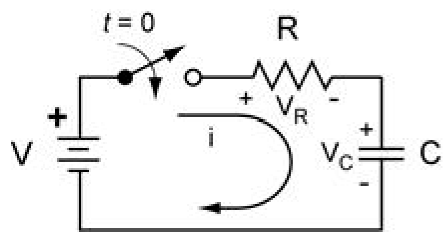
RC AND RL CIRCUITS

Resistors, Capacitors, and Inductors

Summary			
	resistors	capacitors	inductors
series	$R_{EQ} = R_1 + R_2$	$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2}$	$L_{EQ} = L_1 + L_2$
parallel	$\frac{1}{R_{EQ}} = \frac{1}{R_1} + \frac{1}{R_2}$	$C_{EQ} = C_1 + C_2$	$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2}$
stored energy	0	$W_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$	$W_M = \frac{1}{2}Li^2$
DC steady state	$V = iR$	$i = 0$ no current through C	$V = 0$ no voltage drop across L
transient	$V = iR$	$i = C\frac{\mathrm{d}V}{\mathrm{d}t}$ $V = \frac{1}{C}\int i\,\mathrm{d}t$	$V = L\frac{\mathrm{d}i}{\mathrm{d}t}$ $i = \frac{1}{L}\int V\,\mathrm{d}t$
time constant		$\tau = RC$	$\tau = \frac{L}{R}$
continuous variable		V	i

First-Order Transient Response in RC and in RL Circuits.
These two circuits illustrate the basic first-order RC and RL circuits.

RC EXAMPLE



Before the switch is closed. $V_C = 0$ and $i = 0$. Because capacitors can store electrical energy, the capacitor could have an initial voltage that is not zero. Clearly no current can flow before the switch is closed. From KVL we note that the voltage across the switch is V.

The initial state (immediately after the switch is closed). A current will begin to flow to charge C. At the instant after the switch is closed, ($t = 0+$) $V_C=0$ so all of the voltage drop appears across R. Thus the initial charging current is $i = V/R$. (To determine the initial state, C is modeled as a voltage source.)

The final state (DC steady state). After the switch has been closed for a long time, the capacitor is completely charged ($V_C = V$) and the current has decayed to zero ($i = 0$). In this limit C is modeled as an open circuit.

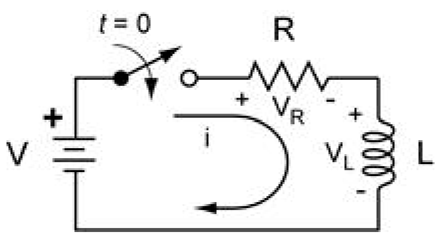
The continuous variable. Capacitors store energy. Because the stored energy cannot be changed instantaneously, change requires time. For capacitors, V_C is the circuit variable directly related to the stored electrical energy. This means that the voltage across the capacitor the instant before the switch is closed and the instant after the switch is closed are the same, $V_C(t=0-) = V_C(t=0+)$.

The transient response. The transient response is the description of how the system evolves from the initial to the final state. We can write a differential equation from KVL, substituting, and then differentiating and dividing by R. The system evolves from the initial to the final state with a characteristic time constant $\tau = RC$.

$$V_R + V_C = V; \qquad iR + \frac{1}{C}\int i\,\mathrm{d}t = V$$

$$RC\frac{\mathrm{d}i}{\mathrm{d}t} + i = 0; \qquad i(t) = i(0+)e^{-t/RC}$$

RL EXAMPLE



Before the switch is closed. $V_L = 0$ and $i = 0$. Although inductors can store magnetic energy, this requires a flow of current through the inductor. Clearly no current is flowing before the switch is closed. From KVL we note that the voltage across the switch is V.

The initial state (immediately after the switch is closed). The current will begin to change, however the inductor opposes the change in current. At the instant after the switch is closed, ($t = 0+$) the current must still be zero, $i = 0$ so all of the voltage drop appears across L. Thus the initial $V_L = V$ (To determine the initial state, L is modeled as a current source.)

The final state (DC steady state). After the switch has been closed for a long time, V_L has decayed to zero ($V_L = 0$) and the current is constant ($i = V/R$). In this limit L is modeled as a short circuit.

The continuous variable. Inductors store energy. Because the stored energy cannot be changed instantaneously, change requires time. For inductors, i_L is the circuit variable directly related to the stored magnetic energy. This means that the current through the inductor the instant before the switch is closed and the instant after the switch is closed are the same, $i_L(t=0-) = i_L(t=0+)$.

The transient response. The transient response is the description of how the system evolves from the initial to the final state. We can write a differential equation from KVL, substituting, and then rearranging and dividing by R. The system evolves from the initial to the final state with a characteristic time constant $\tau = L / R$.

$$V_R + V_L = V; \qquad iR + L\frac{\mathrm{d}i}{\mathrm{d}t} = V$$

$$\frac{L}{R}\frac{\mathrm{d}i}{\mathrm{d}t} + i = \frac{V}{R}; \qquad i(t) = i(\infty) - i(\infty)e^{-tR/L}$$

Source : http://www.nhn.ou.edu/~bumm/ELAB/Lect_Notes/RC_RL_transients_v1_2_3.html