SUPERPOSITION: ANALYZING CIRCUITS WITH MULTIPLE SOURCES

Caveat: Superposition only works for linear circuits. Circuits with R, L, C.

The current or voltage at any point in a circuit containing multiple sources (current and/or voltage) is the superposition (sum) of the currents or voltages imposed separately by each source.

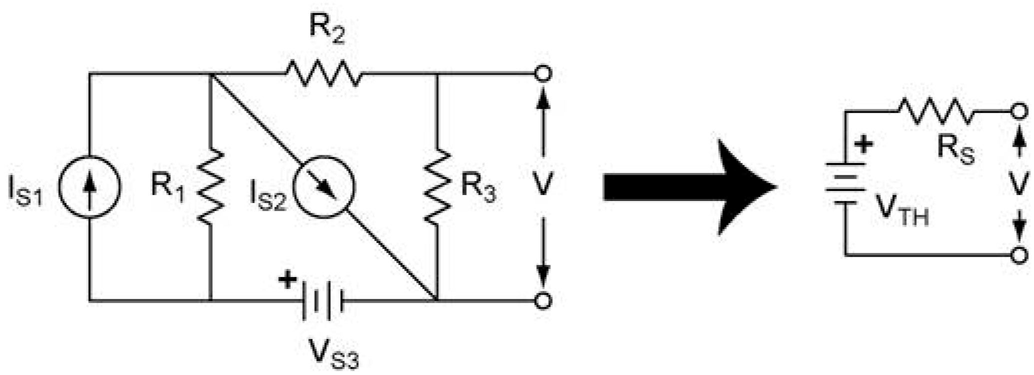
Procedure for analyzing a circuit using superposition.

- Turn on the sources one at a time.
- For each source that is "off", replace it with its characteristic resistance. (Ideal voltage sources R=0, a short circuit. Ideal current sources R=∞, an open circuit.)
- Analyze for the desired quantity due to each source.
- The full solution for all sources active is the sum of the contributions due to each source individually.
- Carefully observe the signs of the individual voltages and currents. The polarity of the voltages and the direction of the currents may not be the same for all sources.

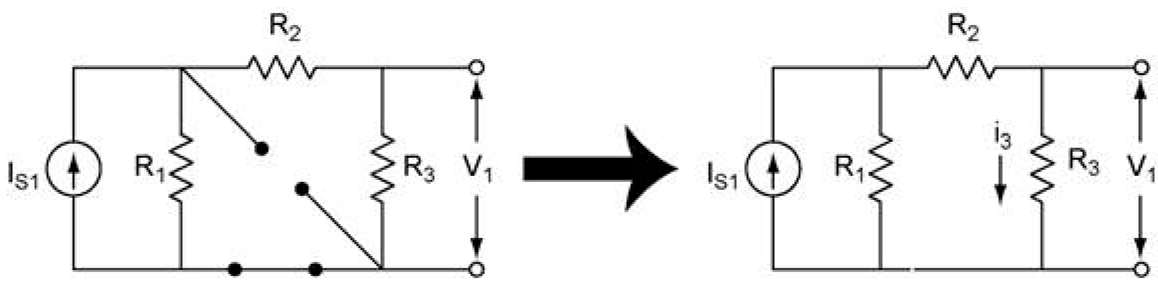
Example: Find the Thevenin equivalent voltage, V_{TH} and the source resistance R_S of the following network as seen at the terminals. Because V_{TH} is the open circuit voltage, we simply need the voltage at the terminals, V, which is the same as the voltage across R_3.

Because the network has 3 internal sources we will use superposition to solve the problem. Superposition says that V is the sum of the voltages contributed from each of the 3 sources. $V = V_1 + V_2 + V_3$. We will determine the voltage contribution from each voltage separately by turning on only one source at a time. Sources that are "off" are replaced with their characteristic resistance.

Find V_{TH} and R_S using superposition.



Solving for V_1 due to I_{S1}.



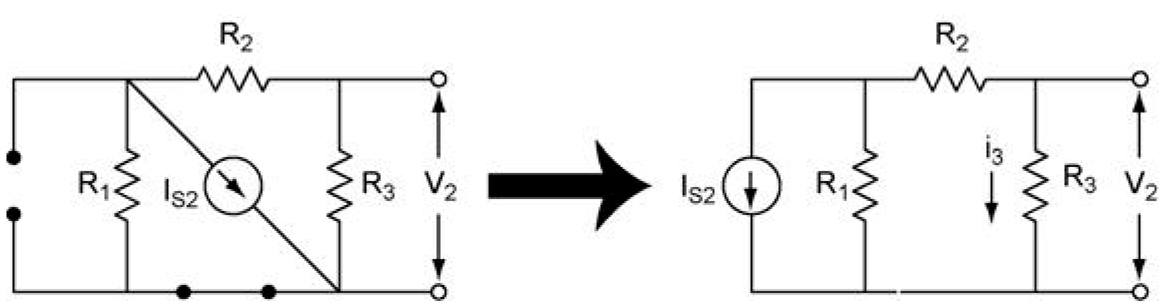
The terminal voltage is the voltage drop across R_3. We also note that R_1 and R_2+R_3 form a current divider. To find V_1 we need i_3 which we can find from the current divider equation.

$$V_1 = i_3 R_3 \qquad i_3 = i_{S1}\left(\frac{R_1 \| (R_2+R_3)}{R_2+R_3}\right) = i_{S1}\left(\frac{R_1(R_2+R_3)}{R_1+(R_2+R_3)}\right)\left(\frac{1}{R_2+R_3}\right) = i_{S1}\left(\frac{R_1}{R_1+R_2+R_3}\right)$$

$$V_1 = i_3 R_3 = i_{S1}\left(\frac{R_1}{R_1+R_2+R_3}\right)R_3 = i_{S1}\frac{R_1 R_3}{R_1+R_2+R_3}$$

$$\boxed{V_1 = i_{S1}\frac{R_1 R_3}{R_1+R_2+R_3}}$$

Solving for V_2 due to I_{S2}.



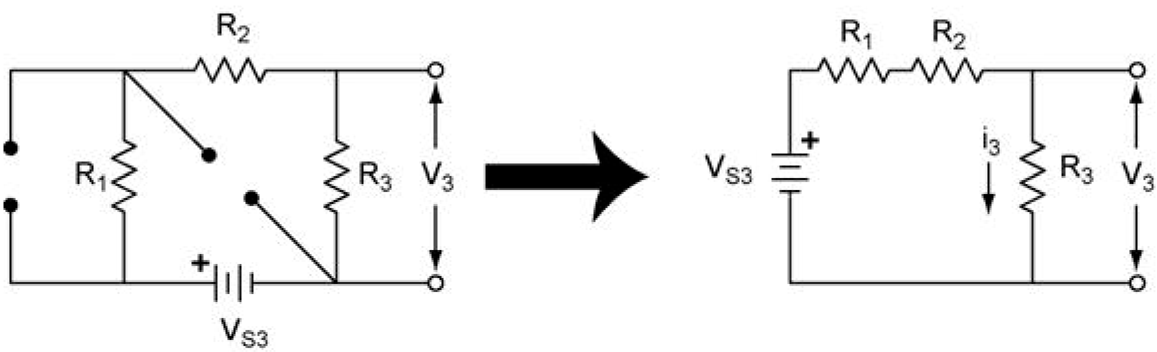
We see that circuit reduces to almost the same problem as for I_{S1}, except that the direction of the current is opposite. We have chosen to keep the same direction of i_3 to remind us that we need a negative sign. The lower terminal will obviously be more positive that the upper terminal. We must be very careful of the signs!

$$V_2 = i_3 R_3 \qquad i_3 = -i_{S2}\left(\frac{R_1 \| (R_2+R_3)}{R_2+R_3}\right) = i_{S2}\left(\frac{R_1(R_2+R_3)}{R_1+(R_2+R_3)}\right)\left(\frac{1}{R_2+R_3}\right) = -i_{S2}\left(\frac{R_1}{R_1+R_2+R_3}\right)$$

$$V_2 = i_3 R_3 = -i_{S2}\left(\frac{R_1}{R_1+R_2+R_3}\right)R_3 = -i_{S2}\frac{R_1R_3}{R_1+R_2+R_3}$$

$$\boxed{V_2 = -i_{S2}\frac{R_1R_3}{R_1+R_2+R_3}}$$

Solving for V_3 due to V_{S3}.



Again, the terminal voltage is the voltage drop across R_3. However we also recognize that R_1+R_2 and R_3 form a voltage divider. To find V_3 we simply use the voltage divider equation.

$$V_3 = V_{S3}\left(\frac{R_3}{(R_1+R_2)+R_3}\right) = V_{S3}\left(\frac{R_3}{R_1+R_2+R_3}\right)$$

$$\boxed{V_3 = V_{S3}\frac{R_3}{R_1+R_2+R_3}}$$

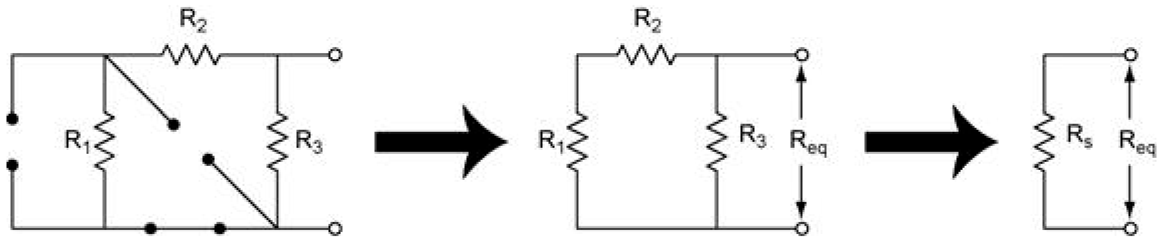
The V_{TH} of this network as seen at the terminals is the sum of each of these component voltages. Be careful of the signs!!

$$V_{TH} = V_1 + V_2 + V_3$$

$$= i_{S1}\frac{R_1R_3}{R_1+R_2+R_3} - i_{S2}\frac{R_1R_3}{R_1+R_2+R_3} + V_{S3}\frac{R_3}{R_1+R_2+R_3} = \frac{R_3}{R_1+R_2+R_3}\left[R_1(i_{S1}-i_{S2})+V_{S3}\right]$$

$$V_{TH} = \frac{R_3}{R_1 + R_2 + R_3}\left[R_1\left(i_{S1} - i_{S2}\right) + V_{S3}\right]$$

To solve for the source resistance, *R*$_S$, we simply turn off ALL of the sources and solve for the equivalent resistance of the resulting resistor network.



$$R_S = \left(R_1 + R_2\right)\| R_3 = \frac{\left(R_1 + R_2\right)R_3}{\left(R_1 + R_2\right) + R_3} = \frac{\left(R_1 + R_2\right)R_3}{R_1 + R_2 + R_3}$$

$$R_S = \frac{\left(R_1 + R_2\right)R_3}{R_1 + R_2 + R_3}$$

Source : http://www.nhn.ou.edu/~bumm/ELAB/Lect_Notes/Superposition_v1_1_2.html