Module 5

Carrier Modulation

Lesson 28

Performance of ASK and binary FSK in AWGN Channel

After reading this lesson, you will learn about

- ***Error Performance of Binary FSK;***
- ***Performance indication for M-ary PSK;***
- ***Approx BER for QPSK;***
- ***Performance Requirements;***

Error Performance of Binary FSK

As we discussed in Lesson #23, BFSK is a two-dimensional modulation scheme with two time-limited signals as reproduced below:

$$s_i(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}} \cos 2\pi f_i t, & 0 \le t \le T_b,\ i = 1,2 \\ 0, & \text{elsewhere.} \end{cases} \qquad 5.28.1$$

We assume appropriately chosen 'mark' and 'space' frequencies such that the two basis functions are orthonormal:

$$\varphi_l(t) = \sqrt{\frac{2}{T_b}} \cos 2\pi f_i t \quad ; \quad 0 \le t \le T_b \text{ and } j = 1,2 \qquad 5.28.2$$

We will consider the coherent demodulator structure [**Fig. 5.28.1(b)**] so that we can apply similar procedure as in Lesson #27 and obtain the optimum error performance for binary FSK. For ease of reference, the signal constellation for BFSK is reproduced in **Fig. 5.28.1(a)**. As we can see, the two signal vectors are $\overline{s_1} = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$ and $\overline{s_2} = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$ while the two associated scalars are $s_{11} = s_{22} = \sqrt{E_b}$. The decision zones are shown by the discontinuous line.

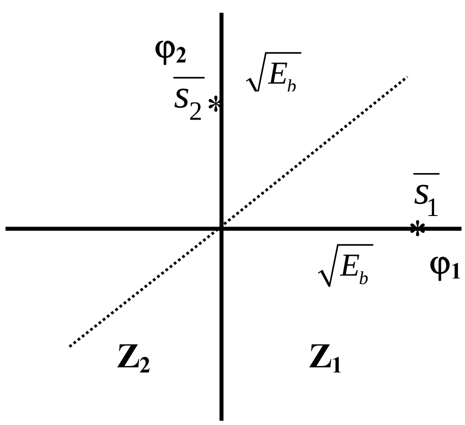

Fig. 5.28.1(a) *Signal constellation for BFSK showing the decision zones Z_1 and Z_2*

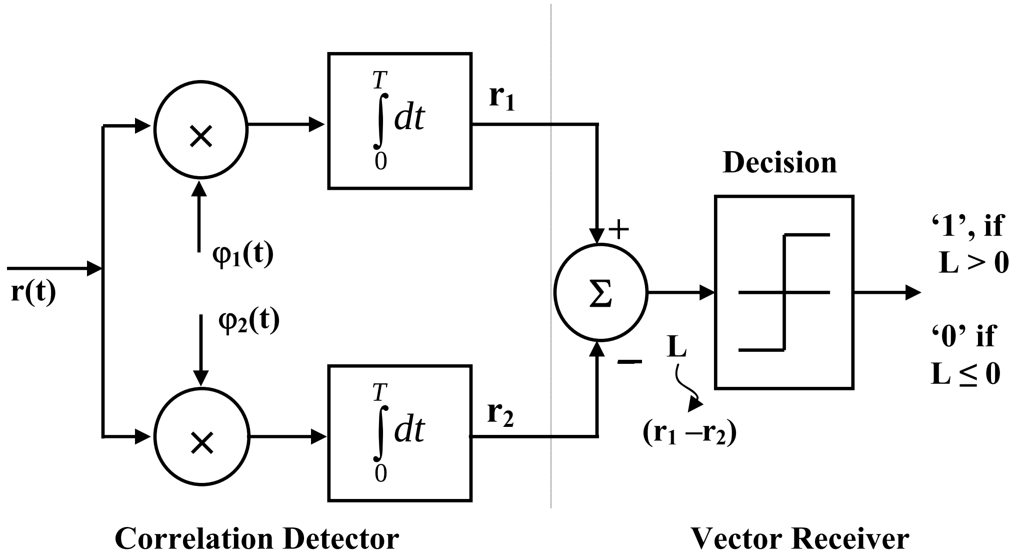

Fig. 5.28.1(b) *Coherent demodulator structure for BFSK highlighting the decision process*

Now, suppose $\bar{s}_1$ represents logic '1' and $\bar{s}_2$ represents logic '0'. If $s_1(t)$ is transmitted and if noise is absent, the output of the upper correlator in **Fig. 5.28.1(b)**, i.e. r_1 is $\sqrt{E_b}$ while the output of the lower correlator, i.e. r_2 is zero. So, we see that the intermediate parameter $L = (r_1 - r_2) > 0$. Similarly, it is easy to see that if $s_2(t)$ is transmitted, $L < 0$. Now, from **Fig. 5.28.1(a)** we see that the decision boundary is a straight line with unit slope. This implies that, if the received vector $\bar{r}$ at the output of the correlator bank is in decision zone Z_1, then $L > 0$ and otherwise it is in zone Z_2.

When we consider additive noise, 'L' represents a random variable whose mean is $+\sqrt{E_b}$ if message '1' is transmitted. For message '0', the mean of 'L' is $-\sqrt{E_b}$.
Further, as we noted in our general analysis earlier in Lesson #19 (Module #4), r_1 and r_2 are independent and identically distributed random variables with the same variance $\frac{N_o}{2}$.

So, variance of 'L' = variance of 'r_1' + variance of 'r_2' = $\frac{N_o}{2} + \frac{N_o}{2} = N_o$ 5.28.3

Now, assuming that a '0' has been transmitted, the likelihood function is:

$$f_L(l\,|\,0) = \frac{1}{\sqrt{2\pi N_0}}.\exp\left[-\frac{\left\{l-\left(-\sqrt{E_b}\right)\right\}^2}{2N_0}\right]$$

$$= \frac{1}{\sqrt{2\pi N_0}} .\exp\left[-\frac{\left(l+\sqrt{E_b}\right)^2}{2N_0}\right] \qquad 5.28.4$$

In the above expressions, 'l' represents a sample value of the random variable 'L'.

From the above expression, we can determine the average probability of error when '0'-s are transmitted as:

$P_e(0)$ = Average probability of error when '0'– s are transmitted $= \int_0^{\infty} f_L\left(l \mid 0\right) dl$

$$= \frac{1}{\sqrt{2\pi N_0}} .\int_0^{\infty} \exp\left[-\frac{\left(l+\sqrt{E_b}\right)^2}{2N_0}\right] dl \qquad 5.28.5$$

Putting $\frac{l+\sqrt{E_b}}{\sqrt{2N_0}} = Z$ in the above expression, we readily get,

$$\text{P}\, e(0) = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{E_b}{2N_0}}}^{\infty} \exp\left(-Z^2\right) dz$$

$$= \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) \qquad 5.28.6$$

Following similar approach, we get,

$P_e(1)$ = Average probability of error when '1'-s are transmitted

$$= \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right) \qquad 5.28.7$$

Now, using the justification that '1' and '0' are equally likely to occur at the input of the modulator, the overall BER = $P_e = \frac{1}{2}.\ P_e(0) + \frac{1}{2}.\ P_e(1) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{2N_0}}\right)$ 5.28.8

Fig. 5.28.2 shows the error performance of binary FSK for coherent demodulation. For comparison, the performance curve for BPSK is also included. Observe that the FSK modulation scheme performs significantly poorer.

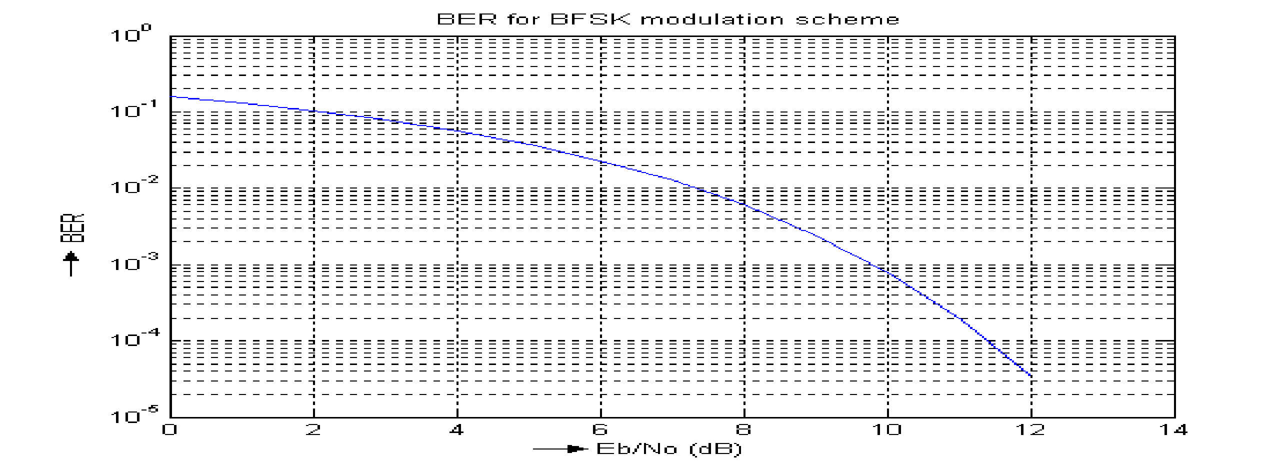

Fig. 5.28.2 *Error performance of binary FSK modulation schemes*

Performance indication for M-ary PSK

Fig. 5.28.3(a) shows the signal constellation for M-ary PSK with M = 2^3. As we have discussed earlier, an M-ary PSK modulated signal over a symbol duration 'T' can be expressed as:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(w_c t + \frac{2\pi i}{M}\right), \text{ i = 0,1,….m M-1 and M} = 2^m \qquad 5.28.9$$

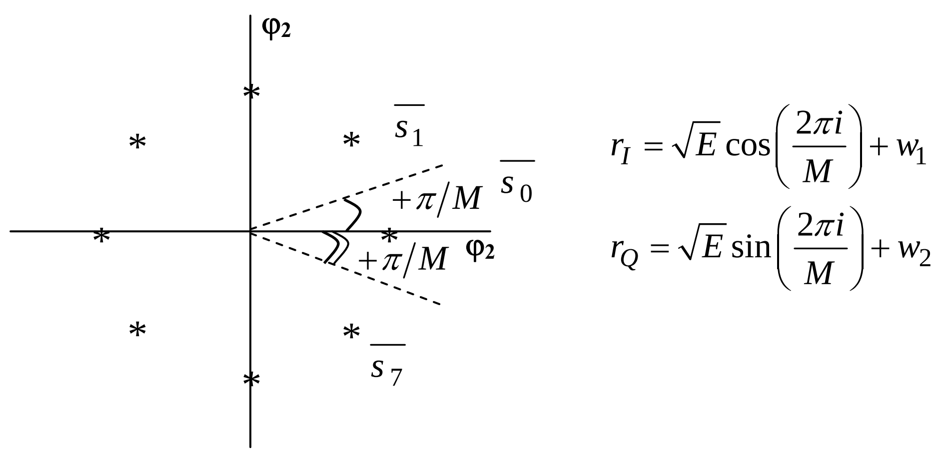

Fig. 5.28.3(a) *Signal constellation for M-ary PSK showing the decision zone Z_0*

The basis functions are :

$$\varphi_1(t) = \sqrt{\frac{2}{T}} \cos w_c t \text{ and } \varphi_2(t) = \sqrt{\frac{2}{T}} \sin w_c t; \qquad 0 \le t \le T \qquad 5.28.10$$

The decision zone Z_0 for the vector $\bar{s}_0$, making an angle of $\pm \pi^c/8 = \pm \pi^c/M$ at the origin, is also shown in the figure. The I-Q demodulator structure is reproduced in **Fig. 5.28.3(b)** for ease of reference. In case of PSK modulation, the Maximum Likelihood decision procedure can be equivalently framed in terms of a 'phase discrimination' procedure. This is usually followed in practice. The phase discriminator in **Fig. 5.28.3(b)** determines $\hat{\theta} = \tan^{-1}\left(\frac{r_Q}{r_I}\right)$ from r_I and r_Q as obtained from the correlation detector. The sign and magnitude of $\hat{\theta}$ produces the optimum estimate of a transmitted symbol.

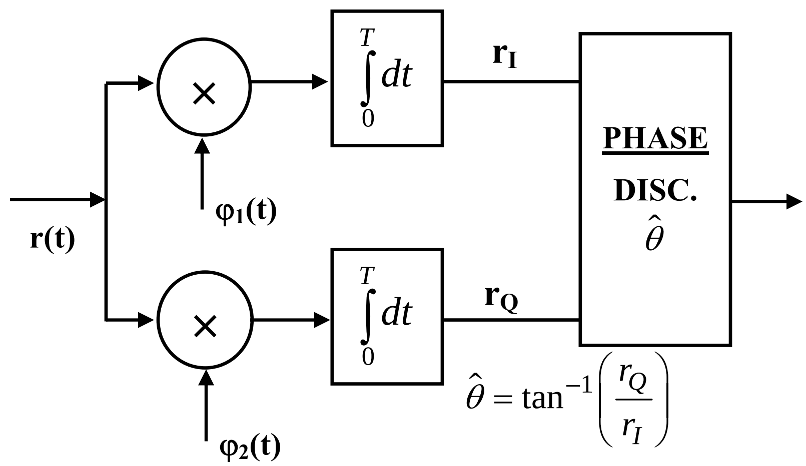

Fig. 5.28.3(b) *I-Q structure for coherent demodulation of M-ary PSK signal*

Determination of the expression for average Symbol Error Rate (SER) will be avoided here for simplicity. However, the approach is general and similar to what we have followed earlier. For example, consider the decision zone of $\overline{s_0}$ where $-\frac{\pi}{M} < \hat{\theta} < +\frac{\pi}{M}$. Now, if $f_\theta\left(\hat{\theta}\right)$ denotes the likelihood function for $\overline{s_0}$, the probability of correct decision for $\overline{s_0}$ ($P_{c_{s0}}$)is:

$$P_{c_{s0}} = \int_{-\pi/M}^{\pi/M} f_\theta\left(\hat{\theta}\right) d\hat{\theta} \qquad 5.28.11$$

So, the probability for erroneous decision when $\overline{s_0}$ is transmitted = $P_{e_{s0}}$ = 1 - $P_{c_{s0}}$.

Fig. 5.28.4 shows the probability of symbol error of M-ary PSK modulation schemes for M = 8, 16 and 32. For comparison, the performance of QPSK modulation is also included. Note that the error performance degrades significantly with increase in the number of symbols (M). This is intuitively acceptable because, with increase in M, the decision region of a symbol narrows down and hence, smaller amount of noise may cause decision error.

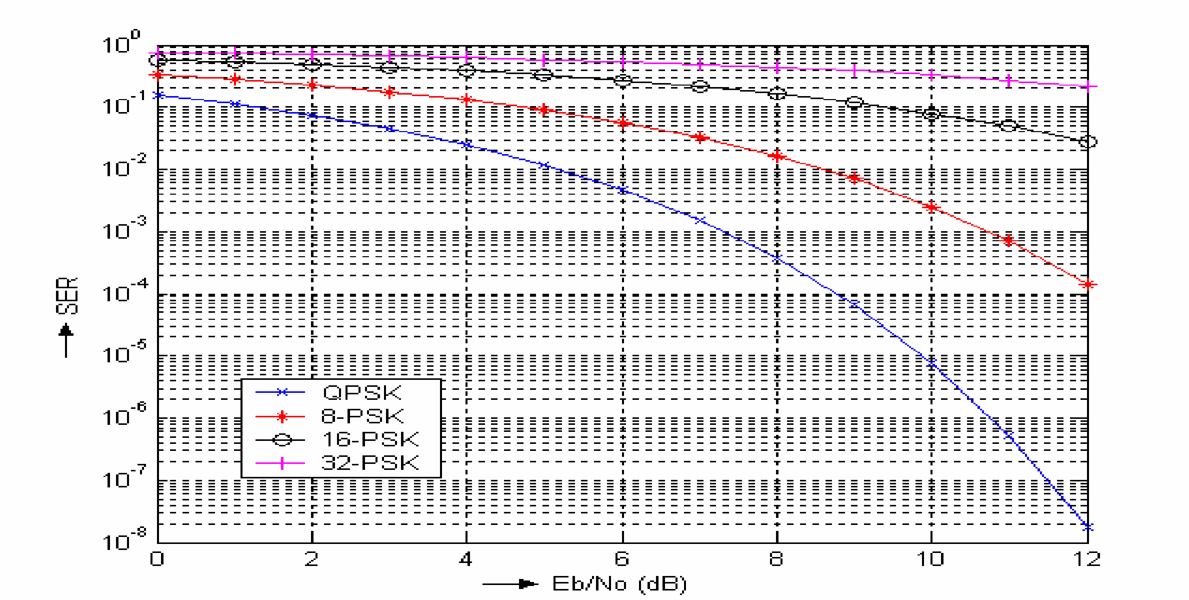

Fig. 5.28.4 *Optimum error performance of M-ary PSK demodulators for M= 4, 8, 16 and 32*

Another issue, which is of great significance in practice, is the need for precise recovery of carrier phase at the receiver so that the optimum coherent demodulation approach can be adopted. Usually, the required level of accuracy pushes up the complexity (and cost) of an M-ary PSK receiver for higher values of M. Still, M-ary modulation schemes have been popular for moderate values of 'M' such as 8 and16 because of their bandwidth efficiency. Another related family of modulation scheme, known as Quadrature Amplitude Modulation (QAM) has become attractive in terms of performance-bandwidth tradeoff especially for large number of signal points (M). We will briefly discuss about QAM in the next lesson.

Problems

Q5.28.1) Briefly mention how a binary FSK modulated signal can be demodulated non-coherently.

Q5.28.2) Compare Binary FSK & binary PSK modulation scheme.