2}} B_{q,j} \eta_{p-2i, q-2j} \]  
(22-2)

Legendre descriptors of the third- and fourth-order are used to test the invariance. These selected descriptors are:

Third-order:
\[ \phi_1 = \frac{35}{8} \eta_{3,0} , \]
\[ \phi_2 = \frac{45}{8} \eta_{2,1} , \]
\[ \phi_3 = \frac{45}{8} \eta_{1,2} , \]
\[ \phi_4 = \frac{35}{8} \eta_{0,3} . \]

Fourth-order:
\[ \phi_5 = \frac{9}{4} \left( \frac{35}{8} \eta_{4,0} - \frac{30}{8} \eta_{2,0} + \frac{3}{8} \eta_{0,0} \right) , \]
\[ \phi_6 = \frac{105}{8} \eta_{3,1} , \]
\[ \phi_7 = \frac{25}{4} \left( \frac{9}{4} \eta_{2,2} - \frac{3}{4} \eta_{2,0} - \frac{3}{4} \eta_{0,2} + \frac{1}{4} \eta_{0,0} \right) , \]
\[ \phi_8 = \frac{105}{8} \eta_{1,3} , \]
\[ \phi_9 = \frac{9}{4} \left( \frac{35}{8} \eta_{0,4} - \frac{30}{8} \eta_{0,2} + \frac{3}{8} \eta_{0,0} \right) . \]

IV. NUMERICAL EXPERIMENTS

In this section, numerical experiments with symmetric and asymmetric images are performed. Standard asymmetric image of baboon of size 256×256 is used in the first experiment, while the symmetric image of recycle logo of size 128×128 in the second experiment. This image is of 3-fold rotation symmetry.
Legendre descriptors of third- and fourth-order are used to test the invariance. Classical non-rotational Legendre moments [8] and the rotational proposed ones are evaluated. The absolute errors for the three different rotations are plotted in figures (2.a-c). It is clear that, the absolute error of the non-rotational descriptors show disturbed behavior and increased as the increasing of the order. On the other side, the absolute error of the proposed descriptors shows steady behavior. Their values decreased and approach zero values as the descriptor's order increased.

Similar to the first numerical experiment, the symmetric image of recycle logo is rotated by three different angles as shown in figures (3.b-d). The invariance absolute errors of the different rotation angles are plotted in figures (4.a-c). It's clear that, the errors of the proposed descriptors are steady and approach zero as the order increase. Based on these numerical experiments, both sets of plotted absolute errors ensure the validity of the proposed method.
This work proposes a new set of rotationally Legendre moment invariants. The proposed indirect method is fast where the tremendous reduction in the computational complexity enables the proposed set of descriptors to deal with large images and objects. The proposed Legendre descriptors are computed for symmetrical as well as non-symmetrical images. Numerical experiments clearly show the accuracy of these descriptors.

V. CONCLUSION

Fig. 4 Absolute invariance errors of the rotated images of the recycle logo

REFERENCES


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