Plane Strain Deformation of a Poroelastic Half-space in Welded Contact with Transversely Isotropic Elastic Half-Space

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Abstract
The Biot linearized theory for fluid saturated porous materials is used to study the plane strain deformation of an isotropic, homogeneous, poroelastPic half space in welded contact with a homogeneous, transversely isotropic, elastic half space caused by an inclined line-load in elastic half space. The integral expressions for the displacements and stresses in the two half spaces in welded contact are obtained from the corresponding expressions for an unbounded transversely isotropic elastic and poroelastic medium by applying boundary conditions at the interface. The integrals for the inclined line-load are solved analytically for the limiting case i.e. undrained conditions in high frequency limit. The undrained displacements, stresses and pore pressure are shown graphically. Expression for the pore pressure is also calculated for undrained conditions in the high frequency limit.

Key words: Inclined line-load, Transversely Isotropic, Plane strain, Poroelastic, Welded half-spaces

Introduction

Poroelasticity is the mechanics of poroelastic solids with fluid filled pores. Its mathematical theory deals with the mechanical behaviour of an elastic porous medium which is either completely filled or partially filled with pore fluid and study the time dependent coupling between the deformation of the rock and fluid flow within the rock. The study of deformation by buried sources of a fluid saturated porous medium is very important because of its applications in earthquake engineering, soil mechanics, seismology, hydrology, geomechanics, geophysics etc. Biot ([1],[2]) developed linearized constitutive and field equations for poroelastic medium which has been used by many researchers (see e.g. Wang [3] and the references listed there in).

When the source surface is very long in one direction in comparison with the others, the use of two dimensional approximation is justified and consequently calculations are simplified to a great extent and one gets a closed form analytical solution. A very long strip-source and a very long line-source are examples of two dimensional sources. Love [4] obtained expressions for the displacements due to a line-source in an isotropic elastic medium. Maruyama[5] obtained the displacements and stress fields corresponding to long strike-slip faults in a homogeneous isotropic half-space. The two dimensional problem has also been discussed by Rudnicki[6], Rudnicki and Roeloffs[7],Singh and Rani [8] Rani and Singh[9], Singh et al. [10].

Different approaches and methods like boundary value method, displacement discontinuity method, Galerkin vector approach, displacement function approach and eigen value approach, Biot stress function approach etc. have been made to study the plane strain (two dimensional) problem of poroelasticity. The use of eigen value approach has the advantage of finding the solutions of the governing equations in the matrix form notations that avoids the complicated nature of the problem. Kumar et al.[11][12], Garg et al.[13], Kumar and Ailwalia [14],Selim and Ahmed [15] , Selim[16], Chugh et al[17] etc. have used this approach for solving plane strain problem of elasticity and poroelasticity.

In the present paper we study the plane strain deformation of a two phase medium consisting of an isotropic, homogeneous, poroelastic half space in welded contact with transversely isotropic, homogeneous, perfectly elastic half space caused by an inclined line-load in elastic half space. Using Biot stress function(Biot[18],Roeloffs[19]) and Fourier transform ,we find stresses ,displacement and pore pressure for poroelastic unbounded medium in integral form and using eigen value approach following Fourier transform ,we find stresses and displacement for unbounded elastic medium in integral form. Then we obtain the integral expressions for the displacements and stresses in the two half spaces in welded contact from the corresponding expressions for an unbounded elastic and poroelastic medium by applying suitable boundary conditions at the interface. These integrals cannot be solved analytically for arbitrary values of the frequency.We evaluate these integrals for the limiting case i.e. undrained conditions in high frequency limit. The undrained displacements, stresses and pore pressure for poroelastic half space are shown graphically.

Geomechanics problems, such as loading by a reservoir lake or seabed structure that is very extensive in one direction on the earth's surface, can be solved as two dimensional plane strain problem. Bell and Nur[20]) used two dimensional half space models with surface loading to study the change in strength produced by reservoir-induced pore pressure and stresses for thrust, normal and strike-slip faulting.

Formulation of the Problem

Consider a homogeneous, transversely isotropic, elastic half space over an isotropic homogeneous, poroelastic half space which is in welded contact at the interface. A rectangular Cartesian coordinate system oxyz is taken in such a way that a plane x=0 coincides with the intersecting surface of the two half spaces. We take x-axis vertically downwards in the poroelastic half space so that isotropic homogeneous, poroelastic half space becomes the medium-I (x≥0) and the elastic half space becomes the medium-II(x≤0). Further an inclined line-load of magnitude $\mathbf{F_0}$, per unit length, is acting on the z-axis and its inclination with x-axis is δ (Fig. 1) The geometry of the problem as shown in figure 1 and it conforms to the two dimensional approximation.

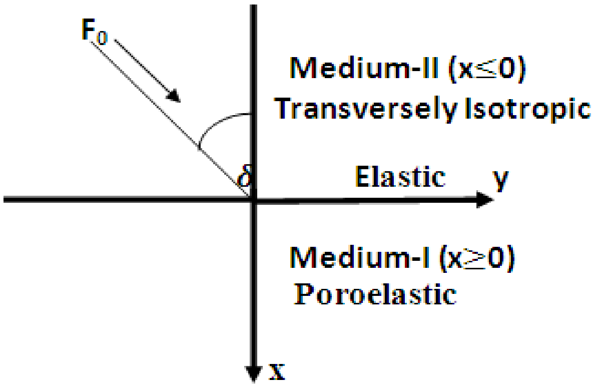

Fig.1 An inclined line load F_0 acting on z-axis with its inclination δ with x-axis

Considering the cartesian coordinates (x, y, z) as (x_1, x_2, x_3), we have $\frac{\partial}{\partial x_3} \equiv 0$ and the displacement components (U_1, U_2, U_3) are independent of the Cartesian coordinate x_3 for the present two dimensional problem. Under this assumption the plane strain problem $(U_3 = 0)$ and the antiplane strain problem $(U_1 = U_2 = 0)$ get decoupled, and can therefore be solved independently. Here, we consider only the plane strain problem.

Solution For Poroelastic Half-space Medium-I (x≥0):

An isotropic, homogeneous, poroelastic medium can be described by five poroelastic parameters; Drained Poisson's ratio (v), undrained Poisson's ratio (v_μ), Shear modulus (G), hydraulic diffusivity (c) and Skempton's coefficient (B). Darcy conductivity (χ) and Biot-willis coefficient α can be written in terms of these five parameters**:**

$$\chi = \frac{9c(1-v_\mu)(v_\mu - v)}{2GB^2(1-v)(1+v_\mu)^2}, \tag{1.1}$$

$$\alpha = \frac{3(v_\mu - v)}{B(1-2v)(1+v_\mu)}, \tag{1.2}$$

Following the two dimensional plane strain problem for an isotropic poroelastic medium can be solved in terms of Biot's stress function F(Wang [3])as

$$\sigma_{11}= \frac{\partial^2 F}{\partial y^2}, \quad \sigma_{22}=\frac{\partial^2 F}{\partial X^2}, \quad \sigma_{12}=- \frac{\partial^2 F}{\partial X\, \partial y}, \tag{1.3}$$

$$\nabla^2(\nabla^2 F + 2\eta p) = 0, \tag{1.4}$$

$$(c\nabla^2 - \frac{\partial}{\partial t})[\ \nabla^2 F + \frac{3}{(1+v_\mu)B} p] = 0, \tag{1.5}$$

where σ_{ij} denotes the total stress in the fluid saturated porous elastic material, p the excess fluid pore pressure (compression negative) and

$$\eta = \frac{(1-2v)\alpha}{2(1-v)} \tag{1.6}$$

is the poroelastic stress coefficient.

From equations (1.4) and (1.5) we get the following decoupled equations

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2 p = 0, \tag{1.7}$$

$$\left(c\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^4 F = 0, \tag{1.8}$$

The general solution of equation (1.7) may be expressed as

$$p = p_1 + p_2\,, \tag{1.9}$$

where p_1 and p_2 satisfies the following equations

$$c\,\nabla^2 p_1 = \frac{\partial p_1}{\partial t}, \tag{1.10}$$

$$\nabla^2 p_2 = 0. \tag{1.11}$$

Similarly, the general solution of equation (1.8) may be expressed as

$$F = F_1 + F_2, \tag{1.12}$$

where

$$c\nabla^2 F_1 = \frac{\partial F_1}{\partial t}, \tag{1.13}$$

$$\nabla^4 F_2 = 0. \tag{1.14}$$

Separation of time and space variables can be made for each of the four functions, p_1, p_2, F_1 and F_2. Assuming the time dependence as exp $(-i\omega t)$, equations (1.10), (1.11), (1.13) and (1.14) become

$$\nabla^2 p_1 + \frac{i\omega}{c} p_1 = 0, \tag{1.15}$$

$$\nabla^2 p_2 = 0, \tag{1.16}$$

$$\nabla^2 F_1 + \frac{i\omega}{c} F_1 = 0, \tag{1.17}$$

$$\nabla^4 F_2 = 0, \tag{1.18}$$

where p_1, p_2, F_1 and F_2 are now functions of x and y only.

Fourier transforms are now used to get suitable solutions of equations (1.15)-(1.18), which on using equations (1.9) and (1.12), can be written as

$$P = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mx} + A_2 e^{-|k|x} + A_3 e^{mx} + A_4 e^{|k|x}\right] e^{-iky} dk, \tag{1.19}$$

$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + B_4 e^{mx} + (B_2 + B_3|k|x)e^{-|k|x} + (B_5 + B_6|k|x)e^{|k|x}\right]e^{-iky} dk, \tag{1.20}$$

For medium I(x≥0), using the relation conditions, we have

$$p = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[A_1 e^{-mx} + A_2 e^{-|k|x}\right] e^{-iky} dk, \tag{1.21}$$

$$F = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + (B_2 + B_3|k|x)e^{-|k|x}\right] e^{-iky} dk, \tag{1.22}$$

where B_1, B_2, B_3, A_1 and A_2 are functions of k. From (1.4), (1.5), (1.21) and (1.22). We find

$$A_1 = \frac{i\omega}{2\eta c} B_1, \quad A_2 = \frac{2}{3}\left(1 + v_\mu\right) B k^2 B_3, \quad m = \left(\left(\frac{ck^2 - i\omega}{c}\right)\right)^{\frac{1}{2}}, (\mathrm{Re}\, m > 0), \tag{1.23}$$

Using (1.22) in (1.3), the stresses in medium I(x≥0) are obtained as

$$\sigma_{22} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m^2 e^{-mx} + (B_2 - 2B_3 + B_3|k|x)k^2 e^{-|k|x}\right] e^{-iky} dk, \tag{1.24}$$

$$\sigma_{11} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + (B_2 + B_3|k|x)e^{-|k|x}\right] k^2 e^{-iky} dk, \tag{1.25}$$

$$\sigma_{12} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mx} + (B_2 - B_3 + B_3|k|x)|k| e^{-|k|x}\right](-ik) e^{-iky} dk. \tag{1.26}$$

Corresponding to these stresses, the displacements are obtained as (Singh and Rani[8])

$$2GU_2{}^1 = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 e^{-mx} + \left\{B_2 + B_3\left(2v_\mu - 2 + |k|x\right)\right\} e^{-|k|x}\right](-ik) e^{-iky} dk, \tag{1.27}$$

$$2GU_1{}^1 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[B_1 m e^{-mx} + \left\{B_2 + B_3\left(1 - 2v_\mu + |k|x\right)\right\}|k| e^{-|k|x}\right] e^{-iky} dk. \tag{1.28}$$

Also from equation (1.21), we have

$$\frac{\partial p}{\partial x} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[-mA_1 e^{-mx} - |k| A_2 e^{-|k|x}\right] e^{-iky} dk, \tag{1.29}$$

Solution For Elastic Solid Half-space Medium-II(x≤0):

The equilibrium equations in Cartesian coordinate system (x_1, x_2, x_3) in absence of body forces are

$$\frac{\partial\tau_{11}}{\partial x_1} + \frac{\partial\tau_{12}}{\partial x_2} + \frac{\partial\tau_{13}}{\partial x_3} = 0, \tag{1.30}$$
$$\frac{\partial\tau_{21}}{\partial x_1} + \frac{\partial\tau_{22}}{\partial x_2} + \frac{\partial\tau_{23}}{\partial x_3} = 0, \tag{1.31}$$
$$\frac{\partial\tau_{31}}{\partial x_1} + \frac{\partial\tau_{32}}{\partial x_2} + \frac{\partial\tau_{33}}{\partial x_3} = 0, \tag{1.32}$$

where τ_{ij} are components of stress tensor.

The stress-strain relations for an transversely isotropic elastic medium are

$$\begin{bmatrix}\tau_{11}\\ \tau_{22}\\ \tau_{33}\\ \tau_{23}\\ \tau_{13}\\ \tau_{12}\end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & 0 & 0 & 0\\ d_{12} & d_{11} & d_{13} & 0 & 0 & 0\\ d_{13} & d_{13} & d_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & d_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & d_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(d_{11} - d_{12})\end{bmatrix}\begin{bmatrix} e_{11}\\ e_{22}\\ e_{33}\\ 2e_{23}\\ 2e_{13}\\ 2e_{12}\end{bmatrix}, \tag{1.33}$$

where e_{ij} are the components of the strain tensor and are related with displacement components (u_1, u_2, u_3) by the relations

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), 1 \le i, j \le 3, \tag{1.34}$$

The two suffix quantity d_{ij} are the elastic moduli for the transversely isotropic elastic medium. We shall write (x_1, x_2, x_3)=(x,y,z), (u_1, u_2, u_3)=(u,v,w) for convenience.

The equilibrium equations in terms of the displacement components can be obtained from equations (1.30)-(1.32) by using (1.33) and (1.34) and for the present two dimensional problem are

$$d_{11}\frac{\partial^2 u}{\partial x^2} + \frac{1}{2}(d_{11} - d_{12})\frac{\partial^2 u}{\partial y^2} + \frac{1}{2}(d_{11} + d_{12})\frac{\partial^2 v}{\partial x\,\partial y} = 0, \tag{1.35}$$

$$\frac{1}{2}(d_{11} + d_{12})\frac{\partial^2 u}{\partial x\,\partial y} + \frac{1}{2}(d_{11} - d_{12})\frac{\partial^2 v}{\partial x^2} + d_{11}\frac{\partial^2 v}{\partial y^2} = 0, \tag{1.36}$$

We define Fourier transform $\bar{f}$(x, k) of f(x, y) (Debnath,[21]) as

$$\bar{f}(\mathrm{x},\mathrm{k}) = \mathrm{F}[f(\mathrm{x},\mathrm{y})] = \int_{-\infty}^{\infty} f(\mathrm{x},\mathrm{y})\mathrm{e}^{\mathrm{iky}}\,dy, \tag{1.37}$$

So that

$$f(\mathrm{x},\mathrm{y}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(\mathrm{x},\mathrm{k})\mathrm{e}^{-\mathrm{iky}}\,dk, \tag{1.38}$$

where k is the transformed Fourier parameter. We know that (Sneddon,[22])

$$\mathrm{F}\left(\frac{\partial}{\partial \mathrm{y}} f(\mathrm{x},\mathrm{y})\right) = (-\mathrm{ik})\bar{f}(\mathrm{x},\mathrm{k}),$$

$$\mathrm{F}\left(\frac{\partial^2}{\partial \mathrm{y}^2} f(\mathrm{x},\mathrm{y})\right) = (-\mathrm{ik})^2\bar{f}(\mathrm{x},\mathrm{k}), \tag{1.39}$$

Applying Fourier Transform as defined above on equations (1.35) and (1.36), we get

$$\frac{\mathrm{d}^2\bar{\mathrm{u}}}{\mathrm{dx}^2} = \mathrm{k}^2\left(\frac{\mathrm{d}_{11}-\mathrm{d}_{12}}{2\mathrm{d}_{11}}\right)\bar{\mathrm{u}} + \mathrm{ik}\left(\frac{\mathrm{d}_{11}+\mathrm{d}_{12}}{2\mathrm{d}_{11}}\right)\frac{\mathrm{d}\bar{\mathrm{v}}}{\mathrm{dx}}, \tag{1.40}$$

$$\frac{\mathrm{d}^2\bar{\mathrm{v}}}{\mathrm{dx}^2} = \mathrm{k}^2\left(\frac{2\mathrm{d}_{11}}{\mathrm{d}_{11}-\mathrm{d}_{12}}\right)\bar{\mathrm{v}} + \mathrm{ik}\left(\frac{\mathrm{d}_{11}+\mathrm{d}_{12}}{\mathrm{d}_{11}-\mathrm{d}_{12}}\right)\frac{\mathrm{d}\bar{\mathrm{u}}}{\mathrm{dx}}, \tag{1.41}$$

The above equations (1.40)-(1.41) can be written in the following vector matrix equation form as

$$\frac{\mathrm{dN}_1}{\mathrm{dx}} = \mathrm{A}_1\mathrm{N}_1 \tag{1.42}$$

where

$$\mathrm{N}_1 = \begin{bmatrix} \bar{\mathrm{u}} \\ \bar{\mathrm{v}} \\ \frac{\mathrm{d}\bar{\mathrm{u}}}{\mathrm{dx}} \\ \frac{\mathrm{d}\bar{\mathrm{v}}}{\mathrm{dx}} \end{bmatrix}, \mathrm{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \mathrm{k}^2\mathrm{R}_1 & 0 & 0 & \mathrm{ikR}_2 \\ 0 & \mathrm{k}^2\mathrm{R}_3 & \mathrm{ikR}_4 & 0 \end{bmatrix}, \tag{1.43}$$

Where R_1 , R_2, R_3 and R_4 are as follow

$$\mathrm{R}_1 = \frac{\mathrm{d}_{11}-\mathrm{d}_{12}}{2\mathrm{d}_{11}}, \quad \mathrm{R}_2 = \frac{\mathrm{d}_{11}+\mathrm{d}_{12}}{2\mathrm{d}_{11}}, \quad \mathrm{R}_3 = \frac{1}{\mathrm{R}_1}, \quad \mathrm{R}_4 = \mathrm{R}_2\mathrm{R}_3 \tag{1.44}$$

Applying eigen value method to solve equation (1.42), we try a solution of the matrix equation (1.42) of the form

$$\mathrm{N}\ (\mathrm{x},\mathrm{k}) = \mathrm{E}\ (\mathrm{k})\ \mathrm{e}^{\mathrm{sx}}, \tag{1.45}$$

where s is a parameter and E(k) is a matrix of the type 4×1.Substituting the value of N from equation (1.45) into equation (1.42),we get the following characteristic equation.

$$\mathrm{s}^4 - 2\mathrm{k}^2\mathrm{s}^2 + \mathrm{k}^4 = 0, \tag{1.46}$$

The solution of the characteristic equation (1.46) gives the repeated eigenvalues as

$$\mathrm{s} = \mathrm{s}_1 = \mathrm{s}_2 = -\mathrm{s}_3 = -\mathrm{s}_4 = |\mathrm{k}|\ ,$$

An eigen vector X_1, corresponding to the eigenvalue $\mathrm{s} = \mathrm{s}_1 = |\mathrm{k}|$ of multiplicity 2 is found to be

$$\mathrm{X}_{1,} = \begin{bmatrix} \mathrm{i}|\mathrm{k}| \\ \mathrm{k} \\ \mathrm{ik}^2 \\ \mathrm{k}|\mathrm{k}| \end{bmatrix}, \tag{1.47}$$

Second eigen vector X_2 corresponding to the eigenvalue $\mathrm{s} = \mathrm{s}_2 = |\mathrm{k}|$ can be obtained as by Ross (1984)

$$X_2 = \begin{bmatrix} i\left\{|k|x - 4\left(\frac{d_{11}}{d_{11}+d_{12}}\right)\right\} \\ k\left(x - \frac{1}{|k|}\right) \\ i|k|\left\{|k|x - \left(\frac{3d_{11}-d_{12}}{d_{11}+d_{12}}\right)\right\} \\ k|k|x \end{bmatrix}, \tag{1.48}$$

An eigen vector X_3, corresponding to the eigenvalue $s = s_3 = -|k|$ of multiplicity 2 is found to be

$$X_{3,} = \begin{bmatrix} -i|k| \\ k \\ ik^2 \\ -k|k| \end{bmatrix}, \tag{1.49}$$

Similarly second eigen vector X_4 corresponding to the eigenvalue $s = s_4 = -|k|$ can be obtained as by Ross (1984)

$$X_4 = \begin{bmatrix} -i\left\{|k|x + 4\left(\frac{d_{11}}{d_{11}+d_{12}}\right)\right\} \\ k\left(x + \frac{1}{|k|}\right) \\ i|k|\left\{|k|x + \left(\frac{3d_{11}-d_{12}}{d_{11}+d_{12}}\right)\right\} \\ -k|k|x \end{bmatrix}, \tag{1.50}$$

Thus, a general solution of (1.42) for a transversely isotropic elastic medium is

$$N_1 = (D_1X_1 + D_2X_2)e^{|k|x} + (D_3X_3 + D_4X_4)e^{-|k|x}, \tag{1.51}$$

Where D_1, D_2, D_3 and D_4 are coefficients to be determined from boundary conditions. From (1.43) and (1.47)-(1.51), we obtain

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$$\bar{u} = i\left[\left\{D_1|k| + D_2\left(|k|x - \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{|k|x} - \left\{D_3|k| + D_4\left(|k|x + \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{-|k|x}\right], \tag{1.52}$$

$$\bar{v} = k\left[\left\{D_1 + D_2\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x} + \left\{D_3 + D_4\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right], \tag{1.53}$$

$$\frac{d\bar{u}}{dx} = i\begin{bmatrix} \left\{D_1k^2 + D_2|k|\left(|k|x - \left(\frac{3d_{11} - d_{12}}{d_{11} + d_{12}}\right)\right)\right\}e^{|k|x} \\ + \left\{D_3k^2 + D_4|k|\left(|k|x + \left(\frac{3d_{11} - d_{12}}{d_{11} + d_{12}}\right)\right)\right\}e^{-|k|x} \end{bmatrix}, \tag{1.54}$$

$$\frac{d\bar{v}}{dx} = k|k|\left[(D_1 + D_2x)e^{|k|x} - (D_3 + D_4x)e^{-|k|x}\right], \tag{1.55}$$

Inversion of equation (1.52) and (1.53) gives the displacements in the following integral forms

$$u(x,y) = \frac{i}{2\pi}\int_{-\infty}^{\infty}\begin{bmatrix} \left\{D_1|k| + D_2\left(|k|x - \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{|k|x} \\ -\left\{D_3|k| + D_4\left(|k|x + \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{-|k|x} \end{bmatrix}e^{-iky}dk, \tag{1.56}$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}k\left[\left\{D_1 + D_2\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x} + \left\{D_3 + D_4\left(x + \frac{1}{|k|}\right)\right\}e^{-|k|x}\right]e^{-iky}dk, \tag{1.57}$$

For the transformed stresses for plane strain deformation parallel to xy plane of a transversely isotropic elastic medium, we make use of equation (1.33)-(1.34) and (1.38)-(1.39), we get

$$\bar{\tau}_{11} = (-ik)d_{12}\bar{v} + d_{11}\frac{d\bar{u}}{dx}, \tag{1.58}$$

$$\bar{\tau}_{12} = \frac{1}{2}\,(d_{11} - d_{12})\left(\frac{d\bar{v}}{dx} - ik\bar{u}\right), \tag{1.59}$$

where $\bar{u}$, $\bar{v}$ used in equations (1.58)-(1.59) are Fourier Transform of u(x, y) and v(x, y).

Putting the values of $\bar{u}$ and $\bar{v}$ from equations (1.52) and (1.53) and their derivatives into equations (1.58)-(1.59) we obtain $\bar{\tau}_{11}$, $\bar{\tau}_{12}$, as

$$\bar{\tau}_{11} = i\left[\begin{array}{c} k^2\left(d_{11}-d_{12}\right)D_1e^{|k|x} + \left\{\left(d_{11}-d_{12}\right)k^2x + |k|\left(\dfrac{-3d_{11}{}^2 + 2d_{11}d_{12} + d_{12}{}^2}{d_{11}+d_{12}}\right)\right\}D_2e^{|k|x} \\ + \text{+quations) and(opic)}k^2\left(d_{11}-d_{12}\right)D_3e^{-|k|x} + \left\{\left(d_{11}-d_{12}\right)k^2x + |k|\left(\dfrac{3d_{11}{}^2 - 2d_{11}d_{12} - d_{12}{}^2}{d_{11}+d_{12}}\right)\right\}D_4e^{-|k|x} \end{array}\right], \tag{1.60}$$

$$\bar{\tau}_{12} = (d_{11}-d_{12})\left[\begin{array}{c} k|k|D_1e^{|k|x} + \left(k|k|x - \frac{2d_{11}k}{d_{11}+d_{12}}\right)D_2e^{|k|x} \\ -k|k|D_3e^{-|k|x} - \left(k|k|x + \frac{2d_{11}k}{d_{11}+d_{12}}\right)D_4e^{-|k|x} \end{array}\right], \tag{1.61}$$

Inversion of equations (1.60)-(1.61) gives the stresses in the following integral forms for a transversely isotropic elastic medium

$$\begin{aligned}\tau_{11} = \frac{i}{2\pi}\int_{-\infty}^{\infty}\Bigg[&k^2(d_{11}-d_{12})D_1e^{|k|x} + \left\{(d_{11}-d_{12})k^2x + |k|\left(\frac{-3d_{11}{}^2 + 2d_{11}d_{12} + d_{12}{}^2}{d_{11}+d_{12}}\right)\right\}D_2e^{|k|x} \\ &+ k^2(d_{11}-d_{12})D_3e^{-|k|x} \\ &+ \left\{(d_{11}-d_{12})k^2x + |k|\left(\frac{3d_{11}{}^2 - 2d_{11}d_{12} - d_{12}{}^2}{d_{11}+d_{12}}\right)\right\}D_4e^{-|k|x}\Bigg]e^{-iky}dk,\end{aligned} \tag{1.62}$$

$$\tau_{12} = \left(\frac{d_{11}-d_{12}}{2\pi}\right)\int_{-\infty}^{\infty}\left[\begin{array}{c} k|k|D_1e^{|k|x} + \left(k|k|x - \dfrac{2d_{11}k}{d_{11}+d_{12}}\right)D_2e^{|k|x} \\ -k|k|D_3e^{-|k|x} - \left(k|k|x + \dfrac{2d_{11}k}{d_{11}+d_{12}}\right)D_4e^{-|k|x} \end{array}\right]e^{-iky}dk, \tag{1.63}$$

The displacements and stress components for transversely isotropic elastic half space medium II ($x \le 0$) are now obtained as from equations (1.56)-(1.57) and (1.62)-(1.63)

$$u(x,y) = \frac{i}{2\pi}\int_{-\infty}^{\infty}\left[\left\{D_1|k| + D_2\left(|k|x - \frac{4d_{11}}{d_{11}+d_{12}}\right)\right\}e^{|k|x}\right]e^{-iky}dk, \tag{1.64}$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}k\left[\left\{D_1 + D_2\left(x - \frac{1}{|k|}\right)\right\}e^{|k|x}\right]e^{-iky}dk, \tag{1.65}$$

$$\tau_{11} = \frac{i}{2\pi}\int_{-\infty}^{\infty}\left[k^2(d_{11}-d_{12})D_1 + D_2\left\{(d_{11}-d_{12})k^2x + |k|\left(\frac{-3d_{11}{}^2+2d_{11}d_{12}+d_{12}{}^2}{d_{11}+d_{12}}\right)\right\}\right]e^{|k|x}e^{-iky}dk, \tag{1.66}$$

$$\tau_{12} = \left(\frac{d_{11}-d_{12}}{2\pi}\right)\int_{-\infty}^{\infty}\left[k|k|D_1 + \left(k|k|x - \frac{2d_{11}k}{d_{11}+d_{12}}\right)D_2\right]e^{|k|x}e^{-iky}dk, \tag{1.67}$$

Case (a) Normal Line-load

Consider a normal line-load F_1 , per unit length, acting in the positive x-direction on the interface x=0 along z-axis. Since the half spaces are assumed to be in welded contact along the plane x=0, the continuity of the stresses and the displacements give the following boundary conditions at x=0:

$$\sigma_{12} = \tau_{12}, \tag{2.1}$$

$$\sigma_{11} - \tau_{11} = -F_1\delta(y), \tag{2.2}$$

$$U_1{}^1(x,y) = u(x,y), \tag{2.3}$$

$$U_2{}^1(x,y) = v(x,y), \tag{2.4}$$

where $\delta(y)$ in equation (2.2) is the Dirac delta function and it satisfies the following properties

$$\int_{-\infty}^{\infty}\delta(y)dy = 1\,, \delta(y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-iky}dk\,, \tag{2.5}$$

Also, if we assume that the interface is impermeable, the hydraulic boundary condition x=0 is

$$\frac{\partial p}{\partial x} = 0. \tag{2.6}$$

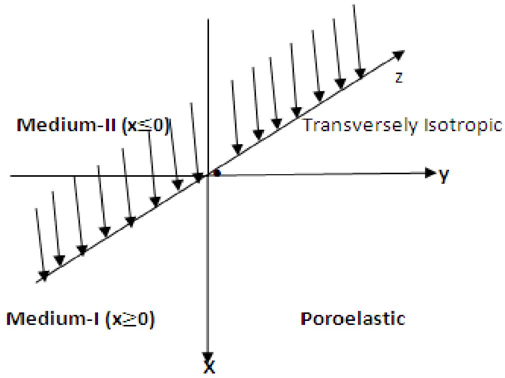


Fig.2 A Normal Line-load F_1, per unit length, acting in the positive x-direction on the interface x=0 along z-axis.

Using equations (1.25)-(1.29), (1.64)-(1.67), and (2.1)-(2.6), we get the following system of equations

$$mB_1 + |k|B_2 - |k|B_3 - m_2 i|k|D_1 + 2m_1 m_2 iD_2 = 0, \tag{2.7}$$

$$B_1 + B_2 + im_2 D_1 + \frac{i}{|k|} m_3 D_2 = \frac{F_1}{K^2}, \tag{2.8}$$

$$B_1 + B_2 + (2v_\mu - 2)B_3 + 2GiD_1 - \frac{2Gi}{|k|} D_2 = 0, \tag{2.9}$$

$$B_1 m + B_2|k| + |k|(1 - 2v_\mu)B_3 - 2Gi|k|D_1 + 8Gim_1 D_2 = 0, \tag{2.10}$$

$$ma^1 B_1 + |k|b^1 B_3 = 0, \tag{2.11}$$

where

$$a^1 = \frac{i\omega}{2\eta c}, b^1 = \frac{2}{3}(1 + v_\mu)Bk^2, \qquad m_1 = \frac{d_{11}}{d_{11} + d_{12}}, \qquad m_2 = d_{11} - d_{12},$$

$$m_3 = \left(\frac{-3{d_{11}}^2 + 2d_{11}d_{12} + {d_{12}}^2}{d_{11} + d_{12}}\right), \tag{2.12}$$

Solving the system of equations (2.7)-(2.11) using (2.12), we obtain

$$B_1 = \frac{-|k|b^1}{ma^1} B_3, \tag{2.13}$$

$$B_2 = -pB_3 + 2GiD_1 - \frac{8Gi}{|k|} m_1 D_2, \tag{2.14}$$

$$B_3 = \frac{4Gi}{(p-q)} D_1 - \frac{2Gi}{|k|}\left(\frac{1+4m_1}{(p-q)}\right) D_2, \tag{2.15}$$

$$D_1 = \frac{F_1}{ik^2}\left\{\frac{F(\alpha-r) - t(q-r)}{(tE-Fs)(q-\alpha)(\alpha-r)}\right\}, \tag{2.16}$$

$$D_2 = \frac{|k|}{it}\left[\frac{F_1}{k^2(q-\alpha)} + isD_1\right], \tag{2.17}$$

where

$$\alpha = \frac{-|k|b^1}{ma^1}, p = \frac{m\alpha}{|k|} + (1 - 2v_\mu), r = \frac{m\alpha}{|k|} - 1, q = \alpha + 2v_\mu - 2,$$

$$E = \left(\frac{2G - m_2}{(q-\alpha)} - \frac{2m_2}{(\alpha - r)}\right) \quad F = \left(\frac{2G + m_3}{(q-\alpha)} + \frac{m_3 - 2m_1 m_2}{(\alpha - r)}\right),$$

$$s = \left[\frac{4G}{p-q} + \frac{2G - m_2}{(q-\alpha)}\right], \quad t = \left[\frac{2G(1+4m_1)}{(p-q)} + \frac{(2G+m_3)}{(q-\alpha)}\right], \tag{2.18}$$

Case (a.1) (Undrained state, $\omega \to \infty$)

Putting the values of B_1, B_2 and B_3 from equations (2.13)-(2.15) into equations (1.24)-(1.29) which corresponds to the stresses ,displacements and pore pressure for poroelastic half space medium 1(x≥ 0) and then taking limit $\omega \to \infty$ and then integrate, we get

$$\sigma_{11}{}^{(N)} = \frac{-F_1}{\pi} P_{10} \frac{x}{x^2+y^2} - \frac{F_1 P_9}{\pi} \frac{x(x^2-y^2)}{(x^2+y^2)^2}, \tag{2.19}$$

$$\sigma_{22}{}^{(N)} = \frac{F_1(P_{10}-2P_9)}{\pi} \frac{x}{x^2+y^2} + \frac{F_1 P_9}{\pi} \frac{x(x^2-y^2)}{(x^2+y^2)^2}, \tag{2.20}$$

$$\sigma_{12}{}^{(N)} = \frac{-F_1}{\pi}(P_{10}-P_9) \frac{y}{x^2+y^2} - \frac{2F_1 P_9}{\pi} \frac{yx^2}{(x^2+y^2)^2}, \tag{2.21}$$

$$2GU_2{}^{1(N)} = \frac{F_1}{\pi}\left(\left(P_{10} + P_9(2v_\mu - 2)\right)\right) \tan^{-1}\frac{y}{x} + \frac{F_1 P_9}{\pi} \frac{xy}{x^2+y^2}, \tag{2.22}$$

$$2GU_1{}^{1(N)} = -\frac{F_1}{2\pi}\left(P_{10} + P_9(1-2v_\mu)\right) \log(x^2+y^2) + \frac{F_1 P_9}{\pi} \frac{x^2}{x^2+y^2}, \tag{2.23}$$

$$p^{(N)} = \frac{2F_1\left(1+v_\mu\right)BP_9}{3\pi} \frac{x}{x^2+y^2}, \tag{2.24}$$

Similarly Putting the values of D_1, and D_2 from equations (2.16)-(2.17) into equations (1.64)-(1.67) which corresponds to the stresses and displacements for transversely isotropic half space medium II(x≤ 0) for normal line-load ,we get

$$u^{(N)}(x,y) = \frac{-F_1}{2\pi}\{P_7 - 4m_1 P_8\} \log(x^2+y^2) - \frac{F_1 P_8}{\pi} \frac{x^2}{x^2+y^2}, \tag{2.25}$$

$$v^{(N)}(x,y) = \frac{F_1}{\pi}(P_7 - P_8) \tan^{-1}\frac{y}{x} - \frac{F_1 P_8}{\pi} \frac{xy}{x^2+y^2}, \tag{2.26}$$

$$\tau_{11}{}^{(N)} = \frac{-F_1}{\pi}(m_2 P_7 + m_3 P_8) \frac{x}{x^2+y^2} + \frac{F_1 m_2 P_8}{\pi} \frac{x(x^2-y^2)}{(x^2+y^2)^2}, \tag{2.27}$$

$$\tau_{12}{}^{(N)} = \frac{-F_1}{\pi}(m_2 P_7 - 2m_1 m_2 P_8) \frac{y}{x^2+y^2} + \frac{2F_1 m_2 P_8}{\pi} \frac{yx^2}{(x^2+y^2)^2}, \tag{2.28}$$

Where P_7, P_8, P_9, P_{10} are as follow

$$P_1 = \left[\frac{4G}{3-4v_\mu} + \frac{2G-m_2}{2v_\mu-2}\right],\ P_2 = \left[\frac{2G(1+4m_1)}{3-4v_\mu} + \frac{2G+m_3}{2v_\mu-2}\right],\ P_3 = \left[\frac{2G-m_2}{2v_\mu-2} - 2m_2\right],$$

$$P_4 = \left[\frac{2G+m_3}{2v_\mu-2} + (m_3 - 2m_1 m_2)\right],\ P_5 = \frac{\{P_4 - P_2(2v_\mu-1)\}}{2v_\mu-2},\ P_6 = P_2P_3 - P_1P_4,\ P_7 = \frac{P_5}{P_6},$$

$$P_8 = \frac{\{1 + P_1P_7(2v_\mu-2)\}}{P_2(2v_\mu-2)}, P_9 = \frac{\{4GP_7 - 2G(1+4m_1)P_8\}}{(3-4v_\mu)},$$

$$P_{10} = \{-(1-2v_\mu)P_9 + 2GP_7 - 8Gm_1P_8\}, \tag{2.29}$$

Case (b) Tangential Line Load

Consider a tangential line-load F_2 , per unit length, is acting at the origin in the positive y-direction. Since the half spaces are assumed to be in welded contact along the plane x=0, the continuity of the stresses and the displacements give the following boundary conditions at x=0:

$$\sigma_{12} - \tau_{12} = -F_2\delta(y), \tag{3.1}$$

$$\sigma_{11} = \tau_{11}, \tag{3.2}$$

$$U_1(x,y) = u(x,y), \tag{3.3}$$

$$U_2(x,y) = v(x,y), \tag{3.4}$$

where $\delta(y)$ in equation (3.1) is the Dirac delta function.

Also, if we assume that the interface is impermeable, the hydraulic boundary condition x=0 is

$$\frac{\partial p}{\partial x} = 0. \tag{3.5}$$

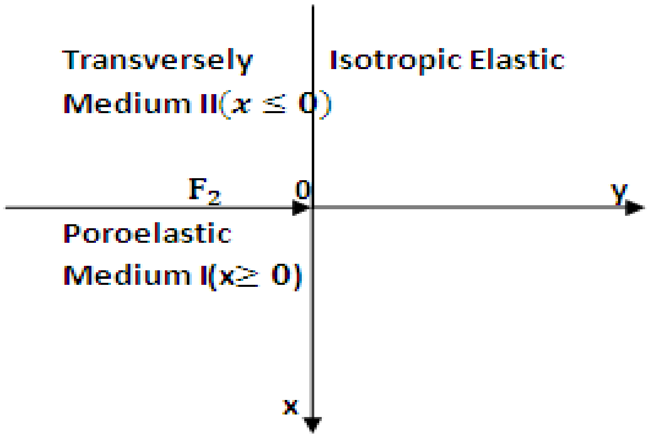


Fig. 3: A Tangential line-load F_2 , per unit length, is acting at the origin in the positive y-direction.

Now using equations (1.25)-(1.29), (1.64)-(1.67)) and (3.1)-(3.5) we get the following system of equations

$$mB_1 + |k|B_2 - |k|B_3 - m_2 i|k|D_1 + 2m_1 m_2 iD_2 = \frac{F_2}{ik}, \tag{3.6}$$

$$B_1 + B_2 + im_2D_1 + \frac{i}{|k|}m_3D_2 = 0, \tag{3.7}$$

$$B_1 + B_2 + (2v_\mu - 2)B_3 + 2GiD_1 - \frac{2Gi}{|k|}D_2 = 0, \tag{3.8}$$

$$B_1 m + B_2|k| + |k|(1 - 2v_\mu)B_3 - 2Gi|k|D_1 + 8Gim_1D_2 = 0, \tag{3.9}$$

$$ma^1B_1 + |k|b^1B_3 = 0, \tag{3.10}$$

where

a^1, b^1, are same as defined in (2.12)

Solving the system of equations (3.6)-(3.10) using (2.12) we obtain

$$B_1 = \frac{-|k|b^1}{ma^1}B_3, \tag{3.11}$$

$$B_2 = -pB_3 + 2GiD_1 - \frac{8Gi}{|k|}m_1D_2, \tag{3.12}$$

$$B_3 = \frac{4Gi}{(p-q)}D_1 - \frac{2Gi}{|k|}\left(\frac{1+4m_1}{(p-q)}\right)D_2, \tag{3.13}$$

$$D_1 = \frac{-F_2}{k|k|}\left\{\frac{1}{(tE-Fs)(\alpha-r)t}\right\}, \tag{3.14}$$

$$D_2 = \frac{s|k|}{t}D_1, \tag{3.15}$$

where $\alpha, p, r, q, E, F, s, t$ are same as defined in equation (2.18)

Putting the values of B_1, B_2 and B_3 from equations (3.11)-(3.13) into equations (1.24)-(1.29) which corresponds to the stresses ,displacements and pore pressure for poroelastic half space medium 1($x \ge 0$) and then taking limit $\omega \to \infty$ and then integrate, we get

$$\sigma_{11}{}^{(T)} = \frac{-F_2}{\pi}q_{10}\frac{y}{x^2+y^2} + \frac{2F_2q_9}{\pi}\frac{yx^2}{(x^2+y^2)^2}, \tag{3.16}$$

$$\sigma_{22}{}^{(T)} = \frac{F_2(q_{10}+2q_9)}{\pi}\frac{y}{x^2+y^2} - \frac{2F_2q_9}{\pi}\frac{yx^2}{(x^2+y^2)^2}, \tag{3.17}$$

$$\sigma_{12}{}^{(T)} = \frac{F_2}{\pi}(q_{10}+q_9)\frac{x}{x^2+y^2} - \frac{F_2q_9}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}, \tag{3.18}$$

$$2GU_1{}^{1(T)} = \frac{F_2}{\pi}\left(\left(q_{10} - q_9(1-2v_\mu)\right)\tan^{-1}\frac{y}{x} - \frac{F_2q_9}{\pi}\frac{xy}{x^2+y^2}\right), \tag{3.19}$$

$$2GU_2{}^{1(T)} = \frac{F_2}{2\pi}\left(q_{10} - q_9(2v_\mu - 2)\right)\log(x^2+y^2) + \frac{F_2 q_9}{\pi}\frac{x^2}{x^2+y^2}, \tag{3.20}$$

$$p^{(T)} = \frac{-2F_2\left(1+v_\mu\right)Bq_9}{3\pi}\frac{y}{x^2+y^2}, \tag{3.21}$$

Similarly Putting the values of D_1, and D_2 from equations(3.14)-(3.15) into equations (1.64)-(1.67) which corresponds to the stresses and displacements for transversely isotropic half space medium II($x \leq 0$) for tangential line-load are and then taking limit $\omega \to \infty$ and then integrate, we get

$$u^{(T)}(x,y) = \frac{F_2}{\pi}(q_7 - 4q_8 m_1)\tan^{-1}\frac{y}{x} - \frac{F_2 q_8}{\pi}\frac{xy}{x^2+y^2}, \tag{3.22}$$

$$v^{(T)}(x,y) = \frac{F_2}{2\pi}\{q_7 - q_8\}\log(x^2+y^2) + \frac{F_2 q_8}{\pi}\frac{x^2}{x^2+y^2}, \tag{3.23}$$

$$\tau_{11}{}^{(T)} = \frac{-F_2}{\pi}(m_2 q_7 + m_3 q_8)\frac{y}{x^2+y^2} + \frac{2F_2 m_2 q_8}{\pi}\frac{yx^2}{(x^2+y^2)^2}, \tag{3.24}$$

$$\tau_{12}{}^{(T)} = \frac{F_2}{\pi}(m_2 q_7 - 2m_1 m_2 q_8)\frac{x}{x^2+y^2} - \frac{F_2 m_2 q_8}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}, \tag{3.25}$$

where q_7, q_8, q_9, q_{10} are as follow

$$q_5 = \frac{1}{P_2},\ q_7 = \frac{q_5}{P_6}, q_8 = \frac{P_1 q_7}{P_2},\ q_9 = \left[\frac{4Gq_7 - 2G(1+4m_1)q_8}{(3-4v_\mu)}\right],$$

$$q_{10} = \left\{\left(1-2v_\mu\right)q_9 - 2Gq_7 + 8Gm_1 q_8\right\}, \tag{3.26}$$

Case(c) Inclined Line-Load

For an inclined line-load $\mathbf{F_0}$,per unit length, we get (Saada,[23])

$$F_1 = F_0\cos\delta, F_2 = F_0\sin\delta, \tag{4.1}$$

The stresses and displacements subjected to inclined line-load can be obtained by superposition of the vertical and tangential cases. The final deformation of the formulated problem is given by

$$U_1{}^{1(IN)}(x,y) = U_1{}^{1(N)}(x,y) + U_1{}^{1(T)}(x,y), \tag{4.2}$$

$$U_2{}^{1(IN)}(x,y) = U_2{}^{1(N)}(x,y) + U_2{}^{1(T)}(x,y), \tag{4.3}$$

$$\sigma_{11}{}^{(IN)}(x,y) = \sigma_{11}{}^{(N)}(x,y) + \sigma_{11}{}^{(T)}(x,y), \tag{4.4}$$

$$\sigma_{12}{}^{(IN)}(x,y) = \sigma_{12}{}^{(N)}(x,y) + \sigma_{12}{}^{(T)}(x,y), \tag{4.5}$$

$$\sigma_{22}{}^{(IN)}(x,y) = \sigma_{22}{}^{(N)}(x,y) + \sigma_{22}{}^{(T)}(x,y), \tag{4.6}$$

$$p^{(IN)}(x,y) = p^{(N)}(x,y) + p^{(T)}(x,y), \tag{4.7}$$

Where deformation due to a normal line-load F_1 and a tangential line-load F_2 have been obtained in **case (a) Normal Line-load** and **case (b) Tangential Line Load**. The superscript (IN) indicates results due to inclined line-load F_0. N indicates results due to Normal line-load F_1 and T indicates results due to Tangential line-load F_2.

Stresses and displacements due to inclined line-load F_0, per unit length, in medium I ($x \geq 0$) are given by

$$\sigma_{11}{}^{(IN)} = \cos\delta\left(\frac{-F_0 P_7}{\pi}\frac{x}{x^2+y^2} - \frac{F_0 P_6}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right) + \sin\delta\left(\frac{-F_0}{\pi}q_7\frac{y}{x^2+y^2} - \frac{2F_0 q_6}{\pi}\frac{yx^2}{(x^2+y^2)^2}\right), \tag{4.8}$$

$$\sigma_{12}{}^{(IN)} = \cos\delta\left(\frac{F_0}{\pi}(P_6 - P_7)\frac{y}{x^2+y^2} - \frac{2F_0 P_6}{\pi}\frac{yx^2}{(x^2+y^2)^2}\right) + \sin\delta\left(\frac{F_2}{\pi}(q_7 - q_6)\frac{x}{x^2+y^2} + \frac{F_2 q_6}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right), \tag{4.9}$$

$$\sigma_{22}{}^{(\mathrm{IN})} = \cos\delta\left\{\frac{F_0(P_{10}-2P_9)}{\pi}\frac{x}{x^2+y^2}+\frac{F_0P_9}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right\} + \sin\delta\left\{\frac{F_0(q_{10}+2q_9)}{\pi}\frac{y}{x^2+y^2}-\frac{2F_0q_9}{\pi}\frac{yx^2}{(x^2+y^2)^2}\right\}, \quad (4.10)$$

$$U_1{}^{1(\mathrm{IN})}(x,y) = \frac{1}{2G}\left\{\cos\delta\left(-\frac{F_0}{2\pi}\left(P_7+P_6(1-2v_\mu)\right)\log(x^2+y^2)+\frac{F_0P_6}{\pi}\frac{x^2}{x^2+y^2}\right) + \sin\delta\left(\frac{F_0}{\pi}\left(\left(q_7+q_6(1-2v_\mu)\right)\tan^{-1}\frac{y}{x}+\frac{F_0q_6}{\pi}\frac{xy}{x^2+y^2}\right)\right\}, \quad (4.11)$$

$$U_2{}^{1(\mathrm{IN})}(x,y) = \frac{1}{2G}\left\{\cos\delta\left(\frac{F_0}{\pi}\left(\left(P_7+P_6(2v_\mu-2)\right)\tan^{-1}\frac{y}{x}+\frac{F_0P_6}{\pi}\frac{xy}{x^2+y^2}\right) + \sin\delta\left(\frac{F_0}{2\pi}\left(q_7+q_6(2v_\mu-2)\right)\log(x^2+y^2)-\frac{F_0q_6}{\pi}\frac{x^2}{x^2+y^2}\right)\right\}, \quad (4.12)$$

$$p^{(\mathrm{IN})}(x,y) = \cos\delta\left\{\frac{2F_1(1+v_\mu)BP_9}{3\pi}\frac{x}{x^2+y^2}\right\}+\sin\delta\left\{\frac{-2F_2(1+v_\mu)Bq_9}{3\pi}\frac{y}{x^2+y^2}\right\}, (4.13)$$

Stresses and displacements due to inclined line-load F_0, per unit length, in medium II ($x\leq 0$) are given by

$$\tau_{11}{}^{(\mathrm{IN})} = \cos\delta\left\{\frac{-F_0}{\pi}(m_2P_7+m_3P_8)\frac{x}{x^2+y^2}+\frac{F_0m_2P_8}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right\} + \sin\delta\left\{\frac{-F_0}{\pi}(m_2q_7+m_3q_8)\frac{y}{x^2+y^2}+\frac{2F_0m_2q_8}{\pi}\frac{yx^2}{(x^2+y^2)^2}\right\}, \quad (4.14)$$

$$\tau_{12}{}^{(\mathrm{IN})} = \cos\delta\left\{\frac{-F_0}{\pi}(m_2P_7-2m_1m_2P_8)\frac{y}{x^2+y^2}+\frac{2F_0m_2P_8}{\pi}\frac{yx^2}{(x^2+y^2)^2}\right\} + \sin\delta\left\{\frac{F_0}{\pi}(m_2q_7-2m_1m_2q_8)\frac{x}{x^2+y^2}-\frac{F_0m_2q_8}{\pi}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right\}, \quad (4.15)$$

$$u^{(\mathrm{IN})}(x,y) = \cos\delta\left\{\frac{-F_0}{2\pi}\{P_7-4m_1P_8\}\log(x^2+y^2)-\frac{F_0P_8}{\pi}\frac{x^2}{x^2+y^2}\right\} + \sin\delta\left\{\frac{F_0}{\pi}(q_7-4q_8m_1)\tan^{-1}\frac{y}{x}-\frac{F_0q_8}{\pi}\frac{xy}{x^2+y^2}\right\}, \quad (4.16)$$

$$v^{(\mathrm{IN})}(x,y) = \cos\delta\left\{\frac{F_0}{\pi}(P_7-P_8)\tan^{-1}\frac{y}{x}-\frac{F_0P_8}{\pi}\frac{xy}{x^2+y^2}\right\} + \sin\delta\left\{\frac{F_0}{2\pi}\{q_7-q_8\}\log(x^2+y^2)+\frac{F_0q_8}{\pi}\frac{x^2}{x^2+y^2}\right\}, \quad (4.17)$$

Numerical Results and Discussion

For numerical computation, we use poroelastic constants correspond to Ruhr Sandstone Wang [3] and values of transversely isotropic constants given by Love [4] for Topaz material. We define the following dimensionless quantities

$X = \frac{x}{h}$, $Y = \frac{y}{h}$, And $\Sigma_{ij} = \frac{\sigma_{ij}}{G}$, ,$1\leq i,j \leq 2$, $U_{\mathbf{i}} = \frac{U_{\mathbf{i}}{}^1}{h}$, i=1,2.

And to make dimensionless to displacements, we divide displacements by h.

We have plotted graphs in figures (4-9) for the variation of the displacements and stresses against the horizontal distance X for a fixed value of X=1/**2**. Each figure has four curves corresponding to four different values of δ,namely; $\delta = 0^0, 45^0, 60^0, 90^0$. The case $\delta = 0^0$ corresponds to a normal line-load and $\delta = 90^0$ for a tangential line load. Figures (4-5) show variation of normal displacement (U_1) and the variation of tangential displacement (U_2) respectively. These figures show that the displacements for $\delta = 45^0, 60^0$ lie between the corresponding displacements for a normal line-load and tangential line-load. Figure (6-8) correspond to the variation of stresses. In figure (6-7) the curves for different values of δ changes steadily. Figure (8) points frequent inter crossing of various curves for different values of δ. Figure (9) shows the variation of the pore pressure against the horizontal distance X for a fixed value of X=1/**2**.

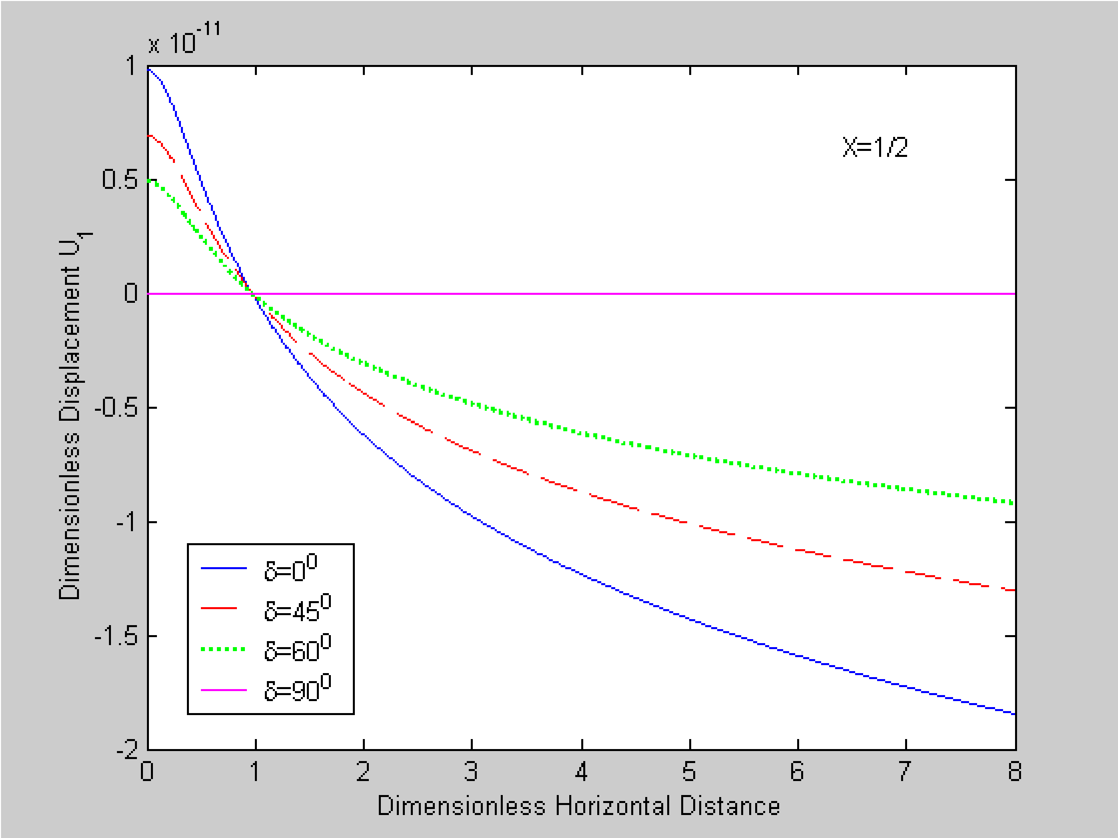
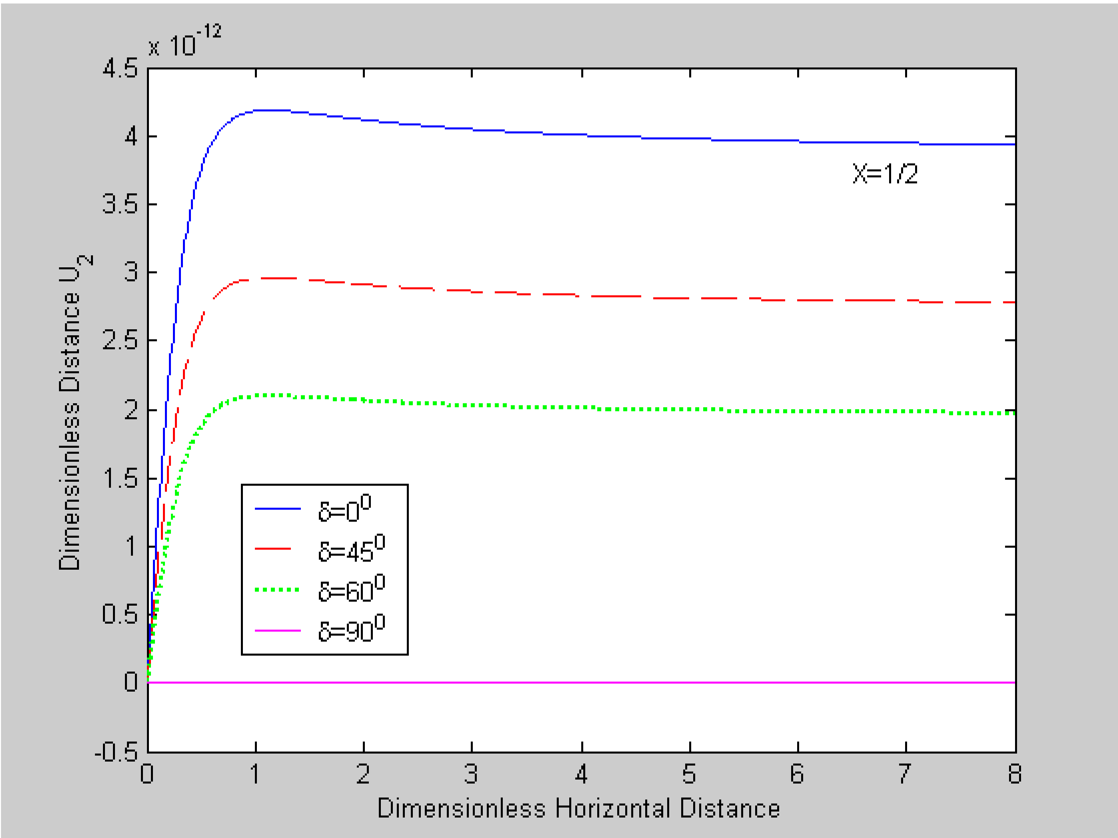


Fig.4 above and Fig 5 below



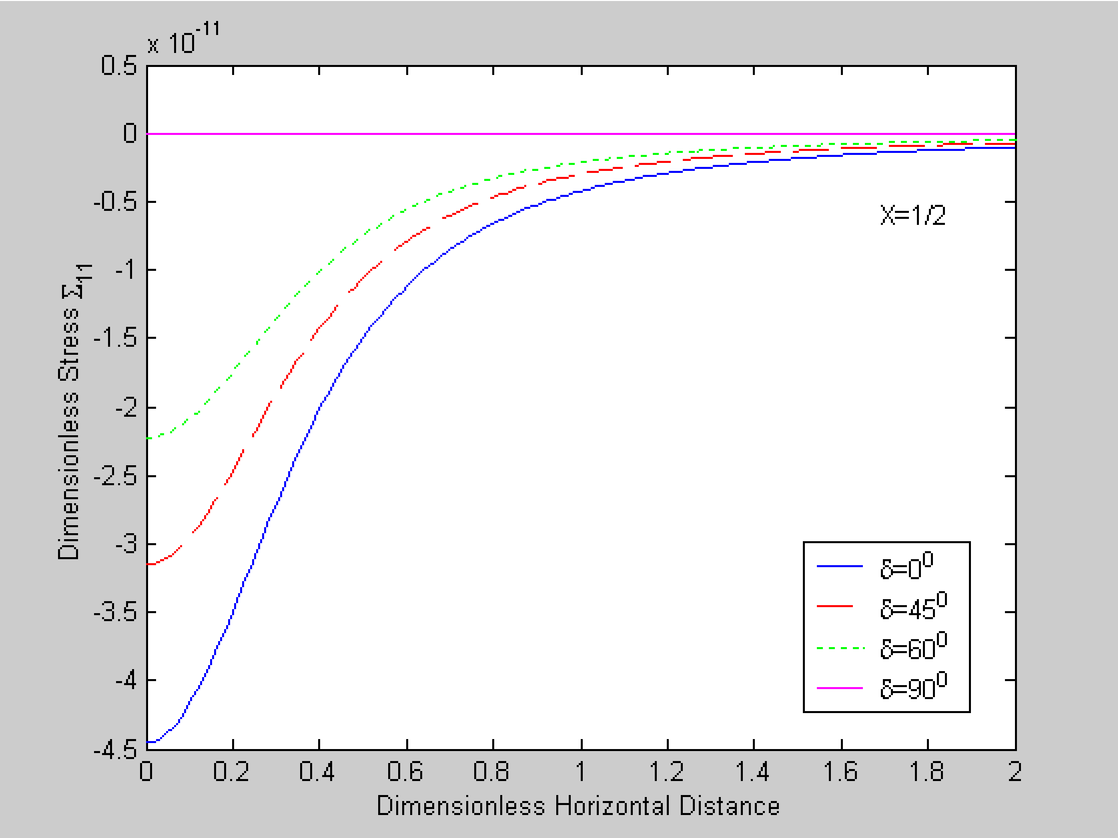
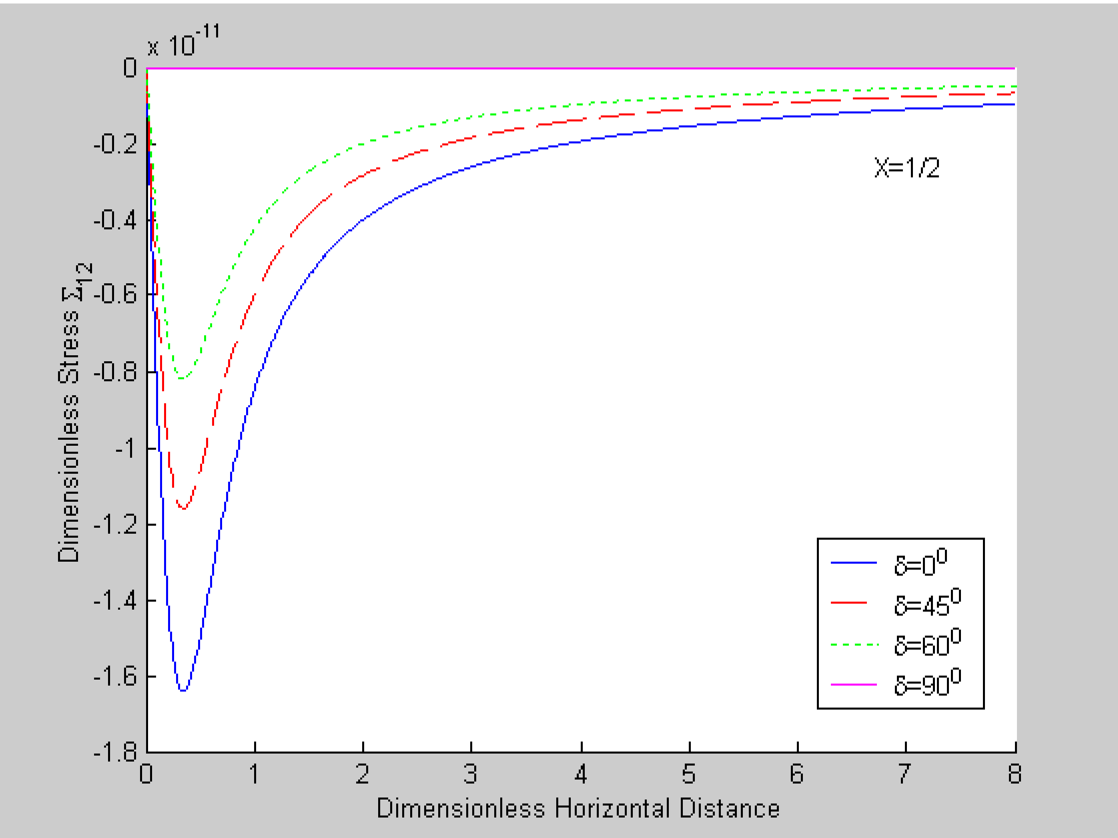


Fig.6 above and Fig 7 below



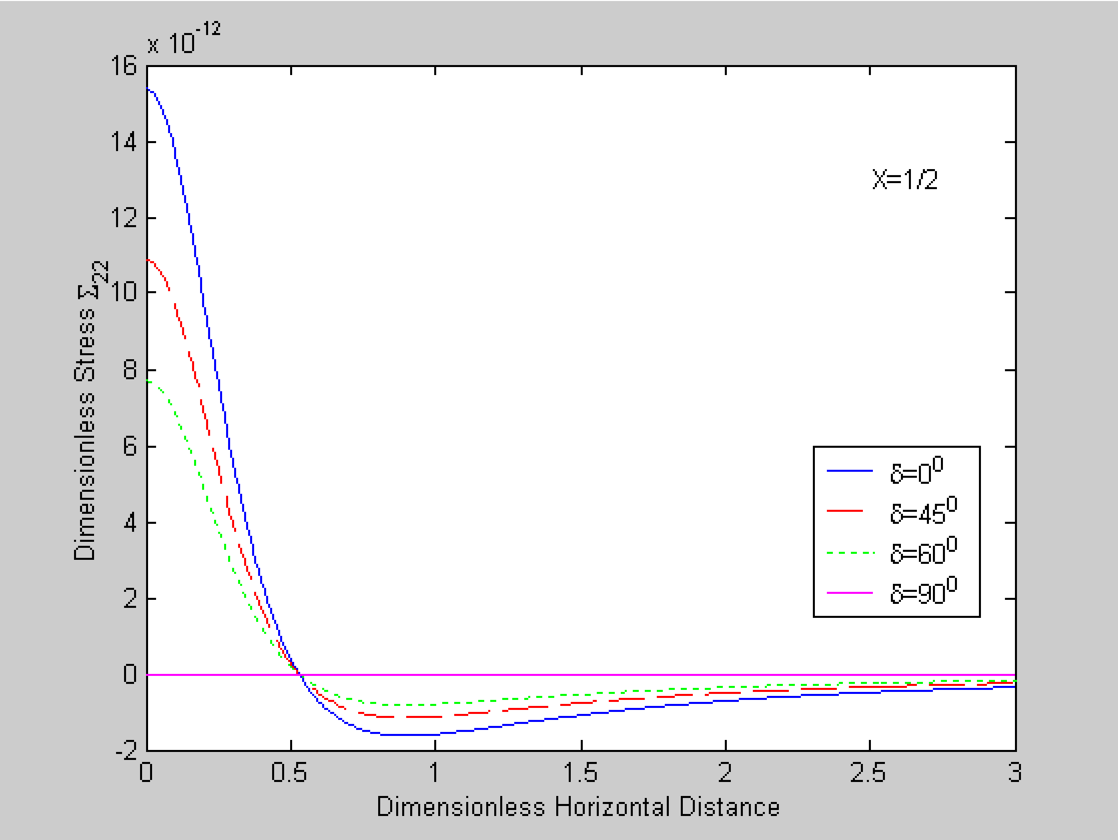
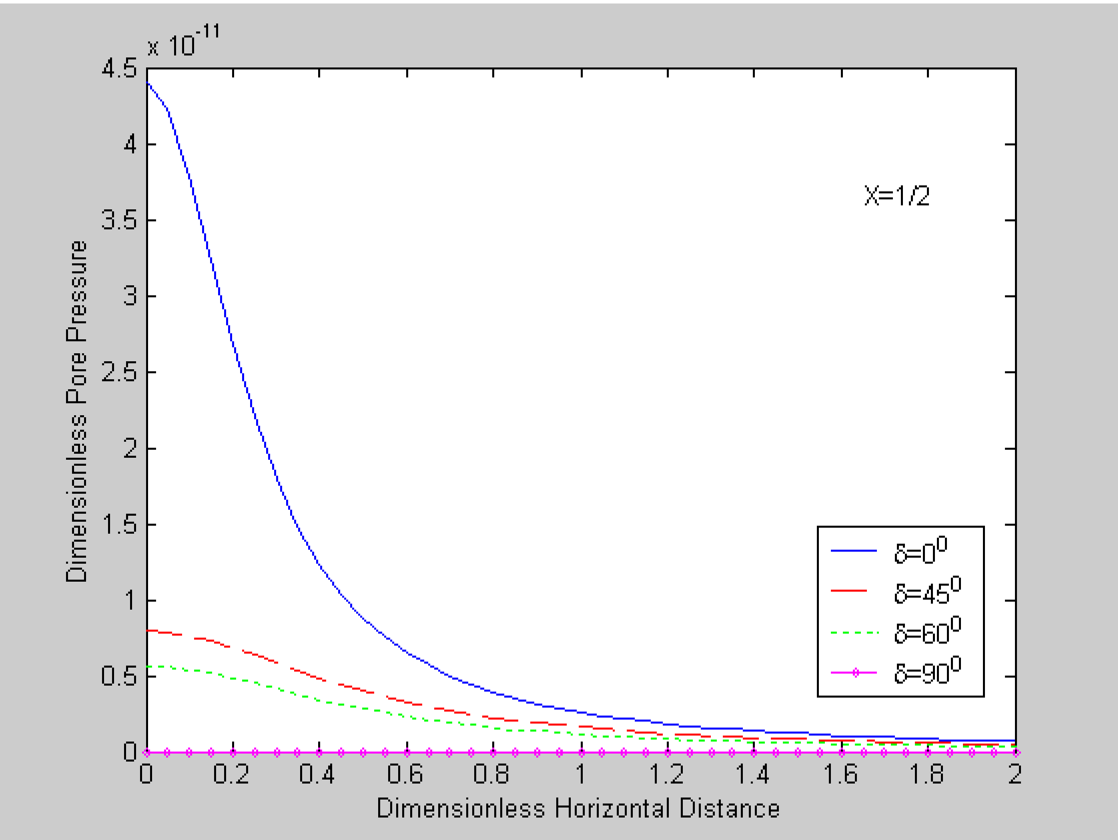


Fig.8 above and Fig 9 below



Appendix $(x > 0)$

$$\int_{-\infty}^{\infty} e^{-|k|x} e^{-\iota k y}\, dk = \frac{2x}{y^2 + x^2},$$

$$\int_{-\infty}^{\infty} |k| e^{-|k|x} e^{-\iota k y}\, dk = \frac{2(x^2 - y^2)}{(y^2 + x^2)^2},$$

$$\int_{-\infty}^{\infty} \frac{k}{|k|} e^{-|k|x} e^{-\iota k y}\, dk = \frac{-2\iota y}{y^2 + x^2},$$

$$\int_{-\infty}^{\infty} k e^{-|k|x} e^{-\iota k y}\, dk = \frac{-4\iota x y}{(y^2 + x^2)^2},$$

$$\int_{-\infty}^{\infty} \frac{1}{k} e^{-|k|x} e^{-\iota k y}\, dk = -2\iota \tan^{-1}\left(\frac{y}{x}\right),$$

$$\int_{-\infty}^{\infty} \frac{1}{|k|} e^{-|k|x} e^{-\iota k y}\, dk = -\log(y^2 + x^2)$$

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