MEASURING L: SINUSOIDAL RESPONSE AND RLC RESONANCE

1. **Exercise**

   Consider the series RL circuit of Figure 5 driven by a sinusoidal voltage.

   Using your favorite method, derive an expression for the ratio $S = \left|\frac{V_{out}}{V_{in}}\right|$ of the magnitude of the voltage across the capacitor/inductor to the magnitude of the voltage across the voltage source as a function of frequency. To model a real inductor, include an internal resistance R_L in series with the inductor, and include this resistor in the voltage drop across the inductor. Use your expression for S to find an expression for the inductance L in terms of S, R, R_L, and ω. It should be noted that there is nothing about this general approach that is specific to inductors. It works just as well for measuring capacitance.

2. **Simulation**

   Use a small signal AC analysis to generate a plot of S vs. frequency for the RL circuit you analyzed in the Exercise above. Include the internal resistance of the inductor. If you use a source amplitude of 1 V, then values of the voltage across the inductor and its internal resistance (with units removed) are also values of S. Set up the simulation, guessing values for L and R_L. You will refine it as you work through the following experiment.

3. **Experiment**

First, measure the internal resistance R_L of the inductor. Give an uncertainty with your result. (Presumably, you know the resistance R of the resistor you are using. If not, measure that, too.) Then, use the function generator to drive your series RL circuit with a sinusoidal wave form. Use one channel of the oscilloscope to measure V_{in} and the other to measure V_{out}.

Compare V_{in} and V_{out} at various frequencies. How well do your measurements compare with your theoretical calculation and spice simulation? *Remember that* $\omega = 2\pi f$. Use a measurement of S in your theoretical calculation to find L. Report an uncertainty with your result. Be sure to measure your S value at good frequency. What does ``good" mean here?[2] How far off were you in your measurement of L of Section 1? Can this discrepancy be attributed to ignoring the internal resistance of the inductor? Explain.

RLC Resonance

1. **Exercise**

   At resonance, the capacitive and inductive reactances cancel, the phase angle ϕ is 0, and the voltage across the resistor is maximal (as are the current and S). Derive an expression for the resonant frequency f_o of a series RLC circuit in terms of L and C.

2. **Simulation**

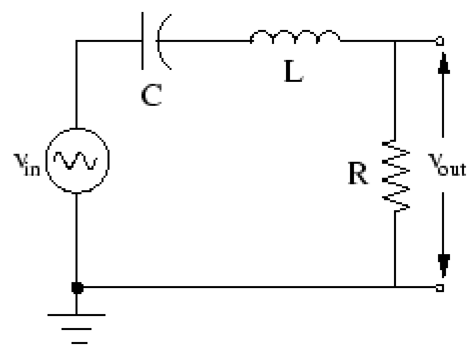

Figure 6: Series RLC circuit.

3. Set up an AC analysis of the circuit shown in Figure 6. Produce plots of S and ϕ as a function of frequency, and note the predicted resonant frequency. Investigate and report on the dependence of the plots and the resonant frequency on the resistance R, and explain.

4. **Experiment**

Set up the circuit shown in Figure 6. Measure the voltage across the source V_{in} with one channel the oscilloscope and V_{out} with the other. Dial up X-Y mode on the time base knob to plot V_{in} vs. V_{out}. At resonance, these two signals are in phase. In X-Y mode, this looks like a straight line. At any other frequency, the oscilloscope trace resembles an ellipse.

In time base mode, you can look for the frequency at which the voltage across the resistor V_{out} is maximal. Report an uncertainty in your result. Compare your result with your theoretical prediction using your best measured values of C and L.

Source: http://webpages.ursinus.edu/lriley/ref/circuits/node3.html