Module
5
Carrier Modulation

Lesson
29

Minimum Shift Keying (MSK) Modulation

After reading this lesson, you will learn about

- ***CPFSK and MSK Modulation Schemes;***
- ***MSK Modulator;***
- ***Demodulation of MSK Signal;***
- ***Differential Detector;***

Linear modulation schemes without memory like QPSK, OQPSK, DPSK and FSK exhibit phase discontinuity in the modulated waveform. These phase transitions cause problems for band limited and power-efficient transmission especially in an interference limited environment. The sharp phase changes in the modulated signal cause relatively prominent side-lobe levels of the signal spectrum compared to the main lobe. In a cellular communication system, where frequency reuse is done extensively, these side-lobe levels should be as small as possible. Further in a power-limited environment, a non-linear power amplifier along with a band pass filter in the transmitter front-end results in phase distortion for the modulated signal waveform with sharp phase transitions. The abrupt phase transitions generate frequency components that have significant amplitudes. Thus the resultant power in the side-lobes causes co-channel and inter-channel interference.

Consequently, in a practical situation, it may be necessary to use either a linear power amplifier or a non-linear amplifier using extensive distortion compensation or selective pre-distortion to suppress out-of-band frequency radiation. However, high power amplifiers may have to be operated in the non-linear region in order to improve the transmission power. Continuous phase modulation schemes are preferred to counter these problems.

Continuous Phase Frequency Shift Keying (CPFSK) refers to a family of continuous phase modulation schemes that allow use of highly power-efficient non-linear power amplifiers. Minimum Shift Keying (MSK) modulation is a special subclass of CPFSK modulation and MSK modulation is free from many of the problems mentioned above. In this lesson, we briefly describe the various features of MSK modulation and demodulation through a general discussion of CPFSK modulation.

CPFSK and MSK Modulation Schemes

Extending the concepts of binary FSK modulation, as discussed earlier, one can define an M-ary FSK signal which may be generated by shifting the carrier by an amount $f_n = I_n /2\Delta f$. where $I_n = \pm 1, \pm 3, \ldots \pm (M-1)$. The switching from one frequency to another may be done by having $M = 2^k$ separate oscillators tuned to the desired frequencies and selecting one of the M frequencies according to the particular k bit symbol transmitted in a duration of T = k/R seconds. But due to such abrupt switching the spectral side-lobes contain significant amount of power compared to the main lobe and hence this method requires a large frequency band for transmission of the signal.

This may be avoided if the message signal frequency modulates a single carrier continuously. The resultant FM signal is phase-continuous FSK and the phase of the carrier is constrained to be continuous. This class of continuous phase modulated signal is may be expressed as:

$$s(t) = \sqrt{(2E/T)\cos\left(2\pi f_c t + \Phi(t;I) + \Phi_0\right)} \qquad (5.29.1)$$

where $\Phi(t;I)$ is the time-varying phase of the carrier defined as:

$$\Phi(t;I) = 2\pi T f_d \int_{-\infty}^{t} d(\tau)\,d\tau \qquad (5.29.2)$$

f_d = peak frequency deviation ; Φ_0 = initial phase.

Let, $$d(t) = \sum_{-\infty}^{\infty} I_n g(t - nT) \qquad (5.29.3)$$

where $\{I_n\}$ is the sequence of amplitudes obtained by mapping k bit blocks of binary digits from the information sequence $\{a_n\}$ into amplitude levels $\pm1, \pm3, \ldots \pm (M-1)$. g(t) is a rectangular pulse of amplitude 1/2T and duration T. The signal d(t) is used to frequency modulate the carrier.

Substituting equation (5.29.3) in (5.29.2),

$$\Phi(t;I) = 2\pi T f_d \int_{-\infty}^{\infty} \left[\sum_{-\infty}^{\infty} I_n g(\tau - nT) \right] d\tau \qquad (5.29.4)$$

It is evident from (5.29.4) that though d(t) contains discontinuities, the integral of d(t) is continuous. The phase of the carrier in the interval $nT \le t \le (n+1)T$ is determined by integrating (5.29.4):

$$\Phi(t;I) = 2\pi f_d T \sum_{-\infty}^{\infty} I_k + 2\pi f_d (t - nT) I_n$$

$$= \theta_n + 2\pi h I_n q(t - nT) \qquad (5.29.5)$$

where h, θ and q(t) are defined as

$$h = 2f_d T \qquad (5.29.6)$$

$$\theta_n = \pi h \sum_{-\infty}^{\infty} I_k \qquad (5.29.7)$$

and

$$q(t) = \begin{cases} 0 & t < 0 \\ 1/2T & (0 \le t \le T) \\ 1/2 & (t > T) \end{cases} \qquad (5.29.8)$$

As is evident from (5.29.7), θ_n represents the accumulated phase due to all previous symbols up to time $(n-1)$ T.

Minimum Shift Keying (MSK) is a special from of binary CPFSK where the modulation index h = 1/2. Substituting this value of h in (5.29.5) we get the phase of the carrier in the interval $nT \le t \le (n + 1)T$ as

$$\Phi(t;I) = 1/2\pi\sum_{-\infty}^{\infty} I_k + \pi I_n q(t - nT)$$

$$= \theta_n + 1/2\pi I_n (t - nT) \; ; \quad nT \le t \le (n+1)T \qquad (5.29.9)$$

The above equation is obtained by substituting the value of q(t) by

$$q(t) = \int_{-\infty}^{t} g(\tau) d\tau$$

where $g(\tau)$ is some arbitrary pulse. Thus the carrier-modulated signal can be represented as

$$s(t) = A\cos\left[2\pi f_c t + \theta_n + 1/2\pi I_n (t - nT)/T\right]$$

$$= A\cos\left[2\pi\left(f_c + 1/4TI_n\right)t - 1/2n\pi I_n + \theta_n\right] \; ; \; nT \le t \le (n+1)T \qquad (5.29.10)$$

From the above expression it may be noted that binary CPFSK signal has one of the two possible frequencies in the interval $nT \le t \le (n + 1)T$ as:

$$f_1 = f_c - ¼ T \quad \text{and} \quad f_2 = f_c + ¼ T \qquad (5.29.11)$$

Equation (5.29.10) can also be written as

$$s_i(t) = A\cos\left[2\pi f_i t + \theta_n \_ 1/2 n\pi (-1)^{(i-1)}\right] \quad , i = 1, 2 \qquad (5.29.12)$$

The frequency separation $\Delta f = f_2 - f_1 = {}^1\!/_2 T$ is the minimum necessary to ensure the orthogonality of the signals $s_1(t)$ and $s_2(t)$ over a signaling interval of length T. Hence binary CPFSK with h = ½ is called Minimum Shift Keying (MSK) modulation.

MSK Modulator

The block schematic diagram of an MSK modulator is shown in **Fig. 5.29.1**. This structure is based on Equation (5.29.1). However, this cannot be easily converted to hardware as an exact relation between the symbol rate and modulation index is required necessitating intricate control circuitry. An advantage with this structure is that it can be used for both analog and digital input signals.

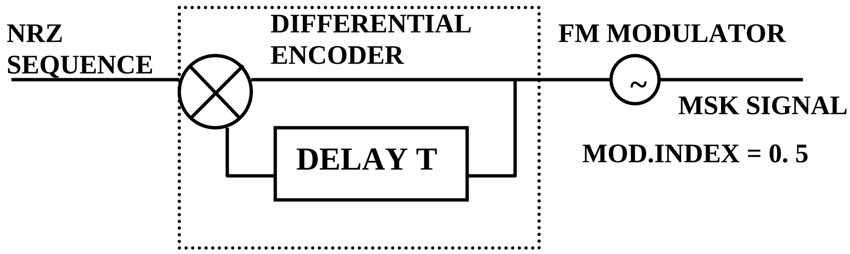

Fig. 5.29.1 An MSK modulator

I denote the sequence of amplitude obtained by mapping binary digits from the information sequence $\{a_n\}$ into amplitude level ±1. This is multiplied with 2π and the resultant product is then used to frequency modulate the carrier.

Demodulation of MSK Signal

An MSK modulated signal can be demodulated either by a coherent or a differential (non coherent) demodulation technique. The coherent demodulator scheme, as we have mentioned earlier for QPSK and other modulations, needs precise reference of carrier, phase, and frequency for optimum demodulation. This is not that easy for an MSK modulated signal. Hence a sub optimal differential modulated technique along with threshold detection is a popular choice in many applications such as in cellular telephony. In the following we briefly discuss a one bit differential demodulation scheme for MSK

Differential Detector

The block diagram for differential detection of MSK signal is shown in **Fig. 5.29.2**.

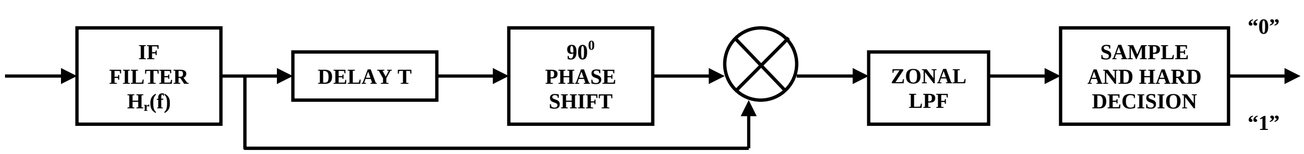

Fig. 5.29.2 *Demodulator for MSK*

A 90^0 phase shifter circuit is included in the delayed arm so that to represent the multiplier output represents the *sine* of the change in the phase of the received signal over one-symbol duration.

Now the received signal r(t) may be expressed as:

$$r(t) = \sqrt{(2P)A(t)\cos\left[\omega_0 t + \phi(t) + n(t)\right]} \tag{5.29.13}$$

where $\sqrt{2PA(t)}$ represents envelope, $\phi(t)$ is the phase and n(t) is the noise.

The noise n(t) can be expressed using quadrature representation of noise by

$$n(t) = n_c(t)\cos\left[\omega_0 t + \phi(t)\right] - n_s(t)\sin\left[\omega_0 t + \phi(t)\right] \qquad (5.29.14)$$

Replacing n(t) of Eqn. (5.29.13) with that of Eqn. (5.29.15) we get

$$r(t) = R(t)\cos\left[\omega_0 t + \phi(t)\right] + \eta(t) \qquad (5.29.15)$$

where

$$R(t) = \sqrt{\left(\sqrt{2P}A(t) + n_c(t)\right)^2 + n_s^2(t)} \qquad (5.29.16)$$

and

$$\eta(t) = -\tan^{-1}\frac{n_s(t)}{\sqrt{2P}A(t) + n_c(t)} \qquad (5.29.17)$$

The one-bit differential detector compares the phase of the received signal r(t) with its one-bit delayed and 90^0 phase shifted version r(t – T). The resultant output of the comparator is given by

$$y(t) = 1/2\left[R(t)R(t-T)\right]\sin\left[\omega_0 T + \Delta\phi(T)\right] \qquad (5.29.18)$$

where the phase difference $\Delta\phi(t)$ is,

$$\Delta\phi(T) = \phi(t) - \phi(t-T) + \eta(t) - \eta(t-T) \qquad (5.29.19)$$

This phase difference indicates the change over symbol duration of the distorted signal phase and phase noise due to the AWGN.

Now let,

$$\omega_0 T = 2\pi K, \text{ where K an integer} \qquad (5.29.20)$$

Then eqn. 5.29.18 reduces to

$$y(t) = 1/2\left[R(t) * R(t-T)\right]\sin\left(\Delta\phi(T)\right) \qquad (5.29.21)$$

The receiver then decides that a '+1' has been sent if y(t) > 0 and a '-1' otherwise. As the envelope R(t) is always positive, actually it is sufficient to determine whether $\sin\left(\Delta\phi(T)\right)$ is ≥ 0.

Problems

Q5.29.1) Justify how MSK can also be viewed as a kind of FSK modulation scheme.

Q5.29.2) Why differential demodulation of MSK is popular?