MANLEY – ROWE RELATIONS:

For the determination of maximum gain of the parametric amplifier, a set of power conservation relations known as "Manley-Rowe" relations are quite useful.

Two sinusoidal signals $f_p$ and $f_s$ applied across a lossless time varying non-linear capacitance $C_j (t)$. At the output of this varying capacitance, harmonics of the two frequencies $f_p$ and $f_s$ are generated.

These harmonics are separated using band-pass filters having very narrow bandwidth.

The power at these harmonic frequencies is dissipated in the respective resistive loads.

From the law of conservation of energy, we have

$$\sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \frac{n P_{mn}}{n f_s + m f_p} = 0$$

$$\sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} \frac{m P_{mn}}{n f_s + m f_p} = 0$$
The above relations are called "Manley-Rowe" power conservation equations. When the power is supplied by the two generators, then Pmn is positive. In this case, power will flow into the non-linear capacitance. If it is the other way, then Pmn is negative.

As an example, let us consider the case when the power output flow is allowed at the sum frequency fp + fs only, with all the remaining harmonics being open circuited. With the above rest ructions, the quantities ‘m’ and ‘n’ can take on values -1,0 and respectively.

\[
\frac{P_{01}}{f_s} + \frac{P_{11}}{f_s + f_p} = 0
\]

and
\[
\frac{P_{10}}{f_p} + \frac{P_{11}}{f_s + f_p} = 0
\]

The powers P01and P10 are considered positive, whereas P11 is considered negative. \( \therefore \) The power gain defined as the power output from the non-linear capacitor delivered to the load at sum frequency to that power received by it at a frequency fs is given by

\[
G_p = \frac{P_{11}}{P_{01}} = \frac{f_s + f_p}{f_s} \quad \text{(for modulator)}
\]
Thus the power gain is the ratio of output to input frequency. This type of parametric device is called "Sum-frequency parametric amplifier" or "up-converter".

On the other hand, if the signal frequency is $f_p + f_s$ and output frequency is $f_s'$ then

$$G_p = \frac{f_s}{f_p + f_s} \text{ (for demodulator)}$$

This type of parametric device will now be called "parametric down-converter" and the power gain becomes power attenuation.

Source: http://elearningatria.files.wordpress.com/2013/10/ece-v-microwaves-and-radar-10ec54-notes.pdf