Module 5

Carrier Modulation

Lesson 25

Quaternary Phase Shift Keying (QPSK) Modulation

After reading this lesson, you will learn about

- ***Quaternary Phase Shift Keying (QPSK);***
- ***Generation of QPSK signal;***
- ***Spectrum of QPSK signal;***
- ***Offset QPSK (OQPSK);***
- ***M-ary PSK;***

Quaternary Phase Shift Keying (QPSK)

This modulation scheme is very important for developing concepts of two-dimensional I-Q modulations as well as for its practical relevance. In a sense, QPSK is an expanded version from binary PSK where in a symbol consists of two bits and two orthonormal basis functions are used. A group of two bits is often called a 'dibit'. So, four dibits are possible. Each symbol carries same energy.

Let, E: Energy per Symbol and T: Symbol Duration = 2. T_b, where T_b: duration of 1 bit. Then, a general expression for QPSK modulated signal, without any pulse shaping, is:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1).\frac{\pi}{4}\right] + 0 ; \quad 0 \le t \le T; \; i = 1,2,3,4 \qquad 5.25.1$$

where, $f_c = n.\frac{1}{T} = n.\frac{1}{2T_b}$ is the carrier (IF) frequency.

On simple trigonometric expansion, the modulated signal s_i (t) can also be expressed as:

$$s_i(t) = \sqrt{\frac{2E}{T}}.\cos\left[(2i-1)\frac{\pi}{4}\right].\cos 2\pi f_c t - \sqrt{\frac{2E}{T}}.\sin\left[(2i-1)\frac{\pi}{4}\right].\sin 2\pi f_c t \,; \; 0 \le t \le T \qquad 5.25.2$$

The two basis functions are:

$$\varphi_1(t) = \sqrt{\frac{2}{T}}.\cos 2\pi f_c t \;; 0 \le t \le T \quad \text{and} \quad \varphi_2(t) = \sqrt{\frac{2}{T}}.\sin 2\pi f_c t \;; 0 \le t \le T \qquad 5.25.3$$

The four signal points, expressed as vectors, are:

$$\overline{s_i} = \left\{\sqrt{E}\cos\left[(2i-1)\pi/4\right] - \sqrt{E}\sin\left[(2i-1)\pi/4\right]\right\} = \begin{bmatrix} s_{i1} \\ s_{i2} \end{bmatrix} \quad ; \quad i = 1,2,3,4 \qquad 5.25.4$$

Fig.5.25.1 shows the signal constellation for QPSK modulation. Note that all the four points are equidistant from the origin and hence lying on a circle. In this plain version of QPSK, a symbol transition can occur only after at least T = $2T_b$ sec. That is, the symbol rate $R_s = 0.5R_b$. This is an important observation because one can guess that for a given binary data rate, the transmission bandwidth for QPSK is half of that needed by BPSK modulation scheme. We discuss about it more later.

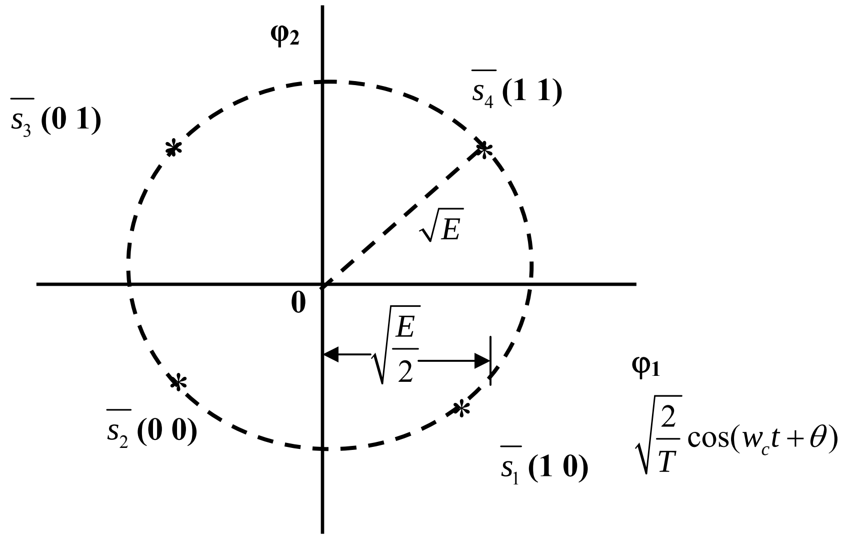

Fig.5.25.1 *Signal constellation for QPSK. Note that in the above diagram θ has been considered to be zero. Any fixed non-zero initial phase of the basis functions is permissible in general.*

Now, let us consider a random binary data sequence: 10111011000110… Let us designate the bits as 'odd' (b_o) and 'even' (b_e) so that one modulation symbol consists of one odd bit and the adjacent even bit. The above sequence can be split into an odd bit sequence (1111001…) and an even bit sequence (0101010…). In practice, it can be achieved by a 1-to-2 DEMUX. Now, the modulating symbol sequence can be constructed by taking one bit each from the odd and even sequences at a time as {(10), (11), (10), (11), (00), (01), (10), …}. We started with the odd sequence. Now we can recognize the binary bit stream as a sequence of signal points which are to be transmitted: { $\bar{s}_1$, $\bar{s}_4$, $\bar{s}_1$, $\bar{s}_4$, $\bar{s}_2$, $\bar{s}_3$, $\bar{s}_1$, …}.

With reference to **Fig.5.25.1**, let us note that when the modulating symbol changes from $\bar{s}_1$ to $\bar{s}_4$, it ultimately causes a phase shift of $\pi^c/2$ in the pass band modulated signal [from $-\pi^c/4$ to $+\pi^c/4$ in the diagram]. However, when the modulating symbol changes from $\bar{s}_4$ to $\bar{s}_2$, it causes a phase shift of π^c in the pass band modulated signal [from $+\pi^c/4$ to $+5\pi^c/4$ in the diagram]. So, a phase change of 0^c or $\pi^c/2$ or π^c occurs in the modulated signal every $2T_b$ sec. It is interesting to note that as no pulse shaping has been used, the phase changes occur almost instantaneously. Sharp phase transitions give rise to significant side lobes in the spectrum of the modulated signal.

Table 5.25.1 summarizes the features of QPSK signal constellation.

Input	Dibit		Phase of QPSK	Coordinates of signal points		
	(b_0)	(b_e)		s_{i1}	s_{i2}	i
$\overline{s_1}$	1	0	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$	1
$\overline{s_2}$	0	0	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$	2
$\overline{s_3}$	0	1	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$	3
$\overline{s_4}$	1	1	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$	4

Table 5.25.1 *Feature summary of QPSK signal constellation*

Fig.5.25.2 shows the QPSK modulated waveform for a data sequence 101110110001. For better illustration, only three carrier cycles have been shown per symbol duration.

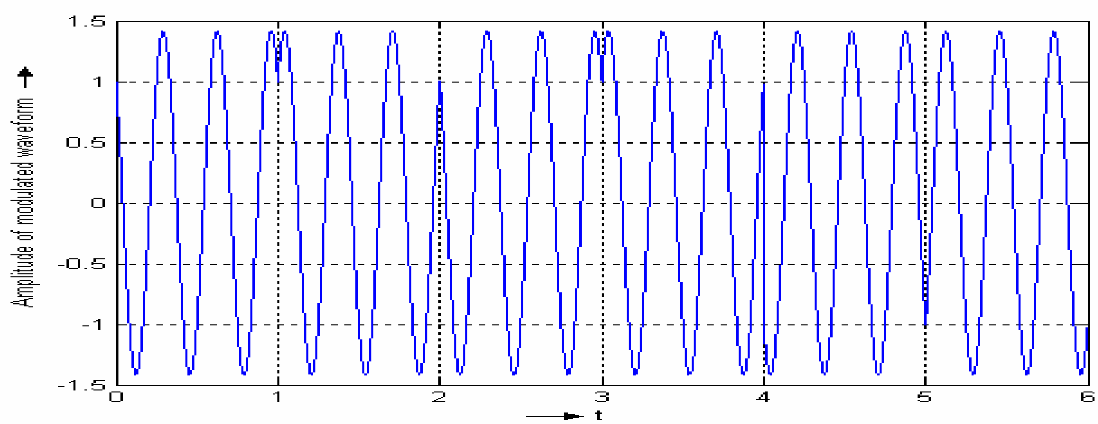

Fig.5.25.2 *QPSK modulated waveform*

Generation of QPSK modulated signal

Let us recall that the time-limited energy signals for QPSK modulation can be expressed as,

$$s_i(t) = \sqrt{\frac{2E}{T}}.\cos\left[(2i-1)\pi/4\right].\cos w_c t - \sqrt{\frac{2E}{T}}.\sin\left[(2i-1)\pi/4\right].\sin w_c t$$

$$= \sqrt{E}.\cos\left[(2i-1)\pi/4\right]\sqrt{\frac{2}{T}}.\cos w_c t - \sqrt{E}\sin\left[(2i-1)\pi/4\right].\sqrt{\frac{2}{T}}\sin w_c t$$

$$= s_{i1}\varphi_1(t) + s_{i2}\varphi_2(t) \qquad \text{i = 1,2,3.4} \qquad 5.25.5$$

The QPSK modulated wave can be expressed in several ways such as:

$$s(t) = \sqrt{E}.d_{odd}(t).\sqrt{\frac{2}{T}}\cos w_c t + \sqrt{E}.d_{even}(t).\sqrt{\frac{2}{T}}\sin w_c t$$

$$= \sqrt{\frac{2E}{T}}.d_{odd}(t)\cos w_c t + \sqrt{\frac{2E}{T}}.d_{even}(t)\sin w_c t$$

$$= \left\{d_{odd}(t).\sqrt{\frac{2E}{T}}\right\}\cos w_c t + \left\{d_{even}(t).\sqrt{\frac{2E}{T}}\right\}\sin w_c t \qquad 5.25.6$$

For narrowband transmission, we can further express s(t) as:

$$s(t) \equiv u_I(t).\cos w_c t - u_Q(t).\sin w_c t$$

where $\tilde{u}(t) = u_I(t) + ju_Q(t)$ is the complex low-pass equivalent representation of s(t).

One can readily observe that, for rectangular bipolar representation of information bits and without any further pulse shaping,

$$u_I(t) = \sqrt{\frac{2E}{T}}.d_{odd}(t) \text{ and } u_Q(t) = \sqrt{\frac{2E}{T}}.d_{even}(t) \qquad 5.25.7$$

Note that while expressing Eq. 5.25.6, we have absorbed the '-' sign, associated with the quadrature carrier 'sin ω_ct' in $d_{even}(t)$. We have also assumed that $d_{odd}(t)$ = +1.0 for 'b_o' ≡1 while $d_{even}(t)$ = -1.0 when b_e ≡ 1. This is not a major issue in concept building as its equivalent effect can be implemented by inverting the quadrature carrier.

Fig. 5.25.3(a) shows a schematic diagram of a QPSK modulator following **Eq. 5.25.6**. Note that the first block, accepting the binary sequence, does the job of generation of odd and even sequences as well as the job of scaling (representing) each bit appropriately so that its outputs are s_{i1} and s_{i2} (**Eq.5.25.5**). **Fig. 5.25.3(b)** is largely similar to **Fig. 5.25.3(a)** but is better suited for simple implementation. Close observation will reveal that both the schemes are equivalent while the second scheme allows adjustment of power of the modulated signal by adjusting the carrier amplitudes. Incidentally, both the in-phase carrier and the quadrature phase carriers are obtained from a single continuous-wave oscillator in practice.

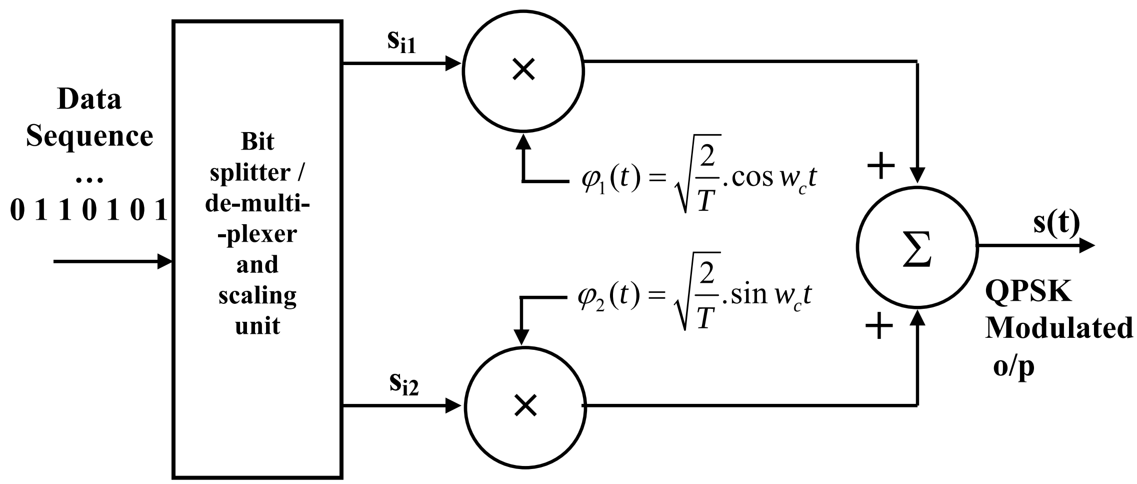

Fig.5.25.3 (a) *Block schematic diagram of a QPSK modulator*

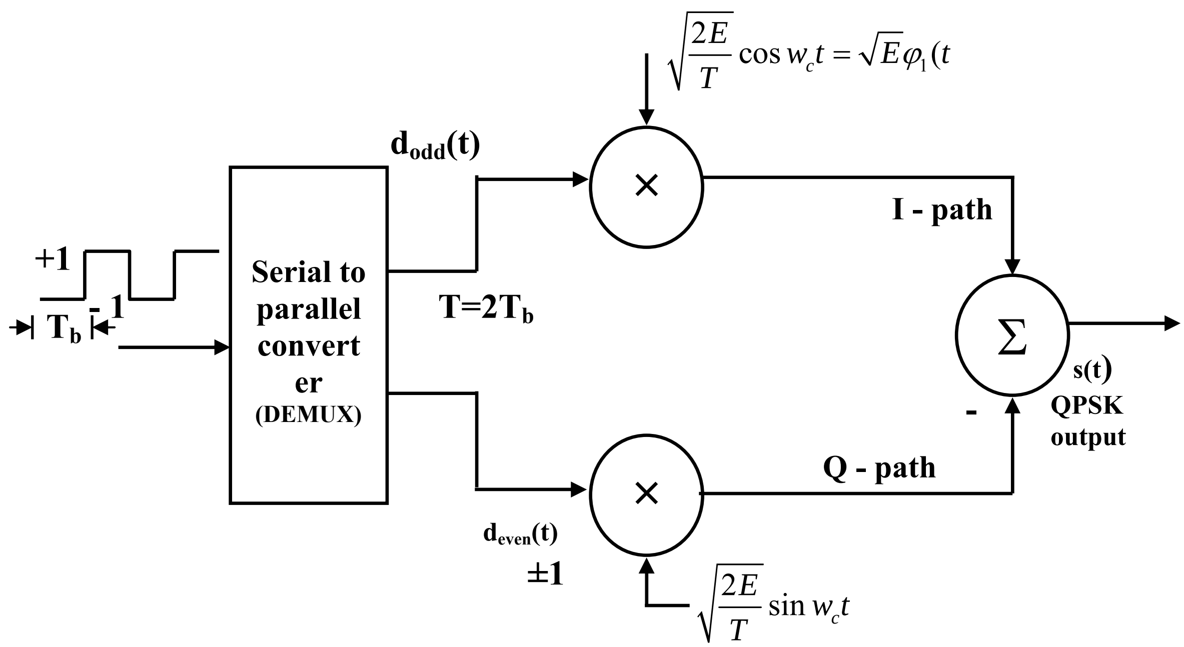

Fig.5.25.3 (b) *Another schematic diagram of a QPSK modulator, equivalent to* ***Fig. 5.23.3(a)*** *but more suitable in practice*

The QPSK modulators shown in **Fig.5.25.3** follow a popular and general structure known as I/Q (In-phase / Quadrature-phase) structure. One may recognize that the output of the multiplier in the I-path is similar to a BPSK modulated signal where the modulating sequence has been derived from the odd sequence. Similarly, the output of the multiplier in the Q-path is a BPSK modulated signal where the modulating sequence is derived from the even sequence and the carrier is a sine wave. If the even and odd bits are independent of each other while occurring randomly at the input to the modulator, the

QPSK modulated signal can indeed be viewed as consisting of two independent BPSK modulated signals with orthogonal carriers.

The structure of a QPSK demodulator, following the concept of correlation receiver, is shown in **Fig. 5.25.4**. The received signal r(t) is an IF band pass signal, consisting of a desired modulated signal s(t) and in-band thermal noise. One can identify the I- and Q- path correlators, followed by two sampling units. The sampling units work in tandem and sample the outputs of respective integrator output every $T = 2T_b$ second, where 'T_b' is the duration of an information bit in second. From our understanding of correlation receiver, we know that the sampler outputs, i.e. r_1 and r_2 are independent random variables with Gaussian probability distribution. Their variance is same and decided by the noise variance while their means are $\pm\sqrt{E/2}$, following our style of representation. Note that the polarity of the sampler output indicates best estimate of the corresponding information bit. This task is accomplished by the vector receiver, which consists of two identical binary comparators as indicated in **Fig.5.25.4.** The output of the comparators are interpreted and multiplexed to generate the demodulated information sequence ($\hat{d}(t)$ in the figure).

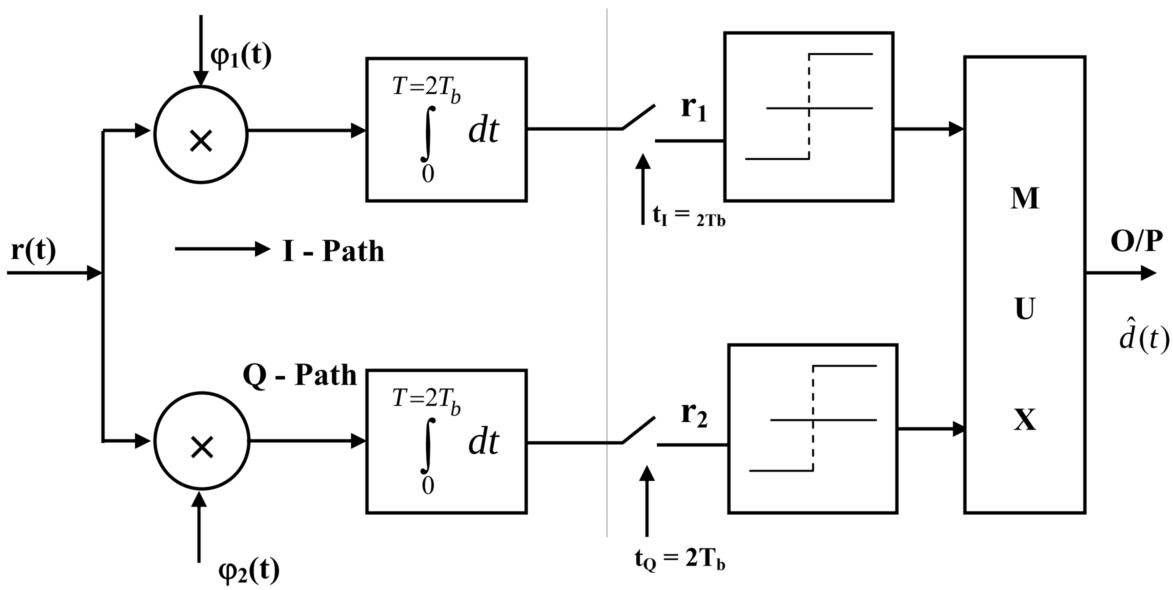

Fig. 5.25.4 *Correlation receiver structure of QPSK demodulator*

We had several ideal assumptions in the above descriptions such as a) ideal regeneration of carrier phase and frequency at the receiver, b) complete knowledge of symbol transition instants, to which the sampling clock should be synchronized, c) linear modulation channel between the modulator output and our demodulator input and so forth. These issues must be addressed satisfactorily while designing an efficient QPSK modem.

Spectrum of QPSK modulated signal

To determine the spectrum of QPSK modulated signal, we follow an approach similar to the one we followed for BPSK modulation in the previous lesson. We assume a long sequence of random independent bits as our information sequence. Without Nyquist filtering, the shaping function in this case can be written as:

$$g(t) = \sqrt{\frac{E}{T}}\ ; \qquad 0 \le t \le T = 2\ T_b \qquad 5.25.8$$

After some straight forward manipulation, the single-sided spectrum of the equivalent complex baseband signal $\tilde{u}(t)$ can be expressed as:

$$U_B(f) = 2E.\sin c^2(Tf) \qquad 5.25.9$$

Here 'E' is the energy per symbol and 'T' is the symbol duration. The above expression can also be put in terms of the corresponding parameters associated with one information bit:

$$U_B\left(f\right) = 4.E_b.\sin c^2(2T_b f) \qquad 5.25.10$$

Fig. 5.25.5 shows a sketch of single-sided baseband spectrum of QPSK modulated signal vs. the normalized frequency (fT_b). Note that the main lobe has a null at fT_b = 0.5.f.T = 0.5 because no Nyquist pulse shaping was adopted. The width of the main lobe is half of that necessary for BPSK modulation. So, for a given data rate, QPSK is more bandwidth efficient. Further, the peak of the first sidelobe is not negligibly small compared to the main lobe peak. The side lobe peak is about 12 dB below the main lobe peak. The peaks of the subsequent lobes monotonically decrease. So, theoretically the spectrum stretches towards infinity. As discussed in Module #4, the spectrum is restricted in a practical system by resorting to pulse shaping. The single-sided equivalent Nyquist bandwidth for QPSK = (1/2) symbol rate (Hz) = $1/2T$ (Hz) = $1/4T_b$ (Hz). So, the normalized single-sided equivalent Nyquist bandwidth = ¼ = 0.25. The Nyquist transmission bandwidth of the real pass band modulated signal s(t) = 2 x single-sided Nyquist bandwidth = $1/2T_b$ (Hz) = $1/T$ (Hz) ≡ The symbol rate.

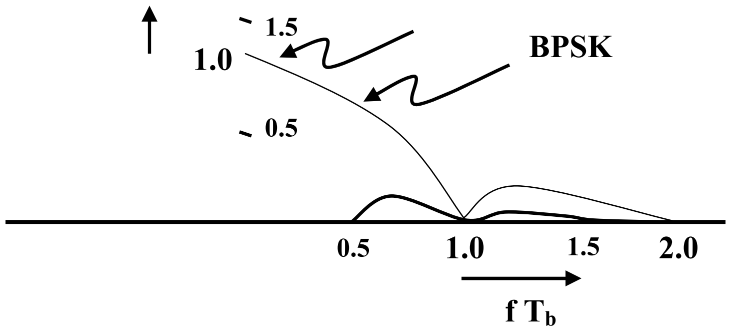

Fig. 5.25.5 *Normalized base band bandwidth of QPSK and BPSK modulated signals*

The actual transmission bandwidth that is necessary = Nyquist transmission bandwidth) x (1 + α) Hz = (1 + α). $\frac{1}{T}$ Hz = (1 + α).R_s Hz, where 'R_s' is the symbol rate in symbols/sec.

Offset QPSK (OQPSK)

As we have noted earlier, forming symbols with two bits at a time leads to change in phase of QPSK modulated signal by as much as 180°. Such large phase transition over a small symbol interval causes momentary but large amplitude change in the signal. This leads to relatively higher sidelobe peaks in the spectrum and it is avoidable to a considerable extent by adopting a simple trick. Offsetting the timing of the odd and even bits by one bit period ensures that the in-phase and quadrature components do not change at the same time instant and as a result, the maximum phase transition will be limited to $\pi^c/2$ at a time, though the frequency of phase changes over a large period of observation will be more. The resultant effect is that the sidelobe levels decrease to a good extent and the demodulator performs relatively better even if the modulation channel is slightly non-linear in behaviour. The equivalent Nuquist bandwidth is not altered by this method. This simple and practical variation of QPSK is known as Offset QPSK.

M–ary PSK

This is a family of two-dimensional phase shift keying modulation schemes. Several bandwidth efficient schemes of this family are important for practical wireless applications.

As a generalization of the concept of PSK modulation, let us decide to form a modulating symbol by grouping 'm' consecutive binary bits together. So, the number of possible modulating symbols is, $M = 2^m$ and the symbol duration $T = m. T_b$. **Fig. 5.25.6** shows the signal constellation for m = 3. This modulation scheme is called as '8-PSK' or 'Octal Phase Shift Keying'. The signal points, indicated by '*', are equally spaced on a circle. This implies that all modulation symbols $s_i(t)$, $0 \le i \le (M-1)$, are of same energy 'E'. The dashed straight lines are used to denote the decision zones for the symbols for optimum decision-making at the receiver.

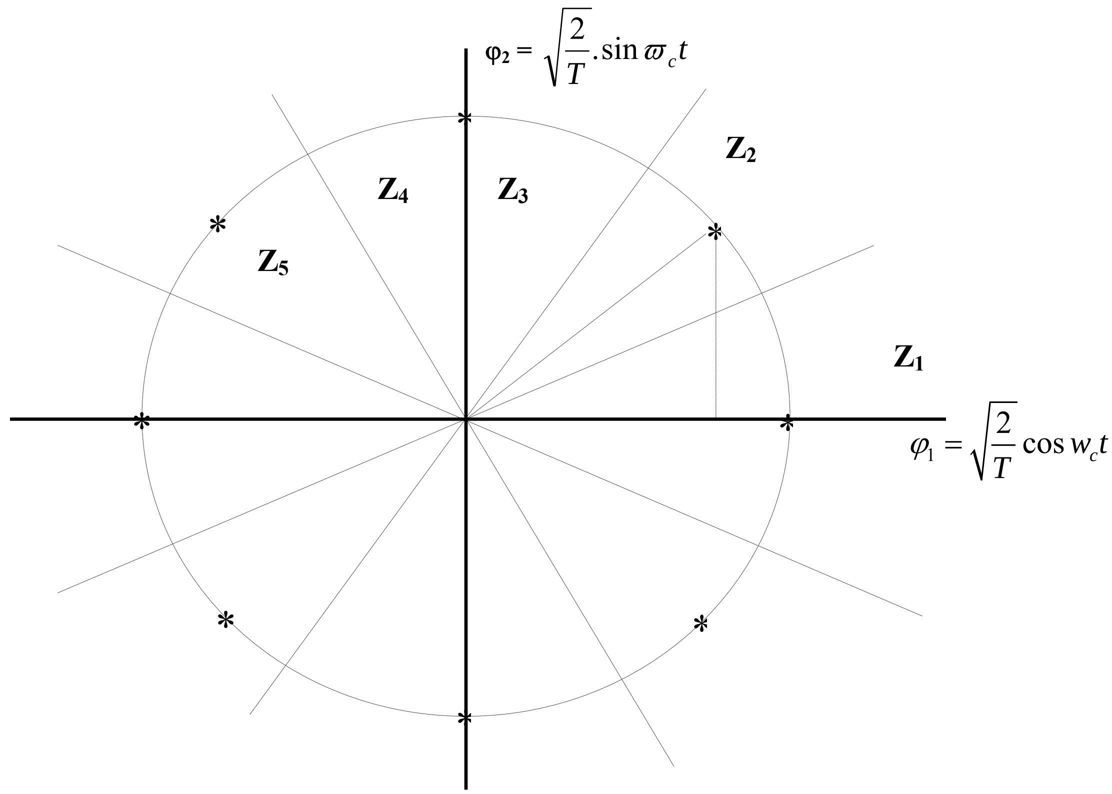

Fig. 5.25.6 *Signal space for 8-PSK modulation*

The two basis functions are similar to what we considered for QPSK, viz.,

$$\varphi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_c t \text{ and } \quad \varphi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_c t \quad ; 0 \le t \le T \qquad 5.25.11$$

The signal points can be distinguished by their angular location:

$$\theta_i = \frac{2\pi_i}{M}; \text{ i} = 0, 1\ldots\ldots \text{ M} - 1 \qquad 5.25.12$$

The time-limited energy signals $s_i(t)$ for modulation can be expressed in general as

$$s_i(t) = \sqrt{\frac{2E}{T}}.\cos\left(2\pi f_c t + \frac{2\pi i}{M}\right) \qquad 5.25.13$$

Considering M-ary PSK modulation schemes are narrowband-type, the general form of the modulated signal is

$$s(t) = u_I(t)\cos w_c t - u_Q(t)\sin w_c t \qquad 5.25.14$$

Fig. 5.25.7 shows a block schematic for an M-ary PSK modulator. The baseband processing unit receives information bit stream serially (or in parallel), forms information symbols from groups of 'm' consecutive bits and generates the two scalars s_{i1} and s_{i2} appropriately. Note that these scalars assume discrete values and can be realized in practice in multiple ways. As a specific example, the normalized discrete values that are to be generated for 8-PSK are given below in **Table 5.25.2**.

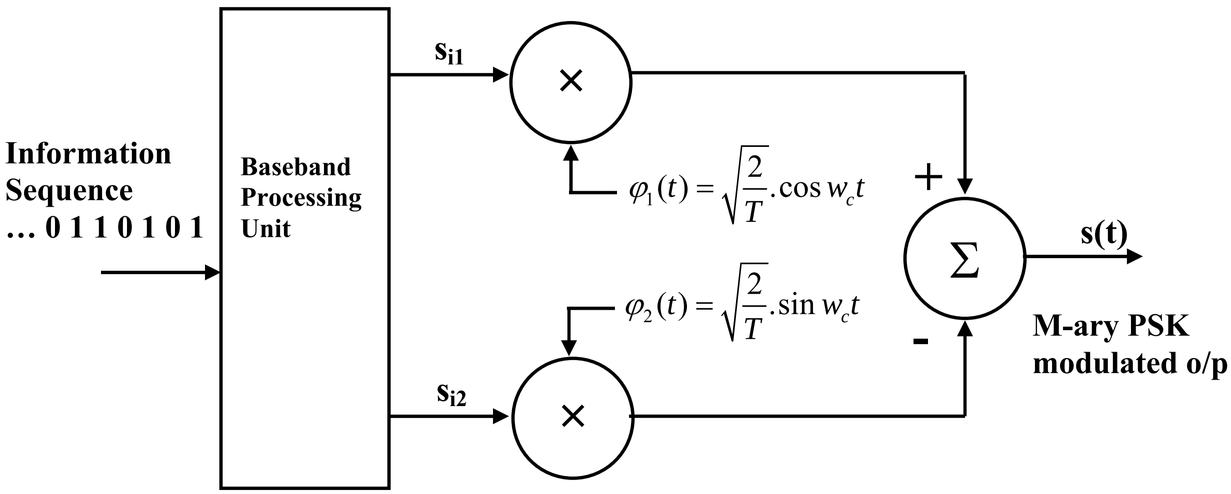

Fig. 5.25.7 *Block schematic diagram of M-ary PSK modulator*

i	0	1	2	3	4	5	6	7
s_{i1}	1	$+\sqrt{1/2}$	0	$-\sqrt{1/2}$	-1	$-\sqrt{1/2}$	0	$+\sqrt{1/2}$
s_{i2}	0	$+\sqrt{1/2}$	1	$+\sqrt{1/2}$	0	$-\sqrt{1/2}$	-1	$-\sqrt{1/2}$

Table 5.25.2 *Normalized scalars for 8-PSK modulation*.

Without any pulse shaping, the $u_I(t)$ and $u_Q(t)$ of **Eq. 5.25.14** are proportional to s_{i1} and s_{i2} respectively. Beside this baseband processing unit, the M-ary PSK modulator follows the general structure of an I/Q modulator.

Fig. 5.25.8 shows a scheme for demodulating M-ary PSK signal following the principle of correlation receiver. The in-phase and quadrature-phase correlator outputs are:

$$r_I = \sqrt{E}\cos\left(\frac{2\pi i}{M}\right) + W_I \text{ , i = 0, 1… M – 1}$$

$$r_Q = -\sqrt{E}\sin\left(\frac{2\pi i}{M}\right) + W_Q \text{, i = 0, 1… M – 1} \qquad 5.25.15$$

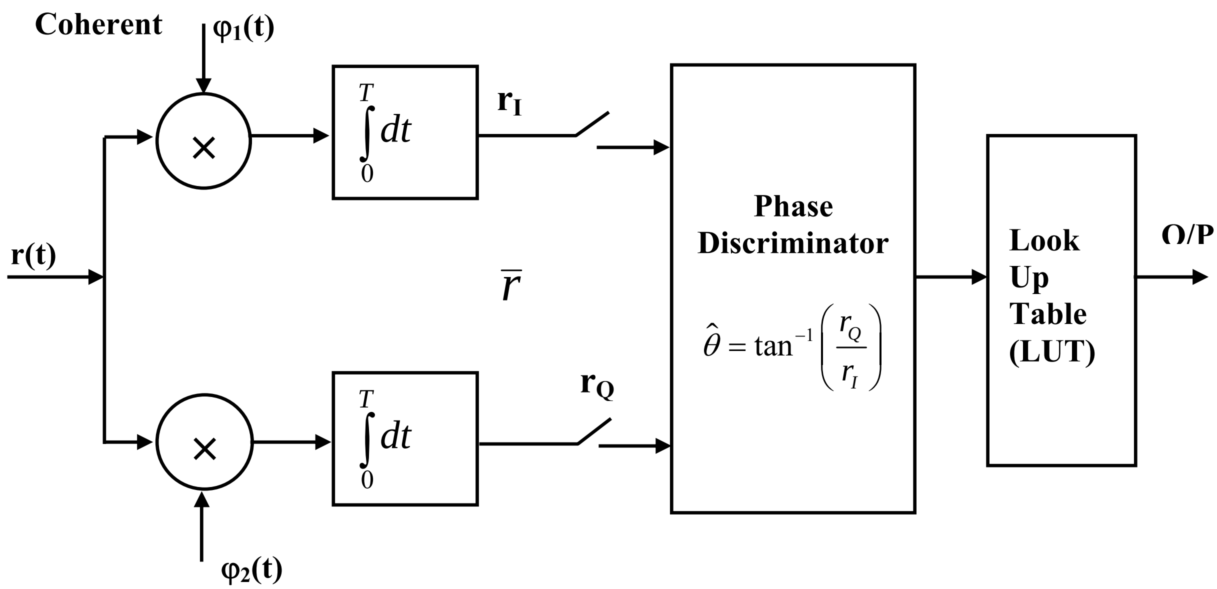

Fig. 5.25.8 *Structure of M-ary PSK demodulator*

W_I represents the in-phase noise sample and W_Q represents the Q-phase noise sample. The samples are taken once every $m.T_b$ sec.

A notable difference with the correlation receiver of a QPSK demodulator is in the design of the vector receiver. Essentially it is a phase discriminator, followed by a map or look-up table (LUT). Complexity in the design of an M-ary PSK modem increases with 'm'.

Problems

Q5.25.1) Write the expression of a QPSK modulated signal & explain all the symbols you have used.

Q5.25.2) What happens to a QPSK modulated signal if the two basis functions are the same that is $\varphi_1(t) = \varphi_2(t)$.

Q5.25.3) Suggest how a phase discriminator can be implemented for an 8-PSK signal?

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