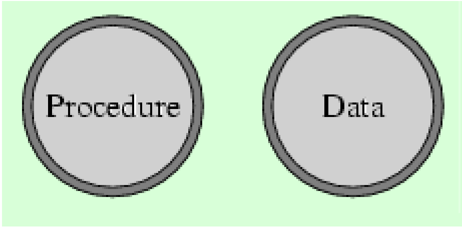
Data & Procedure Introduction

In this book, our goal is to understand *computation*. In particular, we want to be able to take any computational problem and produce a technique for solving it that is both correct and efficient.

1.1. Procedure and data

Computation is built of two basic elements: *procedure* and *data*. (See Figure 1.1.) This fact is succinctly summarized by the title of a 1970s computer science textbook,*Data Structures + Algorithms = Programs* (by Niklaus Wirth, from Prentice Hall, 1976). This binary nature is reflected in object-oriented languages such as Java: Here, each *object* has two major categories of components, *instance methods* (i.e., procedures) and *instance variables* (i.e., data).

Figure 1.1: The two sides of the computation coin.



Like energy and mass are physical phenomena united by Einstein's formula $E = m\,c^2$, procedure and data are to some extent two different ways of viewing the same thing. Computer users understand this intuitively: While we often talk about a program being a procedure, in fact a program is text representing the procedure, and that text is just a long string of characters — that is, data. The compiled program, too, is simply a file on disk containing data. We understand this intuitively today, but understanding programs as just another form of data was a breakthrough in the early hisory of computing: In the ENIAC, introduced in 1946 as the first American computer, each task required rewiring the hardware, until in 1948, when the ENIAC was wired to be able to read programs as data from its own memory. This concept, called the von Neumann computer, quickly caught on. (In the reverse direction, Alonzo Church developed the theory of lambda calculus in the 1930s to study the fundamentals of computing. In his system, each integer and indeed every other piece of data is represented as a procedure.)

Any computational problem will involve both data and procedure in its solution. Sometimes the data portion or the procedure portion will be simple; but it is still present. Take, for example, the computational problem of determining whether a number is prime. Before we can begin a procedure for determining this, we must first have the number in question, which after all is data. The representation of this data is simple but important. You might assume that the number would arrive us in its Arabic representation (e.g., 1003). But the problem is somewhat different if it comes in Roman representation (MIII) or Chinese representation (一千零三).

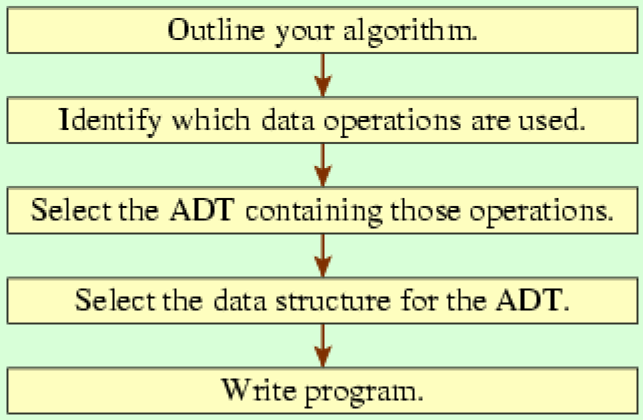
In fact, the representation of data can dramatically affect the efficiency of a procedure. Suppose, for example, if instead of an Arabic representation, the number is instead given us in terms of its prime factorization (17 · 59). Then the procedure will be much more efficient: We simply to check whether the prime factorization contains more than one number.

This theme of how data should be represented will form a major portion of this book. We won't spend too much time on simple data like numbers, though: Our focus will be on representing a large collection of data so that it can be processed efficiently. Such techniques for storing data collections are called data structures, and we'll see several throughout the book.

1.1.1. Abstract data types

But before we get to data structures, we first need the concept of an abstract data type (often abbreviated ADT), which refers to a particular set of operations we want to be able to perform on a collection. Given an algorithm to solve a problem using a collection of data, we should be able to identify which operations are necessary on that collection, and then we find the ADT that matches it. Once we know which ADT to use, we can then select the appropriate data structure based on others' work concerning how best to implement the ADT. This program design process is summarized by the following diagram.

Figure 1.2: The steps to developing a data-intensive program.


Outline your algorithm.
Identify which data operations are used.
Select the ADT containing those operations.
Select the data structure for the ADT.
Write program.

In this book, we'll examine several ADTs and the data structures commonly used for them. One ADT is the Set ADT, intended for algorithms that use a data collection like a mathematical set. Operations in the Set ADT include:

- `size` to get the number of elements in the set.
- `contains(x)` to test whether a particular element `x` lies within the set.
- `add(x)` to insert an element `x` into the set, returning **`true`** if `x` is newly added. (As a set, we wouldn't add `x` if it were already present.)
- `remove(x)` to remove an element `x` from the set.

This ADT is quite useful; our study of it will be deferred, though, to Chapter 5.

Another ADT that we'll study is the List ADT, where elements are stored in a particular order. The List ADT contains the following operations.

- `size` to get the number of elements in the list.
- `add(x)` to add a value `x` to the end of a list.
- `get(i)` to fetch the element at index `i` of the list.
- `set(i, x)` to change the element at index `i` of the list to be `x` instead.
- `add(i, x)` to insert an element `x` into the list at index `i`.
- `remove(i)` to remove an element at index `i` from the list.

Although the List and Set ADTs are similar, they have some important differences. With the List ADT, each element is considered to be at a particular index, whereas the Set ADT has no notion of ordering; on the other hand, the Set ADT includes a `contains` operation that is absent from the List ADT.

In the course of this book, we'll study both of these ADTs along with others that people have found useful over the years. Our primary emphasis, though, will be on the best data structures to use for implementing them. This understanding enables writing efficient algorithms for many important problems. We'll study a sampling of these problems in the course of this book.

1.1.2. Interfaces

You might notice that the ADT concept resembles Java's concept of *interface*: Both are abstract sets of operations. The concepts are not entirely equivalent, though. One difference is that the ADT concept is intended to be language-neutral; the language doesn't need a construct similar to Java interfaces for the ADT concept to be useful. Another difference is that ADTs are really meant only for large masses of data, whereas Java programs use interfaces for other purposes, too.

If we're programming in Java, though, it's easy enough to define an ADT in terms of an interface.

```
public interface Set<E> {
    public int size();
    public boolean contains(E value);
    public boolean add(E value);
    public E remove(E value);
}

public interface List<E> {
    public int size();
    public boolean add(E value);
    public E get(int index);
    public E set(int index, E value);
    public void add(int index, E value);
    public E remove(int index);
}
```

Indeed, these interfaces are already built into Java's java.util package — but they include many more methods beyond those listed above, too. (In this book, we'll study many of the interfaces and classes defined in the java.util package. Figure 1.3 lists all of those we'll study.)

Figure 1.3: Parts of the java.util package appearing in this book.

Interfaces	**Classes**
List	ArrayList
Set	LinkedList
Iterator	TreeSet
ListIterator	TreeMap
Map	HashSet
	HashMap

Perhaps you have not before seen the `<E>` notation used above. These definitions use a feature of Java called *generics*, where we can designate an identifier — `E` in this case — to represent an arbitrary class; that is, the identifier acts something like a variable that stands for a type. When we use the interface, we'll specify what class the variable `E` should be taken to stand for. For List and Set, the individual elements of the collection will be instances of this class. (In fact, the *E* stands for *element*.) As an example of using these generics, if we want to declare a variable `names` to represent a list of strings, we could write the following.

```
List<String> names;
```

We can later create a List of Strings and assign `names` to refer to it. Then, `names`'s `get` method will return a String, since, after all, the `get` method's return type is `E`, and in`names`'s case, `E` refers to the String type.

```
String first = names.get(0);
```

If the notion of generics is unfamiliar to you, then you should read a Java book and become familiar with the concept. (Generics were introduced with Java 5; you won't find much information about the concepts in books preceding its release date of Summer 2004. Previous to Java 5, the java.util classes worked with Objects.)

Before continuing, note quickly that the Set and List interfaces contain several methods in common. The interface designers carefully named them the same so that programmers could remember them more easily. One peculiar thing about the commonality is the `add` method. In the Set interface, the method returns a **`boolean`**, so that it can indicate whether the value needed to be added to the set. (A set doesn't include any elements twice.) Just to be consistent, the designers made List's `add`method return a **`boolean`** also, but in fact `add`'s return value will always be **`true`** with a List.

To use the List interface, we need some way of getting at List objects. Of course, since List is an interface, we can't write new List to create List objects in a program. We can only create List objects through creating instances of an implementation of the List interface. We'll see one such implementation next: the ArrayList class. In the next chapter, we'll see another implementation, the LinkedList class.

1.2. The ArrayList data structure

The *array* is the fundamental data structure built into Java for holding collections of items. Arrays provide two basic operations: You can retrieve the value at an index, or you can change the value at an index. These are quite similar to the List ADT's `get` and `set` operations.

array operation	equivalent List ADT operation
`x = data[i];`	`x = get(i)`
`data[i] = x;`	`set(i, x)`

This similarity is what inspires the implementation of the List interface using an array to store the data elements. There are other List operations that don't correspond to anything we can do with an array (`add` and `remove`); but this is a detail that we can hope to overcome. Overall, it seems like an array is a good candidate for implementing the List interface.

Java's built-in libraries already include such an implementation; specifically, the java.util package includes the ArrayList class. To use it, you need to include a **`import`**statement at the top of your program definition.

```
import java.util.ArrayList;
```

Like the List interface, the ArrayList class is a generic. Inside a method definition, we can declare a `names` variable and then assign it to refer to a newly created ArrayList object.

```
List<String> names;
names = new ArrayList<String>();
```

Once we have a way of referring to an ArrayList object, then we can tell it to do things using the ArrayList methods, which necessarily include all the methods in the List interface. Among these is the `add` method.

```
names.add("Martin");
names.add("Galloway");
```

Suppose we want to display all the strings in the ArrayList. List's `size` and `get` methods are useful for this.

```
for(int i = 0; i < names.size(); i++) {
    System.out.println(names.get(i));
}
```

1.2.1. Relationship to arrays

It's important to remember that despite the name, arrays are completely unrelated to ArrayLists. (Ok, there is something of a relationship: The programmers who wrote the ArrayList class implemented it using arrays. But this is **`private`** information in the class, and normal programmers don't ever look inside the ArrayList implementation, so this relationship doesn't have any direct impact when programming.)

This lack of relationship means that you can't confuse the syntax for accessing them. If you have an ArrayList *`names`*, you cannot use the subscripts used with arrays, as in `names[0] = "Harpo";`; instead, you must use the more cumbersome notation `names.set(0, "Harpo");`. An ArrayList, after all, is a regular object, and thus the only operation you can perform on it is to access its public methods (and public variables, if it had any). Similarly, if `data` is a `String[]` variable — that is, an array of Strings —, you cannot write `data.add("Harpo");` to grow the array. An array's size is fixed at the time of the array's creation.

When we're programming, how should we decide between using an array and an ArrayList? While arrays are more convenient for accessing elements by index, that convenience comes at the price of missing the List methods (particularly the automatic-growth behavior of `add`). The fundamental question to ask, then, is whether you are confident that the array's size will not change once the collection is created. If so, then you should choose to use an array. But if the structure might need to adapt its size, you should choose an ArrayList; you won't get to use the special array syntax for accessing elements, but the automatic growth will make up for it.

ArrayLists offer one other advantage over arrays: They have some additional methods that we haven't covered that can be useful. The most notable is the `indexOf`method, which takes an element as a parameter and returns the index at which that element first occurs in the list — or, if it doesn't occur at all, −1. Even if you have a fixed-length structure, then, the ArrayList may still be a better choice.

1.2.2. Wrapper classes

A significant limitation of ArrayLists (and List and Set implementations generally) is that they can contain only objects. This isn't usually much of a limitation, since virtually every data item in Java is an object. But Java includes a few types that aren't classes (i.e., the type of an object). These are called *primitive types*, and Java includes eight, including **`int`**, **`boolean`**, and **`double`**. (The other five are **`char`**, **`byte`**, **`short`**, **`long`**, and **`float`**.) Java distinguishes the primitive types from the object types by using names starting with lower-case letters, while classes' names are conventionally capitalized.

So we can't put an **`int`** into an ArrayList. But what if we *want* a list of integers? For cases like this, Java provides a wrapper class for each of the eight primitive types; each instance of the wrapper class is an object with a single instance variable, whose type is the corresponding primitive type. The wrapper class for **`int`** is named*Integer*. While `ArrayList<int>` is illegal because **`int`** is not a class, Integer is a class, and so `ArrayList<Integer>` is fine.

```
ArrayList<Integer> primes = new ArrayList<Integer>();
```

We can now add Integers into the list. But remember that an **`int`** is not an Integer, so we cannot write `primes.add(2);`. Fortunately, we can easily create an Integer object wrapped around an **`int`**.

```
primes.add(new Integer(2));
```

Java (since Java 5) has a feature called *autoboxing*, where the compiler will automatically convert between Integers and **`int`**s when necessary. Thus, we actually *can* write `primes.add(2);`. The compiler will notice the bug and silently translate it to `primes.add(new Integer(2));`. Similarly, the compiler will add a call to `intValue` were we to write `primes.get(0) + 1`.

Sounds nice, no? But it has some major problems. One such is that `==` and `!=` work differently for objects, so that `primes.get(0) == 2` is *false*, whereas `primes.get(0).intValue() == 2` is *true*. (In fact, the situation is even more complex: The first example would be *true* on some computers and *false* on others.) For this, and for other reasons, I advise against using autoboxing, and this book will not make use of it.

Integer objects are not eligible for mathematical operations like addition. Thus, when we extract an Integer from the ArrayList, we will probably want to get the **`int`** value it is wrapped around. The Integer class's `intValue` method returns this **`int`**.

```
int three = primes.get(0).intValue() + 1;
```

As you can see, having to use Integers rather than **`int`** is a bit of a pain — but with practice, the pain isn't really much of a problem.

There are no methods in the Integer class for altering the value inside the wrapper object. If you want a wrapper object for a different value, you must create a wholly new instance of the wrapper class.

1.2.3. Using ArrayList

As an example of a problem where the List ADT is useful, consider the problem of finding the number of primes up to and potentially including n. Mathematicians call this function $\pi(n)$; for example, $\pi(11)$ is 5 because four primes (2, 3, 5, 7, 11) are at most 11. I realize that it's a stretch to think of a scenario where somebody's livelihood depends on this quantity, but still it's an interesting problem.

Incidentally, mathematicians have proven that $\pi(n)$ is approximately n / ln n. Figure

1.4 tabulates a few values of π(n) (computed using our program) to check how close this approximation is.

Figure 1.4: A sampling of values for π(n).

n	**π(*n*)**	***n* / ln *n***	**error**
100	25	21.7	13.1%
1,000	168	144.8	13.4%
10,000	1,229	1,085.7	11.7%
100,000	9,592	8,685.9	9.4%
1,000,000	78,498	72,382.4	7.8%
10,000,000	664,579	620,420.7	6.6%

You can see that the error appears to be slowly converging to 0. The mathematical proof, though, is more difficult: Legendre conjectured the formula in 1798, and it was proven nearly a century later in 1896, independently by Hadamard and de la Vallée Poussin.

The algorithm we'll use, implemented in Figure 1.5, involves creating a collection of all the primes up to n. (We'll see later, in Section 4.1, that this isn't the most efficient algorithm.)

We'll start with the collection containing just the number 2, and then we'll iterate through the odd numbers up to n. For each odd number k, we'll determine whether it belongs in the collection by seeing whether any of the primes already present divide into k.

Figure 1.5: Counting primes using the ArrayList class.

```java
// Returns the number of primes that are <= n.
public static int countPrimesTo(int n) {
    List<Integer> primes = new ArrayList<Integer>(n);
    primes.add(new Integer(2));
    for(int k = 3; k <= n; k += 2) {
        if(testPrimality(k, primes)) primes.add(new Integer(k));
    }
    return primes.size();
}

// Tests whether k is prime using the primes listed.
private static boolean testPrimality(int k, List<Integer> primes) {
    for(int i = 0; true; i++) {
        int p = primes.get(i).intValue();
        if(p * p > k) return true;    // we passed sqrt(k); k is prime
        if(k % p == 0) return false; // we found a divisor; k is not prime
    }
}
```

The algorithm used in Figure 1.5 has a small bug. Do you see it? It illustrates a common programming problem: Whenever you write a program, you should thank about how it works for the extreme cases. Frequently, a program that works for most cases will fail on the easiest cases. In this case, the program works wrongly when*n* is given as 1: It will report that there is a prime less than or equal to 1, when in fact the first prime is 2. This bug is easy to fix: We can just add an `if` statement as the first line saying to return 0 if *n* is less than 2.

The implementation takes advantage of a simple but major efficiency help: In testing whether a number *k* is prime, we need only check for factors up to *sqrt*(*k*). After all, if *k* has a factor above *sqrt*(*k*) — call it *p* — then *k* / *p* would also be a factor, and *k* / *p* would be less than *sqrt*(*k*) since *p* > *sqrt*(*k*). Thus, if *k* has any factors, some of them must be less than or equal to *sqrt*(*k*).

Initially, the Set ADT may seem a more appropriate choice for representing the collection of primes, since the index of each individual prime doesn't matter. But for the efficiency improvement to take hold, we would like to be able to go through the collection in ascending order so that we can stop once we reach *sqrt*(*k*). (Also, checking smaller primes first is a good idea because these are more likely to divide large numbers.) The List ADT allows us to keep the primes in order as we would like.

1.2.4. Implementing the ArrayList class

Programmers can theoretically use ArrayLists without really understanding how they work: After all, Java developers have already built the class into the java.util package. *Good* programmers, though, need to understand the class's internals in order to understand the performance of ArrayList's methods.

Figure 1.6 contains a possible way of writing the ArrayList class so that it implements all of the List methods we discussed earlier.

Figure 1.6: An ArrayList implementation.

```
public class ArrayList<E> implements List<E> {
    private E[] elements; // values stored in current list
    private int curSize;  // number of element in current list

    public ArrayList(int capacity) {
```

```java
        elements = (E[]) new Object[capacity]; // ignore this
        curSize = 0;
    }

    public int size() {
        return curSize;
    }

    public E get(int index) {
        return elements[index];
    }

    public E set(int index, E value) {
        E old = elements[index];
        elements[index] = value;
        return old; // should return previous value at index
    }

    public boolean add(E value) {
        // see text about handling case when array is full
        elements[curSize] = value;
        curSize++;
        return true;
    }

    public void add(int index, E value) {
        // see text about handling case when array is full
        for(int i = curSize - 1; i >= index; i--) {
            elements[i + 1] = elements[i];
        }
        elements[index] = value;
        curSize++;
    }

    public E remove(int index) {
        E old = elements[index];
        for(int i = index + 1; i < curSize; i++) {
            elements[i - 1] = elements[i];
        }
        curSize--;
```

```
64          return old; // should return previous value at index
65      }
70  }
```

(The ignore this comment of line 6 refers to the peculiar technique that appears for creating an array of generic objects. We want to say`elements = new E[capacity]`, but Java doesn't handle this relatively obscure construct properly. Thus, line 6 must work around Java's limitation. If the workaround doesn't make sense to you, don't worry about it.)

As you can see, it involves two instance variables: `elements`, an array containing all of the list's current values, and `curSize`, the number of elements currently in the list. Conceptually, the ArrayList starts out empty, but it will grow through calls to the `add` method. Behind the scenes, though, invisible to programmers using the class, the array is created once when the object is created, and rather than grow, the class simply uses more of the array that was created at the outset.

The `size` and `get` methods are straightforward; the `set` method is only slightly more complicated because the List interface specification requires that the method return the value previous to the requested change at that index. The `remove` method is complicated by the need to shift all of the elements forward in the array over the removed item.

The implementation in Figure 1.6 has an important limitation: We can only add as many elements as we specify at the time we construct the array. The array can'tgrow after being created in line 6 — and, indeed, Java arrays have no way of growing once they are created.

The ArrayList class built into the java.util package gets around this by instead using an `add` method similar to that of Figure 1.7.

Figure 1.7: Modifying `add` to extend array when full.

```
public boolean add(E value) {
    ensureElements(curSize + 1);
    elements[curSize] = value;
    curSize++;
    return true;
}

// Ensures that elements array has the desired length by
// creating a longer copy if necessary.
private void ensureElements(int desired) {
    if(elements.length < desired) {
        E[] newElements = (E[]) new Object[2 * elements.length];
        for(int i = 0; i < elements.length; i++) {
            newElements[i] = elements[i];
        }
        elements = newElements;
    }
}
```

With this method, once the current array becomes filled, it switches to a new, longer array, being careful to copy the old values into the new array. As the programmer, you don't need to worry about when this happens: It all happens behind the scenes, transparently. This process of creating a new array and copying all of the old values takes a lot of time, but this happens quite rarely — and, if we double the length of the array each time, it becomes increasingly rare as the array becomes longer. Overall, then, it doesn't really affect performance dramatically.

The ArrayList class also includes a constructor including no arguments, which starts the capacity out at a small value (10).

```
public ArrayList() {
    this(10); // enters the other constructor, giving capacity of 10
}
```

Using this constructor, the programmer doesn't have to guess the list's initial capacity. Instead, using the amended `add` method, the program will simply grow the array to be large enough to fit the actual data.

Source: http://www.toves.org/books/data/ch01-intro/index.html