

# Orthogonal Functions Approach to LQG Control

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**Abstract**—In this paper a unified approach via block-pulse functions (BPFs) or shifted Legendre polynomials (SLPs) is presented to solve the linear-quadratic-Gaussian (LQG) control problem. Also a recursive algorithm is proposed to solve the above problem via BPFs. By using the elegant operational properties of orthogonal functions (BPFs or SLPs) these computationally attractive algorithms are developed. To demonstrate the validity of the proposed approaches a numerical example is included.

**Keywords**—Linear quadratic Gaussian control; Linear quadratic estimator; Linear quadratic regulator; Time-invariant systems; Orthogonal functions; Block-pulse functions; Shifted Legendre polynomials.

## I. INTRODUCTION

**T**HE LQG control problem [1] concerns linear systems disturbed by additive white Gaussian noise, incomplete state information and quadratic costs. The LQG controller is simply the combination of a linear-quadratic estimator (LQE), i.e. Kalman filter with a linear quadratic regulator (LQR). The separation principle guarantees that these can be designed and computed independently.

Orthogonal functions approach [8], [9] has been recognized as an efficient and useful approach computationally to solve variety of problems in systems and control. In [6] the solution of the LQG control design problem was obtained by employing general orthogonal polynomials. In [10] the authors considered the problem of LQG control system and showed its application as an information transmission problem. The discrete time LQG problem was considered in [11] and showed its applications over lossy data networks.

Very recently, applications of orthogonal function approach is extended to different type of systems, i.e. systems described by integro-differential equations [12], multi-delay systems [13], [17], distributed parameter systems [14], delay systems with reverse time functions [15], singular systems [16] and to nonlinear systems [18].

In this paper, we consider linear time-invariant systems and propose a unified approach, based on using BPFs or SLPs, to solve LQG control problem of such systems. We call this approach unified approach because it can be used via SLPs or BPFs. In addition to the unified approach a recursive algorithm is proposed using BPFs. It is very important to note that the LQG control problem is not yet studied via BPFs.

The paper is organized as follows : The next section deals with BPFs and SLPs, and their properties. The LQG control

design problem is discussed in Section 3. The method of obtaining solution of the LQG control design problem is presented in Section 4. A numerical example is considered in Section 5. The last section concludes the paper.

## II. ORTHOGONAL FUNCTIONS AND THEIR PROPERTIES

We consider two classes of orthogonal functions, namely BPFs and SLPs, and discuss their properties.

### A. BPFs and their properties [4], [7]

A set of  $m$  BPFs, orthogonal over  $t \in [t_0, t_f]$ , is defined as

$$B_i(t) = \begin{cases} 1, & t_0 + iT \leq t < t_0 + (i+1)T \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

for  $i = 0, 1, 2, \dots, m-1$ , where

$$T = \frac{t_f - t_0}{m}, \text{ the block-pulse width} \quad (2)$$

A square integrable function  $f(t)$  on  $t_0 \leq t \leq t_f$  can be approximately represented in terms of BPFs as

$$f(t) \approx \sum_{i=0}^{m-1} f_i B_i(t) = \mathbf{f}^T \mathbf{B}(t) \quad (3)$$

where

$$\mathbf{f} = [f_0, f_1, \dots, f_{m-1}]^T \quad (4)$$

is an  $m$  - dimensional block-pulse spectrum of  $f(t)$ , and

$$\mathbf{B}(t) = [B_0(t), B_1(t), \dots, B_{m-1}(t)]^T \quad (5)$$

an  $m$  - dimensional BPF vector.  $f_i$  in Eq. (3) is given by

$$f_i = \frac{1}{T} \int_{t_0+iT}^{t_0+(i+1)T} f(t) dt \quad (6)$$

which is the average value of  $f(t)$  over  $t_0 + iT \leq t \leq t_0 + (i+1)T$ . The product of two BPFs  $B_i(t)$  and  $B_j(t)$  can be expressed as

$$B_i(t)B_j(t) = \begin{cases} 0 & \text{if } i \neq j \\ B_i(t) & \text{if } i = j \end{cases} \quad (7)$$

### Operational matrix of forward integration [4] :

Integrating  $\mathbf{B}(t)$  from  $t_0$  to  $t$  and expressing the result in  $m$ -set of BPFs, we have

$$\int_{t_0}^t \mathbf{B}(\tau) d\tau \approx P_f \mathbf{B}(t) \quad (8)$$

where

$$P_f = T \begin{bmatrix} \frac{1}{2} & 1 & 1 & \dots & 1 \\ 0 & \frac{1}{2} & 1 & \dots & 1 \\ 0 & 0 & \frac{1}{2} & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \frac{1}{2} \end{bmatrix} \quad (9)$$

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is called the operational matrix of forward integration of BPFs and it is an  $m \times m$  upper triangular matrix.

**Operational matrix of backward integration [7] :**

We integrate  $B(t)$  from  $t_f$  to  $t$  and express the result in  $m$ -set of BPFs to obtain

$$\int_{t_f}^t B(\tau)d\tau \approx P_b B(t) \quad (10)$$

where

$$P_b = -T \begin{bmatrix} \frac{1}{2} & 0 & 0 & \dots & 0 \\ 1 & \frac{1}{2} & 0 & \dots & 0 \\ 1 & 1 & \frac{1}{2} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & \frac{1}{2} \end{bmatrix} = -P_f^T \quad (11)$$

We call  $P_b$  the operational matrix of backward integration of BPFs. It is an  $m \times m$  lower triangular matrix.

**B. SLPs and their properties [8]**

SLPs satisfy the recurrence relation

$$L_{i+1}(t) = \frac{(2i+1)}{(i+1)} \varphi(t) L_i(t) - \frac{i}{(i+1)} L_{i-1}(t) \quad (12)$$

for  $i = 1, 2, 3, \dots$  with

$$\varphi(t) = \frac{2(t-t_0)}{(t_f-t_0)} - 1 \quad (13)$$

$$L_0(t) = 1, \text{ and } L_1(t) = \varphi(t) \quad (14)$$

A function  $f(t)$  that is square integrable on  $t \in [t_0, t_f]$  can be represented in terms of SLPs as

$$f(t) \approx \sum_{i=0}^{m-1} f_i L_i(t) = \mathbf{f}^T \mathbf{L}(t) \quad (15)$$

Here  $\mathbf{f}$  is called Legendre spectrum of  $f(t)$ , given in Eq. (4) and

$$\mathbf{L}(t) = [ L_0(t), L_1(t), \dots, L_{m-1}(t) ]^T \quad (16)$$

is called SLP vector.  $f_i$  in Eq. (15) is given by

$$f_i = \frac{(2i+1)}{(t_f-t_0)} \int_{t_0}^{t_f} f(t) L_i(t) dt \quad (17)$$

The product of two SLPs  $L_i(t)$  and  $L_j(t)$  can be expressed as

$$L_i(t) L_j(t) \approx \sum_{k=0}^{m-1} \psi_{ijk} L_k(t) \quad (18)$$

where

$$\psi_{ijk} = \frac{(2k+1)}{(t_f-t_0)} \pi_{ijk} \quad (19)$$

$$\pi_{ijk} = \pi_{ikj} = \pi_{jik} = \pi_{kji} = \pi_{kji} = \pi_{kij} \quad (20)$$

$$\pi_{ijk} = \begin{cases} \frac{a_i a_{j-l} a_{i-j+l}}{a_{i+l}} \frac{(t_f-t_0)}{2^{(i+l)+1}} & \text{if } k = i-j+2l \\ 0 & \text{if } k \neq i-j+2l \end{cases} \quad (21)$$

for  $i \geq j$

$$a_0 = 1, \quad a_{l+1} = \frac{(2l+1)}{(l+1)} a_l \quad (22)$$

for  $l = 0, 1, 2, \dots$

**Operational matrix of forward integration [8] :**

Integrating  $L(t)$  from  $t_0$  to  $t$ , and expressing the result in terms of the same set of SLPs, we have

$$\int_{t_0}^t L(\tau)d\tau \approx P_f L(t) \quad (23)$$

where

$$P_f = \frac{(t_f-t_0)}{2} \begin{bmatrix} 1 & 1 & 0 & 0 & \dots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \dots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \frac{1}{2m-3} \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (24)$$

which is called the operational matrix of forward integration of SLPs.

**Operational matrix of backward integration [5] :**

If we integrate  $L(t)$  from  $t_f$  to  $t$  and express the result in terms of the same set of SLPs, we have

$$\int_{t_f}^t L(\tau)d\tau \approx P_b L(t) \quad (25)$$

where

$$P_b = \frac{(t_f-t_0)}{2} \begin{bmatrix} -1 & 1 & 0 & 0 & \dots & 0 \\ -\frac{1}{3} & 0 & \frac{1}{3} & 0 & \dots & 0 \\ 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \dots & \frac{1}{2m-3} \\ 0 & 0 & 0 & 0 & \dots & 0 \end{bmatrix} \quad (26)$$

which is called the operational matrix of backward integration of SLPs.

**III. THE LQG CONTROL PROBLEM**

Consider the linear dynamic system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{v}(t) \quad (27)$$

$$\mathbf{z}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{w}(t) \quad (28)$$

where  $\mathbf{x}(t)$  is  $n$  dimensional state vector,  $\mathbf{u}(t)$   $p$  dimensional control vector, and  $\mathbf{z}(t)$   $q$  dimensional output vector, and  $\mathbf{v}(t)$  and  $\mathbf{w}(t)$  the additive zero-mean white Gaussian system noise and measurement noise, respectively, i.e.

$$E \{ \mathbf{v}(t) \mathbf{v}^T(\tau) \} = Q_2 \delta(t-\tau) \quad (29)$$

$$E \{ \mathbf{w}(t) \mathbf{w}^T(\tau) \} = R_2 \delta(t-\tau) \quad (30)$$

where  $Q_2$  is positive semi-definite and  $R_2$  is positive definite symmetric matrices. Also  $\mathbf{v}(t)$  is uncorrelated with  $\mathbf{w}(t)$ , i.e.

$$E \{ \mathbf{v}(t) \mathbf{w}^T(\tau) \} = 0 \quad (31)$$

Assume that the initial condition  $\mathbf{x}(t_0)$  is Gaussian with mean  $\bar{\mathbf{x}}(t_0)$  and covariance matrix

$$P = P_2(t_0) = E \{ [\mathbf{x}(t_0) - \bar{\mathbf{x}}(t_0)] [\mathbf{x}(t_0) - \bar{\mathbf{x}}(t_0)]^T \}$$

which is symmetric positive semi-definite, and

$$\mathbb{E}\{\mathbf{v}(t)\mathbf{x}_0^T\} = \mathbb{E}\{\mathbf{w}(t)\mathbf{x}_0^T\} = 0 \quad \text{for } t \geq t_0 \quad (32)$$

Given this system, the objective is to find the control input  $\mathbf{u}(t)$  which at every time  $t$  may depend only on the past measurements  $\mathbf{z}(t_1)$ ,  $t_0 \leq t_1 < t$  such that the cost function

$$J = \mathbb{E}\left\{\frac{1}{2}\mathbf{x}^T(t_f)S\mathbf{x}(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [\mathbf{x}^T(t)Q_1\mathbf{x}(t) + \mathbf{u}^T(t)R_1\mathbf{u}(t)] dt\right\} \quad (33)$$

is minimized, where the matrix  $R_1$  is positive definite symmetric, and  $S$  and  $Q_1$  are symmetric positive semi-definite matrices.

The LQG controller that solves the LQG control problem is specified by the equations

$$\dot{\hat{\mathbf{x}}}(t) = A\hat{\mathbf{x}}(t) + B\mathbf{u}(t) + K_2(t)[\mathbf{z}(t) - C\hat{\mathbf{x}}(t)] \quad (34)$$

$$\hat{\mathbf{x}}(t_0) = \mathbb{E}[\mathbf{x}(t_0)] = \bar{\mathbf{x}}_0$$

$$\mathbf{u}(t) = -K_1(t)\hat{\mathbf{x}}(t) \quad (35)$$

The matrix  $K_2(t)$  is called the Kalman gain of the associated Kalman filter represented by Eq. (34). At each time  $t$  this filter generates estimates  $\hat{\mathbf{x}}(t)$  of the state  $\mathbf{x}(t)$  using the past measurements and inputs. The Kalman gain is determined through the associated matrix Riccati differential equation

$$\dot{P}_2(t) = AP_2(t) + P_2(t)A^T - P_2(t)C^TR_2^{-1}CP_2(t) + Q_2 \quad (36)$$

$$P_2(t_0) = P$$

Given the solution  $P_2(t)$ ,  $t_0 \leq t \leq t_f$  the Kalman gain equals

$$K_2(t) = P_2(t)C^TR_2^{-1} \quad (37)$$

The matrix  $K_1(t)$  is called the feedback gain matrix which is determined through the associated matrix Riccati differential equation

$$-\dot{P}_1(t) = A^TP_1(t) + P_1(t)A - P_1(t)BR_1^{-1}B^TP_1(t) + Q_1 \quad (38)$$

$$P_1(t_f) = S$$

Given the solution  $P_1(t)$ ,  $t_0 \leq t \leq t_f$  the feedback gain equals

$$K_1(t) = R_1^{-1}B^TP_1(t) \quad (39)$$

Observe the similarity of the two matrix Riccati differential Equations (36) and (38); the first one running forward in time, and the second one running backward in time. The first one solves the LQE problem and the second one solves LQR problem. So the LQG problem separates into LQE and LQR problems that can be solved independently.

The block diagram of the LQG problem is presented in Figure 1.

#### IV. ORTHOGONAL FUNCTIONS APPROACH

Integrating the Riccati equation (38) backward in time from  $t_f$  to  $t$ , we obtain

$$-[P_1(t) - S] = \int_{t_f}^t [A^TP_1(\tau) + P_1(\tau)A - P_1(\tau)FP_1(\tau) + Q_1] d\tau \quad (40)$$

where  $F = BR_1^{-1}B^T$ . Expressing  $P_1(t)$ ,  $P_1(t)FP_1(t)$ ,  $Q_1$  and  $S$  in terms of orthogonal functions  $\{\phi_i(t)\}$ , which may be BPFs  $\{B_i(t)\}$  or SLPs  $\{L_i(t)\}$ , we have

$$P_1(t) \simeq \sum_{i=0}^{m-1} P_{1i}\phi_i(t) = \tilde{P}_1(\phi(t) \otimes I_n) \quad (41)$$

where

$$\tilde{P}_1 = [P_{10}, P_{11}, \dots, P_{1,m-1}] \quad (42)$$

Then

$$A^TP_1(t) \simeq \bar{P}_1(\phi(t) \otimes I_n) \quad (43)$$

$$P_1(t)A \simeq \hat{P}_1(\phi(t) \otimes I_n) \quad (44)$$

where

$$\bar{P}_1 = [A^TP_{10}, A^TP_{11}, \dots, A^TP_{1,m-1}] \quad (45)$$

$$\hat{P}_1 = [P_{10}A, P_{11}A, \dots, P_{1,m-1}A] \quad (46)$$

$$P_1(t)FP_1(t) \simeq \sum_{i=0}^{m-1} P_{1i}FP_{1i}\phi_i(t) \quad (47)$$

if BPFs are used

$m-1 \quad m-1 \quad m-1$

$$\simeq \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} \sum_{k=0}^{m-1} P_{1i}FP_{1j}\psi_{ijk}\phi_k(t) \quad (48)$$

if SLPs are used

$$\simeq \tilde{F}(\phi(t) \otimes I_n) \quad (49)$$

where

$$\tilde{F} = [P_{10}FP_{10}, P_{11}FP_{11}, \dots, P_{1,m-1}FP_{1,m-1}] \quad (50)$$

if  $\phi(t)$  is  $\mathbf{B}(t)$

$$= \left[ \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} \psi_{ij0}P_{1i}FP_{1j}, \dots, \dots, \sum_{i=0}^{m-1} \sum_{j=0}^{m-1} \psi_{ij,m-1}P_{1i}FP_{1j} \right] \quad (51)$$

if  $\phi(t)$  is  $\mathbf{L}(t)$

$$Q_1 = \tilde{Q}_1(\phi(t) \otimes I_n) \quad (52)$$

where

$$\tilde{Q}_1 = [Q_1, Q_1, \dots, Q_1] \quad \text{if } \phi(t) \text{ is } \mathbf{B}(t) \quad (53)$$

$$= [Q_1, 0, \dots, 0] \quad \text{if } \phi(t) \text{ is } \mathbf{L}(t) \quad (54)$$

and

$$S = \tilde{S}(\phi(t) \otimes I_n) \quad (55)$$

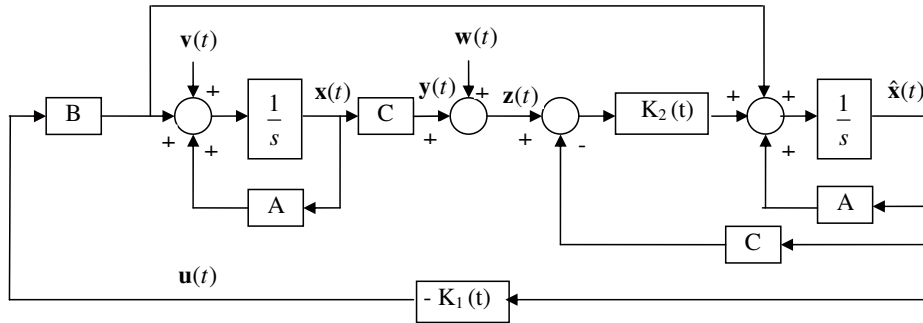


Fig. 1. Optimum linear combined estimation and control

where

$$\tilde{S} = [S, S, \dots, S] \text{ if } \phi(t) \text{ is } \mathbf{B}(t) \quad (56)$$

$$= [S, 0, \dots, 0] \text{ if } \phi(t) \text{ is } \mathbf{L}(t) \quad (57)$$

and  $\otimes$  is the Kronecker product [3].

Substituting Eqs. (41), (43), (44), (49), (52) and (55) into Eq. (40) and making use of the backward integration operational property in Eq. (10) or (25), we have

$$-\tilde{P}_1 + \tilde{S} = [\tilde{P}_1 + \hat{P}_1 - \tilde{F} + \tilde{Q}_1] (P_b \otimes I_n)$$

$$\Rightarrow \tilde{P}_1 + [\tilde{P}_1 + \hat{P}_1 - \tilde{F}] (P_b \otimes I_n) = \tilde{S} - \tilde{Q}_1 (P_b \otimes I_n) \quad (58)$$

which is to be solved for the spectrum of  $P_1(t)$ . Similarly, the spectrum of  $P_2(t)$  can also be found from the Riccati equation (36), and is given by

$$\tilde{P}_2 - [\tilde{P}_2 + \hat{P}_2 - \tilde{G}] (P_f \otimes I_n) = \tilde{P} + \tilde{Q}_2 (P_f \otimes I_n) \quad (59)$$

where

$$\tilde{P}_2 = [AP_{20}, AP_{21}, \dots, AP_{2,m-1}] \quad (60)$$

$$\hat{P}_2 = [P_{20}A^T, P_{21}A^T, \dots, P_{2,m-1}A^T] \quad (61)$$

and  $G = C^T R_2^{-1} C$

Notice that both the Riccati equations are thus reduced to the non-linear algebraic equations (58) and (59), which can easily be solved using Newton-Raphson method.

#### A. Recursive algorithm via BPFs

For a scalar system it is possible to obtain a recursive algorithm if BPFs are used. This point is discussed here. Substituting the operational matrix of backward integration  $P_b$  in Eq. (11) into Eq. (58) and simplifying, we obtain the following recursive algorithm : \_\_\_\_\_

#### V. NUMERICAL EXAMPLE

Consider the linear system [2], [6]

$$\dot{x}(t) = -0.5x(t) + u(t) + v(t)$$

$$\bar{x}(0) = 10$$

with the measurement

$$z(t) = x(t) + w(t)$$

and the cost function

$$J = E \left\{ 0.5x^2(t_f)S + 0.5 \int_0^{t_f} [2x^2(t) + u^2(t)] dt \right\}$$

where

$$E \{v(t)v(\tau)\} = 2\delta(t - \tau)$$

$$E \{w(t)w(\tau)\} = 0.25\delta(t - \tau)$$

$$E \{[x(0) - \bar{x}(0)]^2\} = 0$$

If  $S = 0$  and  $t_f = 1$ , the exact solutions of  $P_1(t)$  and  $P_2(t)$  are given by

$$P_1(t) = -0.5 + 1.5 \tanh(-1.5t + 1.8465736) \quad \text{and}$$

$$P_2(t) = -0.125 + 0.125\sqrt{33} \tanh\left\{0.5\sqrt{33}t + \tan^{-1}\left(1/\sqrt{33}\right)\right\}$$

So with  $m = 4$  and 24 the above recursive algorithm via BPFs and with  $m = 4$  the nonrecursive approach in Section 4 via SLPs are applied, and  $P_1(t)$  and  $P_2(t)$  are computed as shown in Figs. 2 and 3. The exact solutions are also shown in the same figures for comparison sake. The results are quite satisfactory even with four SLPs.

#### VI. CONCLUSION

A unique method to determine the filter gain and the regulator gain in LQG control problem is proposed. It is shown that the application of orthogonal functions (BPFs and SLPs) reduces differential calculus to algebra. A BPF based recursive algorithm is presented to solve the LQG control problem of linear time-invariant scalar systems. An illustrative example is

Recursive algorithm :

$$P_{1,m-1} = -\frac{1}{F} \left( \frac{1}{T} - A \right) + \sqrt{\left[ \frac{1}{F} \left( \frac{1}{T} - A \right) \right]^2 + \frac{Q_1}{F}} \tag{62}$$

$$P_{1,j} = -\frac{1}{F} \left( \frac{1}{T} - A \right) + \sqrt{\left[ \frac{1}{F} \left( \frac{1}{T} - A \right) \right]^2 + \frac{2Q_1}{F} + \frac{2}{F} \left( \frac{1}{T} + A \right) P_{1,j+1} - P_{1,j+1}^2} \tag{63}$$

for  $j = m - 2, m - 3, \dots, 1, 0$ .

Similarly, substituting the operational matrix of forward integration  $P_f$  in Eq. (9) into Eq. (59) we have

$$P_{2,0} = -\frac{1}{G} \left( \frac{1}{T} - A \right) + \sqrt{\left[ \frac{1}{G} \left( \frac{1}{T} - A \right) \right]^2 + \frac{Q_2}{G}} \tag{64}$$

$$P_{2,j} = -\frac{1}{G} \left( \frac{1}{T} - A \right) + \sqrt{\left[ \frac{1}{G} \left( \frac{1}{T} - A \right) \right]^2 + \frac{2Q_2}{G} + \frac{2}{G} \left( \frac{1}{T} + A \right) P_{2,j-1} - P_{2,j-1}^2} \tag{65}$$

for  $j = 1, 2, \dots, m - 1$ . Such a recursive algorithm is not possible with SLPs.

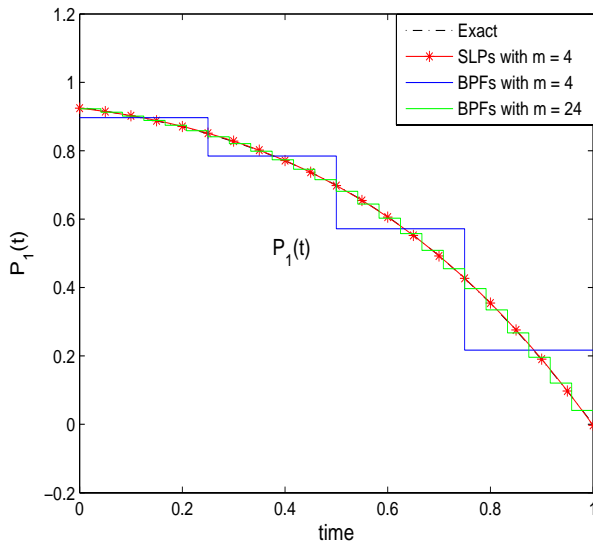


Fig. 2. Exact, SLP and BPF solutions of  $P_1(t)$

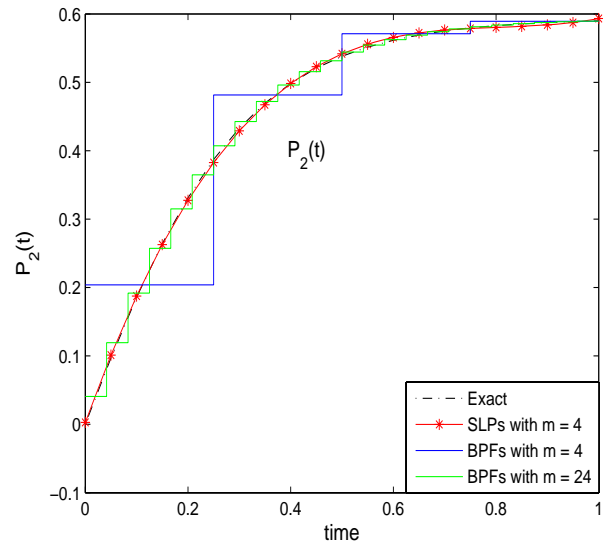


Fig. 3. Exact, SLP and BPF solutions of  $P_2(t)$

included to demonstrate the usefulness of the unified approach via SLPs and the recursive algorithm via BPFs. As can be seen from Figs. 2 and 3, only four SLPs are good enough to obtain the result which is almost following the exact solution. One has to consider a large number of BPFs to improve upon the accuracy. This is because we are using piecewise constant functions (BPFs) to represent the smooth functions  $P_1(t)$  and  $P_2(t)$  in the present context.

Every approach (SLP or BPF) has its own advantage and disadvantage. SLP method does not require large number of polynomials in series expansion to represent smooth functions, but computationally it is not as much attractive as BPF

method because SLPs are to be computed and used for signal representation while it is not so in BPF method as BPFs are all unity and disjoint.

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