Multi User Diversity Evaluation in MIMO HSDPA Downlink Channels

G.Vamshi Krishna, Ch. Suresh Kumar, U. Kiran and K. Rajesh Reddy
Department of ECE
Vaagdevi College of Engineering
Warangal, India.
Email:krish.vamshee@gmail.com, snani1119@gmail.com
Kiranuday411@gmail.com and Rajeshreddy_488@yahoo.co.in

Abstract— **A multiple transmit antenna, single receive antenna (per receiver) downlink channel with limited channel feedback is considered. Given a constraint on the total system-wide channel feedback, the following question is considered: is it preferable to get low-rate feedback from a large number of receivers or to receive high-rate/high-quality feedback from a smaller number of (randomly selected) receivers. Acquiring feedback from many users allows multi-user diversity to be exploited, while highrate feedback allows for very precise selection of beamforming directions. It is shown that systems in which a limited number of users feedback high-rate channel information significantly outperform low-rate/many user systems. The marginal benefit of channel feedback is very significant up to the point where the CSI is essentially perfect.**

keywords: Multi-user multi-input multi-output, Base Station, Channel Feedback, Channael State Information(CSI), Random Beforming.

I. INTRODUCTION

Multi-user multiple-input, multiple-output (MU-MIMO) communication is very powerful and has recently been the subject of intense research. A transmitter equipped with M_t antennas can serve up to M_t users simultaneously over the same time-frequency resource, even if each receiver has only a single antenna. Such a model is very relevant to many applications, such as the cellular downlink from base station (BS) to mobiles (users). However, knowledge of the channel is required at the BS in order to fully exploit the gains offered by MU-MIMO.

In systems without channel reciprocity (such as frequency-division duplexed systems), the BS obtains Channel State Information (CSI) via channel feedback from mobiles. In the single antenna per mobile setting, feedback strategies involve each mobile quantizing its M_t-dimensional channel vector and feeding back the corresponding bits approximately every channel coherence time. Although there has been considerable prior work on this issue of channel feedback, e.g., optimizing feedback contents and quantifying the sensitivity of system throughput to the feedback load, almost all of it has been performed from the perspective of the per-user feedback load. Given that channel feedback consumes considerable uplink resources (bandwidth and power), the aggregate feedback load, summed across users, is more meaningful than the per-user load from a system design perspective. However, it is not yet well understood how an aggregate feedback budget is best utilized.

Thereby motivated, in this paper we ask the following fundamental design question:

For a fixed aggregate feedback load, is a larger system sum rate achieved by collecting a small amount of per-user feedback from a large number of users, or by collecting a larger amount of per-user feedback from a smaller subset of users?

Assuming an aggregate feedback load of T_{fb} bits, we consider a system where T_{fb}/K users quantize their channel direction to K bits each and feedback these bits along with one real number (per user) representing the channel quality. The BS then selects, based upon the feedback received from the T_{fb}/K users, up to M_t users for transmission using multi-user beamforming. A larger value of K corresponds to more accurate CSI but fewer users and reduced multi-user diversity. By comparing the sum rates for different values of K, we reach the following striking conclusion: for almost any number of antennas M_t, average SNR, and feedback budget T_{fb}, sum rate is maximized by choosing K (feedback bits per user) such that near-perfect CSI is obtained for each of the T_{fb}/K users that do feedback. In other words, accurate CSI is more valuable than multi-user diversity. In a 4 antenna ($M_t = 4$) system operating at 10 dB with $T_{fb} = 100$ bits, for example, it is nearoptimal to have 5 users (arbitrarily chosen from a larger user set) feedback K=20 bits each. This provides a sum rate of 9.9 bps/Hz, whereas 10 users with K = 10 with and 25 users with K = 4 (i.e., operating with less accurate CSI) provide sum rates of only 8.5 and 4.6, respectively.

Our finding is rather surprising in the context of prior work on schemes with a very small per-user feedback load. Random beamforming (RBF), which requires only $\log_2 M_t$ feedback bits per user, achieves a sum rate that scales with the number of users in the same manner as the perfect-CSI sum rate [1], and thus appears to be a good technique when there are a large number of users. On the contrary, we find that RBF achieves a significantly smaller sum rate than a system using a large value of K. This is true even when T_{fb} is extremely large, in which case the number of users who feedback is very large (and thus multi-user diversity is plentiful) if RBF is used. Although perhaps not initially apparent, the problem considered here has very direct relevance to system design. The designer must specify how often (in time) mobiles feedback CSI and the portion of the channel response (in frequency) that the CSI feedback corresponds to. If each mobile feeds back CSI for essentially every time/frequency coherence block, then the BS will have many users to select from (on every block) but, assuming a constraint on the total feedback, the CSI accuracy will be rather limited, thereby corresponding to a small value of K in our setup. On the other hand, mobiles could be grouped in frequency and/or time and thus only feedback information about a subset of time/frequency coherence blocks; this corresponds to fewer users but more accurate CSI (i.e., larger K) on each resource block. Our results imply a very strong preference towards the latter strategy.

I. SYSTEM MODEL

We consider a multi-input multi-output (MIMO) Gaussian broadcast channel Where N transmitting antennas at the transmitter (the BS or transmitter) and K single antenna users.

The channel output y_a at user a is given by,

$$y_a = h_a^B x + c_a \text{, a=1, 2,……..,K} \tag{1}$$

Where C_a~CN(0,1) models Additive White Gaussian Noise (AWGN), h_a= $[h_{a,1},……h_{a,Q}]$is the vector of channel coefficients from the k^{th} user antenna to the transmitter antenna array and x is the vector of channel input symbols transmitted by the base station. The channel input is subject to an average power constraint $E\left[\|x\|_2^2\right]$≤SNR. We assume that the channel state, given by the collection of all channel vectors, varies in time according to a block-fading model, where the channels are constant within a block but vary independently from block to block. The entries of each channel vector are Gaussian with elements ~CN(0,1) . Each user is assumed to know its own channel perfectly.

At the beginning of each block, each user quantizes its channel to K bits and feeds back the bits, in an error- and delay-free manner, to the BS. Vector quantization is performed using a codebook N that consists of 2^K M_t-dimensional unit norm vectors $N \overset{\Delta}{=} \{w_1, . . . ,w_{2^K}\}$. Each user quantizes its channel vector to the quantization vector that forms the minimum angle to it. Thus, user *a* quantizes its channel to $\hat{h}_a$ and feeds the K-bit index back to the transmitter, where $\hat{h}_a$ is chosen according to:

$$\hat{h}_a = \arg\min_{w \in N} \sin^2(\angle(h_a, w)) \tag{2}$$

Where

$$\cos^2(\angle(h_a, w)) = \frac{\left|h_a^B w\right|^2}{\|h_a\|^2 \|w_k\|^2} = 1 - \sin^2(\angle(h_a, w))$$

The specification of the quantization codebook is discussed later. Each user also feeds back a single real number, which can be the channel norm or some other Channel Quality Indicator (CQI). We assume that this CQI is known perfectly to the BS, i.e., it is not quantized, and thus CQI feedback is not included in the feedback budget; this simplification is investigated.

For a total aggregate feedback load of T_{fb} bits, we are interested in the sum rate (of the different feedback/beamforming strategies described later in this section) when T_{fb}/K users feedback K bits each. The T_{fb}/K users who feed back are arbitrarily selected from a larger user set.[1] Furthermore, in our block fading setting, only those users who feed back in a particular block/coherence time are considered for transmission in that block; in other words, we are limited to transmitting to a subset of only the T_{fb}/K users.

II. COMPARISON OF DIFFERENT TECHNIQUES

A. Zero Forcing Beamforming

When Zero-Forcing (ZF) is used, each user feeds back the K-bit quantization of its channel direction as well as the channel norm $\|h_a\|$ representing the channel quality. The BS then uses the greedy user selection algorithm described in [10], adopted to imperfect CSI by treating the vector $\|h_a\| \cdot \hat{h}_a$ (which is known to the BS) as if it were user *a*'s true channel. The algorithm first selects the user with the largest CQI. In the next step the ZF sum rate is computed for every pair of users that includes the first selected user (where the rate is computed assuming $\|h_a\| \cdot \hat{h}_a$ is the true channel of user a), and the additional user that corresponds to the largest sum rate is selected next. This process of adding one user at a time, in greedy fashion, is continued until M_t users are selected or there is no increase in sum rate. Unlike [10], we do not optimize power and instead equally split power amongst the selected users.

We denote the indices of selected users by Π(1), . . . ,Π(n), where n≤M_t is the number of users selected (*n* depends on the particular channel vectors). By the ZF criteria, the unit-norm beamforming vector $\hat{v}_{\Pi(a)}$ for user $\Pi(a)$ is chosen in the direction of the projection of $\hat{h}_{\Pi(a)}$ on the null space of $\left\{\hat{h}_{\Pi(j)}\right\}_{j\neq a}$. Although ZF beamforming is used, there is residual interference because the beamformers are based on imperfect CSI. The (post-selection) SINR for selected user Π(a) is

$$\mathrm{SINR}_{\Pi(a)} = \frac{\frac{SNR}{n}\left\|h_{\Pi(a)}\right\|^2 \cos^2\left(\angle\left(h_{\Pi(a)}, \hat{v}_{\Pi(a)}\right)\right)}{1 + \frac{SNR}{n}\left\|h_{\Pi(a)}\right\|^2 \sum_{j\neq a} \cos^2\left(\angle\left(h_{\Pi(a)}, \hat{v}_{\Pi(j)}\right)\right)} \tag{3}$$

And the corresponding sum rate is

$$\sum_{a=1}^{n} \log_2\left(1 + \mathrm{SINR}_{\Pi(a)}\right) \tag{4}$$

For the sake of analysis and ease of simulation, each user utilizes a quantization codebook *C* consisting of unit-vectors independently chosen from the isotropic distribution on the

M_t-dimensional unit sphere [11] (Random Vector Quantization or RVQ). Each user's codebook is independently generated, and sum rate is averaged over this ensemble of quantization codebooks although we focus on RVQ, we show that our conclusions are not dependent on the particular quantization scheme used.

In [12] it is shown that the sum rate of ZF beamforming with quantized CSI but without user selection (i.e. M_t users are randomly selected) is lower bounded by:

$$R^{CSI}_{ZFB-\text{no selection}}(SNR) - M_t \log 2\left(1 + SNR.2^{-\frac{K}{M_t-1}}\right) \quad (5)$$

Where $R^{CSI}_{ZFB-\text{no selection}}(SNR)$ is the perfect CSI rate. This bound, which is quite accurate for large values of K [13], indicates that ZF beamforming is very sensitive to the CSI accuracy. With $M_t = 4$ and SNR = 10 dB, for example, K = 10 corresponds to a sum rate loss of 4 bps/Hz (relative to perfect CSI) and 17 bits are required to reduce this loss to 1bps/Hz. Equation 4 is no longer a lower bound when user selection is introduced, but nonetheless it is a reasonable approximation and hints at the importance of accurate CSI.

B. Random Beamforming

Random beamforming (RBF) was proposed in [1][3], wherein each user feeds back $\log_2 M_t$ bits along with one real number. In this case, there is a common quantization codebook *C* consisting of M_t orthogonal unit vectors and quantization is performed according to (2). In addition to the quantization index,each user feedbacks a real number representing its SINR. If w_m ($1 \le m \le M_t$) is the selected quantization vector for user *a*, then

$$SINR_a = \frac{\frac{SNR}{M_t}\left|h_a^B w_m\right|^2}{1 + \frac{SNR}{N_t}\sum_{n \neq m}\left|h_a^B w_n\right|^2} = \frac{\left\|h_a\right\|^2 \cos^2(\angle h_a, w_m)}{\frac{N_t}{SNR} + \left\|h_a\right\|^2 \sin^2(\angle h_a, w_m)} \quad (6)$$

After receiving the feedback, the BS selects the user with the largest SINR on each of M_t beams ($w_1, \ldots, w_{M_t}$), and beamforming is performed along these same vectors.

C. PU²RC

Per unitary basis stream user and rate control (PU²RC), proposed in [10], is a generalization of RBF in which there is a common quantization codebook N consisting of $2^{K-\log_2 M_t}$ 'sets' of orthogonal codebooks, where each orthogonal codebook consists of M_t orthogonal unit vectors, and thus a total of 2^K vectors. Quantization is again performed according to (2), and each user feeds back the same SINR statistic as in RBF. User selection is performed as follows: for each of the orthogonal sets the BS repeats the RBF user selection procedure and computes the sum rate (where the per-user rate is $\log_2(1 + SINR)$, after which it selects the orthogonal set with the highest sum rate. If $K=\log_2 M_t$, there is only a single orthogonal set and the scheme reduces to ordinary RBF. The primary difference between PU²RC and ZF is the user selection algorithm: PU²RC is restricted to selecting users within one of the orthogonal sets and thus has very low complexity, whereas the described ZF technique has no such restriction.

III. Optimization of Zero Force Beamforming

Let the sum rate for a system using ZF with M_t antennas at the transmitter is $R_{ZFB}\left(SNR, M_t, \frac{T_{fb}}{K}, K\right)$, signal-to-noise ratio SNR, and $\frac{T_{fb}}{K}$ users each feeding back K bits. K is varied within $1 + \log_2 M \le K \le \frac{T_{fb}}{K}$. We are interested in the number of feedback bits per user $K^{OPT}_{ZFB}\left(SNR, M_t, \frac{T_{fb}}{K}, K\right)$ that maximizes this sum rate for total feedback bits of T_{fb}.

Then, the optimum K can be written as

$$B^{OPT}_{ZFB}(SNR, M_t, T_{fb}) \stackrel{\Delta}{=} \arg\max_{\log_2 M_t \le B \le \frac{T_{fb}}{N_t}} R_{ZFB}\left(SNR, M_t, \frac{T_{fb}}{K}, K\right) \quad (7)$$

Then , the sum rate can be written as from (4)

$$R_{ZFB}\left(SNR, M_t, \frac{T_{fb}}{K}, K\right) = E\left[\sum_{a=1}^{n} \log_2\left(1 + \frac{\frac{SNR}{n}\left\|h_{\Pi(a)}\right\|^2 \cos^2\left(\angle\left(h_{\Pi(a)}, \hat{v}_{\Pi(a)}\right)\right)}{1 + \frac{SNR}{n}\left\|h_{\Pi(a)}\right\|^2 \sum_{j \neq a} \cos^2\left(\angle\left(h_{\Pi(a)}, \hat{v}_{\Pi(a)}\right)\right)}\right)\right] \quad (8)$$

The above expression for sum rate can be approximated

$$\hat{R}_{ZFB}\left(SNR, M_t, \frac{T_{fb}}{K}, K\right)$$

$$\stackrel{\Delta}{=} \quad M_t \log_2\left[1+\frac{\left(\frac{SNR}{M_t}\right)\log\left(\frac{T_{fb}M_t}{K}\right)}{1+\left(\frac{SNR}{M_t}\right)2^{-\frac{K}{M_t-1}}\log\left(\frac{T_{fb}M_t}{K}\right)}\right] \qquad (9)$$

The expression for approximated sum rate (9) from equation (8) is obtained by[11][12] the following changes to the expression (8).

(i) Replacing $\left\|h_{\Pi(a)}\right\|^2$ with $\log\left(\frac{T_{fb}M_t}{K}\right)$.

(ii) Assumed that the maximum number of users are selected i.e.,n=M.

(iii) Approximating the $\cos^2\left(\angle\left(h_{\Pi(a)}, \hat{v}_{\Pi(a)}\right)\right)$ in the SINR expression's (A) numerator with unity.

(iV) Replacing $\cos^2\left(\angle\left(h_{\Pi(a)}, \hat{v}_{\Pi(a)}\right)\right)$ in the SINR expression's (6) denominator with its expected value $\frac{2^{-\frac{K}{M_t-1}}}{(M_t-1)}$.

In the above approximated sum rate expression (9), the received signal power is $\left(\frac{SNR}{M_t}\right)\log\left(\frac{T_{fb}M_t}{K}\right)$, and the interference power $2^{-\left(\frac{K}{M_t-1}\right)}$ times the signal power. $2^{-\left(\frac{K}{M_t-1}\right)}$ term in the interference power is the evidence of imperfect CSI. Multi user diversity is evidenced in the term $\log\left(\frac{T_{fb}M_t}{K}\right)$.

From the above approximated sum rate expression, the approximated optimum K can be written as,

$$K_{ZFB}^{OPT}\left(SNR, M_t, T_{fb}\right) \approx \hat{K}_{ZFB}^{OPT}\left(SNR, M_t, T_{fb}\right)$$

$$\stackrel{\Delta}{=} \underset{\log_2 M_t \le K \le \frac{T_{fb}}{M_t}}{\arg\max}\ \hat{R}_{ZFB}\left(SNR, M_t, \frac{T_{fb}}{K}, K\right) \qquad (10)$$

By delivering $\hat{R}_{ZFB}\left(SNR, M_t\frac{T_{fb}}{K}, K\right)$ expression (9) and equating it to zero gives the expression in terms of K_{ZFB}^{OPT}, from which the optimum K can be solved.

$$\left(\frac{SNR}{M_t}\right)2^{\frac{\hat{K}_{ZFB}^{OPT}\log 2}{M_t-1}}\left(\log\frac{T_{fb}M_t}{K_{ZFB}^{OPT}}\right)^2 = 1 \qquad (11)$$

The above expression is solved for K.

In figure1, the sum rates of optimized Zero Force Beamforming and PU²RC are plotted versus total system feedback load T_{fb} bits, with M =4, SNR=10dB and with optimized K. In figure 2, the sum rates of optimized Zero Force Beamforming and PU²RC are plotted versus per user feedback load K bits with M=4, SNR=10dB and T_{fb}=300 bits.

IV. SIMULATION RESULTS

The simulation results are observed for M= 4 transmitting antennas, SNR=10dB for the various values of total feedback bits T_{fb} per user feedback bits for both feedback strategies Optimized Zero force beamforming and PU²RC. In case of T_{fb}=100bits, K=25bits the sum rate observed is 9.56bps/Hz and maximum sum rate observed is 9.87bps/Hz for optimum K=16. So the number of bits per user needed for maximum rate is 16 from each of the 6 users instead of 25 feedback bits from each of the 4 users. Similarly, for T_{fb}=400bits, K=50bits the sum rate observed is 11.39bps/Hz and maximum sum rate observed is 12.88bps/Hz for optimum K=25. So the proposed scheme chooses the optimum K such that the sum rate is maximized. In case of PU²RC it is observed that for T_{fb}=100bits, the maximum sum rate is observed 6.75bps/Hz, which is less than the sum rate is achieved in case of optimized zero force beamforming. As the total feedback load increases, the sum rate is also increasing rapidly in case of optimized zero force beamforming, whereas in PU²RC it is increasing slowly. And also observed that for a given total feedback load T_{fb}, in case of optimized zero force beam forcing scheme, as the number of per user feedback bits increases(upto optimum K) the sum rate also increasing sharply and after that decreasing very slowly. While in PU²RC scheme the sum rate is decreasing very sharply as K increases. Simulation results show that the proposed optimized zero force beamforming scheme has higher user diversity gain than the other scheme PU²RC.

In figure 1, the sum rates of ZFB and PU²RC are compared for values of SNR, T_{fb} and M. It is seen that ZFB maintains a significant advantages over PU²RC for M=4. In addition, the advantage of ZFB increases extremely rapidly with M and SNR. The basic conclusion is that optimized ZFB significantly outperforms PU²RC. This holds for essentially all system parameters (M, SNR, T_{fb}) of interest. Sum rate is plotted versus K (for PU²RC) in figure 2. Very different from ZFB, the sum rate does not increase rapidly with K for small K, and it begins to decrease for even moderate values of K. If K is too large, the number of orthogonal sets $2^K/M$ becomes comparable to the number of users T_{fb}/K and thus it is likely that there are fewer than M users on every set(there are on average $T_{fb}M/K2^K$ users per set). Hence, the BS likely fewer than schedules much fewer than M users, thereby leading to a reduced sum rate. Thus, large values of K are not preferred. As K increases the quantization quality increases, but because there are only $T_{fb}M/K2^K$ users per set (on average) the multi-user diversity (in each set) decreases sharply, so much so that the rate per set in fact decreases with K. For ZFB there is also a loss in multi-user diversity as K is increased, but the number of users is inversely proportional to K, whereas here it is inversely proportional to $K2^K$. The BS does choose the best set amongst the $2^K/M$ sets, but this is not enough to compensate for the decreasing per-set rate.

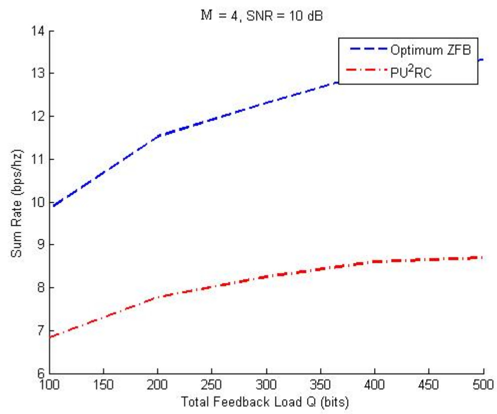

Figure 1. Sum rate Vs Total feedback load for Optimized Zero Force beamforming and PU^2RC feedback strategies with M=4 and SNR=10dB.

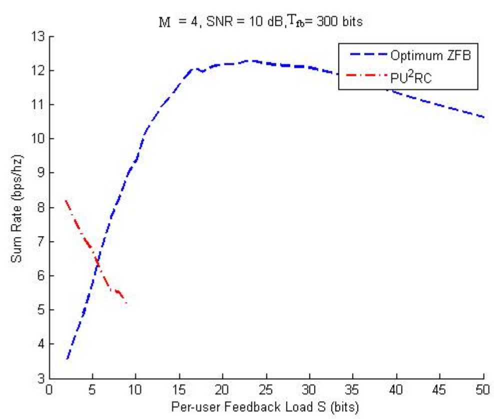

Figure 2. Sum rate Vs per user feedback load for Optimized Zero Force beamforming and PU^2RC feedback strategies with M=4,SNR=10dB and T_{fb}=300 bits.

V. CONCLUSION

The user selection schemes that draw advantage of multiuser diversity to achieve a significant fraction of the multi antenna downlink sum capacity. The SINR distributions and the sum-rates under quantized channel state information (CSI) are derived. We have proved that the optimized zero-forcing beamforming strategy can achieve better asymptotic sum rate than the other user selection scheme, PU^2RC as the number of users goes to infinity. We have proposed a simple method for such a user group selection, and presented a fair scheduling scheme based on the algorithm. The basic design insight is that feedback resources should first be used to obtain accurate CSI and only afterwards be used to exploit multi-user diversity. Numerical results show that optimized zero-forcing beamforming strategy is asymptotically optimal and has a fairly good performance.

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