

# Low-complexity Integer Frequency Offset Synchronization for OFDMA System

Young-Jae Kim and Young-Hwan You

**Abstract**—This paper presents a integer frequency offset (IFO) estimation scheme for the 3GPP long term evolution (LTE) downlink system. Firstly, the conventional joint detection method for IFO and sector cell index (CID) information is introduced. Secondly, an IFO estimation without explicit sector CID information is proposed, which can operate jointly with the proposed IFO estimation and reduce the time delay in comparison with the conventional joint method. Also, the proposed method is computationally efficient and has almost similar performance in comparison with the conventional method over the Pedestrian and Vehicular channel models.

**Keywords**—LTE, OFDMA, primary synchronization signal (PSS), IFO, CID.

## I. INTRODUCTION

**L**ONG term evolution (LTE) has been standardized evolved-universal mobile telecommunication system (EUTRA), introduced by 3rd generation partnership project (3GPP) is a 3rd generation of mobile communication standard that uses orthogonal frequency division multiple access (OFDMA) in downlink and single carrier-frequency division multiple access (SC-FDMA) in uplink [1].

The LTE physical layer employs some advanced technologies that are new to cellular applications. The downlink transmission utilizes OFDMA as the physical layer technique which enables high data rate transmission in frequency selective fading channels. In an OFDMA system, inter-carrier interference (ICI) is avoided through the inherent orthogonality of OFDMA. However, in order to maintain this orthogonality among terminals, tight time and frequency synchronization between the terminal and the base station is required. Therefore, frequency offset caused by frequency differences between the local oscillators in the transmitter and the receiver should be accurately estimated and compensated. Synchronization for general OFDM systems has been studied extensively. Numerous papers [2][3] deal with timing and carrier frequency offset synchronization and its proposed estimators operate before the fast Fourier transform (FFT) block present in the receiver. After coarse synchronization, there might still be present the integer frequency offset (IFO). The IFO produces the cyclic shift of the output of the FFT in the receiver. The impact of IFO as well as IFO compensation schemes were investigated in [4]. However, the method has a high computational complexity because this scheme estimates the IFO and sector cell index (CID) information at the same time.

In this paper, we propose the IFO estimation scheme in LTE downlink system. The proposed IFO estimation without

explicit sector CID information is proposed and also a low-complexity IFO detection method is suggested. Simulation results show advantage of the proposed scheme over the conventional scheme.

This paper is organized as follows. The next section describes the system model. Section III briefly introduces conventional joint detection of IFO and sector CID. In Section IV, the proposed IFO estimation scheme is presented. Section V shows the simulation results verifying the performance of the IFO estimator. Finally we conclude this paper with Section VI.

## II. SYSTEM MODEL

In the LTE downlink system, two synchronization signal, PSS and SSS, are provided for synchronization and cell search, where 504 cell identities (cell IDs) are defined and are packed into 168 cell groups such that each cell group contains three cell IDs. Three different PS signals are used to distinguish three cells within a cell group and 168 different PSS signals are used to differentiate 168 cell groups so that total 504 cells can be identified by the combination of the PSS and SSS. Both signals are transmitted in slot 0 and slot 10 in a frame. In each slot, the OFDM symbol of the SSS comes first, and the symbol of the PSS is the transmitted in the next symbol position. The sequence of PSS is generated from a frequency domain Zadoff-Chu sequence [1] according to

$$PS_u(k) = \begin{cases} e^{-j\pi uk(k+1)/63} & k = 0, 1, \dots, 30 \\ e^{-j\pi u(k+1)(k+2)/63} & k = 31, 32, \dots, 61 \end{cases} \quad (1)$$

where Zadoff-Chu root sequence index  $u (\in 0, 1, 2)$  is given by the cell ID within a group. The same PSS is transmitted in slot 0 and slot 10.

We commence with the signal model of OFDM system and the focus our attention on the PSS of the LTE downlink system. In this paper, we assume that fractional part of frequency offset and timing offset are perfectly estimated and corrected before the estimation of the integer part. Then, the received PSS of the OFDM symbol in frequency domain takes the form of

$$Y(k) = e^{j2\pi\Lambda_i N_e/N} H(k - \Lambda_i) X(k - \Lambda_i) + W(k), \quad (2)$$

where  $X(k - \Lambda_i)$  represents the  $\Lambda_i$ -th cyclic shifted version of the transmitted PSS  $X(k)$ .  $N_e = N + N_{cp}$  and  $\Lambda_i$  is the IFO.  $H(k)$  is the channel's frequency response, and  $W(k)$  is a zero-mean complex Gaussian noise.

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### III. CONVENTIONAL JOINT DETECTION OF INTEGER FREQUENCY OFFSET AND SECTOR CELL INDEX

Although the Zadoff-Chu sequence in PSS originally has excellent properties of constant envelope and zero autocorrelation for each cyclic shift, the complex channel frequency response and the symbol timing error distort the information carried in the phase of the frequency-domain data. In addition, the IFO causes the index shift of the frequency-domain data. All the above factors degrade the performance of frequency-domain direct cross-correlation with the Zadoff-Chu sequence.

Observe that the Zadoff-Chu sequence has phase increment proportional to the square of the subcarrier index. The phase difference of adjacent subcarriers becomes

$$\begin{aligned} PS_u^*(k+1) \cdot PS_u(k) &= e^{j\pi u(k+1)(k+2)/63} e^{-j\pi u k(k+1)/63} \\ &= e^{j2\pi u(k+1)/63} \end{aligned} \quad (3)$$

which contains linear phase increment scaled by the sector cell information  $u$ . The cross-correlation for phase difference of the received frequency-domain data and the PSS is then introduced to jointly detect sector cell information  $u$  and the IFO  $\Lambda_i$ , which is given by [4]

$$\left( \hat{\Lambda}_i, \hat{u} \right) = \arg \max_{|d| \leq D, u \in \{25, 29, 34\}} \{ \mathcal{Re} \{ Z(d, u) \} \} \quad (4)$$

with

$$\begin{aligned} Z(d, u) &= \sum_{k=-31}^{-2} (Y_{pss}(k+1+d)Y_{pss}^*(k+d)) \\ &\quad \cdot (PS_u(k+32)PS_u^*(k+31))^* \\ &\quad + \sum_{k=1}^{30} (Y_{pss}(k+1+d)Y_{pss}^*(k+d)) \\ &\quad \cdot (PS_u(k+31)PS_u^*(k+30))^* \end{aligned} \quad (5)$$

where  $Y_{pss}$  is the received OFDM symbol containing the PSS. Assume that the CFO is in the range of  $\pm 40$ ppm, and thus  $\Lambda_i$  has only five possible values, i.e.  $\{-2, -1, 0, 1, 2\}$ . Meanwhile,  $u$  is chosen from the set of  $\{25, 29, 34\}$ . Hence, the detection can be achieved by correlator banks for 15 hypotheses.

### IV. PROPOSED ESTIMATION OF INTEGER FREQUENCY OFFSET

In the conventional approach, one complex multiplier is needed for the adjacent frequency-domain signal multiplication in (5) and 15 sets of correlator banks each having one multiplier-and-accumulator are required. Obviously, the complexity of the conventional method is much less than a matched filter. However, a new IFO estimation and sector CID detection method can reduce the cell search time more effectively in comparison with conventional method [4]

Assuming that the DC subcarrier index is  $k = N/2$ , the proposed IFO estimator can be expressed as

$$\hat{\Lambda}_i = \arg \max_{|d| \leq D} \left\{ \mathcal{Re} \left\{ \sum_{k=0}^{29} Z_{ls}(k+d)Z_{rs}(29-k+d) \right\} \right\} \quad (6)$$

TABLE I  
COMPLEXITY COMPARISON OF CONVENTIONAL AND PROPOSED METHODS

Procedure of Methods	IFO Estimation	
	Conventional	Proposed
Number of Complex Multiplication	$120 \cdot (2D+1) \cdot 3$	$90 \cdot (2D+1)$
Number of Complex Addition	$59 \cdot (2D+1) \cdot 3$	$29 \cdot (2D+1)$

with

$$\begin{aligned} Z_{ls}(k+d) &= Y_{pss}(N/2-31+k+d)Y_{pss}^*(N/2-30+k+d) \\ &= H_{ls}(N/2-31+k-\Lambda_i+d) \\ &\quad \cdot e^{-j\pi u(k-\Lambda_i+d)(k+1-\Lambda_i+d)/63} \\ &\quad \cdot H_{ls}^*(N/2-30+k-\Lambda_i+d) \\ &\quad \cdot e^{j\pi u(k+1-\Lambda_i+d)(k+2-\Lambda_i+d)/63} + \tilde{W}_{ls}(k) \end{aligned} \quad (7)$$

and

$$\begin{aligned} Z_{rs}(k+d) &= Y_{pss}(N/2+1+k+d)Y_{pss}^*(N/2+2+k+d) \\ &= H_{rs}(N/2+1+k-\Lambda_i+d) \\ &\quad \cdot e^{-j\pi u(k+32-\Lambda_i+d)(k+33-\Lambda_i+d)/63} \\ &\quad \cdot H_{rs}^*(N/2+2+k-\Lambda_i+d) \\ &\quad \cdot e^{j\pi u(k+33-\Lambda_i+d)(k+34-\Lambda_i+d)/63} + \tilde{W}_{rs}(k) \end{aligned} \quad (8)$$

where  $\tilde{W}(k)$  is statistically equivalent to  $W(k)$ .

Here,  $d = \Lambda_i$  and the channel's frequency response of two adjacent subcarriers are assumed to be nearly identical, i.e.,  $H(k) \approx H(k+1)$ . Then,  $Z_{ls}(k+d)Z_{rs}(29-k+d)$  can be approximated as

$$\begin{aligned} Z_{ls}(k)Z_{rs}(29-k) &= |H_{ls}(k)|^2 |H_{rs}(29-k)|^2 e^{j2\pi u} \\ &\quad + |H_{rs}(29-k)|^2 \tilde{W}_{ls}(k) e^{j2\pi u(62-k)/63} \\ &\quad + |H_{ls}(k)|^2 \tilde{W}_{rs}(29-k) e^{j2\pi u(k+1)/63} \\ &\quad + \tilde{W}_{ls}(k)\tilde{W}_{rs}(29-k), \quad 0 \leq k \leq 29, \end{aligned} \quad (9)$$

where  $Z_{ls}(k) = |H_{ls}(k)|^2 e^{j2\pi u(k+1)/63} + \tilde{W}_{ls}(k)$  and  $Z_{rs}(k) = |H_{rs}(k)|^2 e^{j2\pi u(k+33)/63} + \tilde{W}_{rs}(k)$ .

From the above equations, the mean of the signal component of (6) can be approximated by

$$\begin{aligned} \frac{1}{29} \sum_{k=0}^{29} Z_{ls}(k)Z_{rs}(29-k) &\approx \\ E [H_{rs}(k)H_{rs}^*(k+1)H_{rs}(29-k)H_{rs}^*(30-k)e^{j2\pi u}] \\ &\quad + E [H_{ls}(k)H_{ls}^*(k+1)\tilde{W}_{rs}(k)e^{j2\pi u(k+1)/63}] \\ &\quad + E [H_{rs}(29-k)H_{rs}^*(30-k)\tilde{W}_{ls}(k)e^{j2\pi u(62-k)/63}] \\ &\quad + E [\tilde{W}_{ls}(k)\tilde{W}_{rs}(29-k)] \\ &= E [H_{rs}(k)H_{rs}^*(k+1)H_{rs}(29-k)H_{rs}^*(30-k)] \end{aligned} \quad (10)$$

where  $E[\tilde{W}(k)] = 0$  and the AWGN is uncorrelated for different channel response. As shown in (10), the proposed

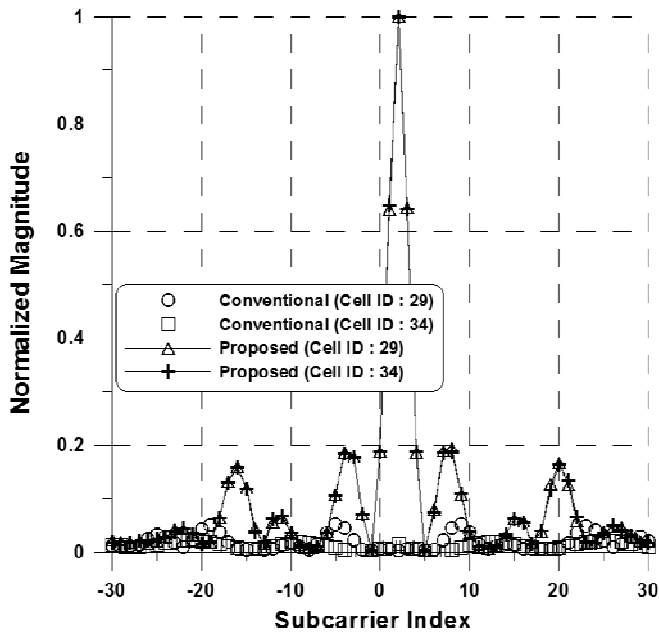


Fig. 1. Detection waveform for IFO estimator when  $\Lambda_i = 3$  and  $u \neq 25$ , i.e.,  $u \in \{29, 34\}$

IFO estimator can estimate the IFO regardless of the sector CID due to  $e^{j2\pi u} = 1, \{\forall u \in \{25, 29, 34\}\}$ . With this provision, the proposed method is a simple way of implementing PSS assisted IFO estimator since it isn't necessary to detect the sector CID prior to the frequency synchronization, i.e., the proposed method only performs 5 correlation procedure when  $D = 2, d \in \{-2, -1, 0, 1, 2\}$ . The proposed detection scheme of IFO has the advantages regarding the complexity. The complexity comparison between the proposed and conventional approaches are summarized in table I when  $(PS_u^*(k+1)) \cdot PS_u(k)$  for the conventional method is known at the receiver.

V. SIMULATION RESULTS

To verify the effectiveness of the proposed algorithm presented in the previous sections, the conventional and proposed methods are simulated under the Pedestrian and Vehicular channel models that has been contributed in [5]. The simulation parameters are considered according to LTE downlink FDD, which are as follows:  $N = 512, N_{cp} = 128$ , and  $N_{RS} = 30$  (the number of the reference signals). Simulations are carried out in order to evaluate the performance of the IFO estimators in terms of the probability of failure,  $\Pr\{\hat{\Lambda}_i \neq \Lambda_i\}$ . The proposed method can directly estimate and compensate the IFO since they use only PSS sequence of one OFDM symbol.

Figure 1 presents the waveform of the frequency-domain phase-difference cross-correlation results. It can be shown that a peak only occurs when  $\hat{u}$  matches  $u$  in conventional case. If sector cell index is incorrect ( $\hat{u} \neq u$ ), the peak of the correlator output diminishes due to the uniformly-distributed random phase term. However, it is clear that the peak of the proposed estimator can be immune to cell index  $u$ .

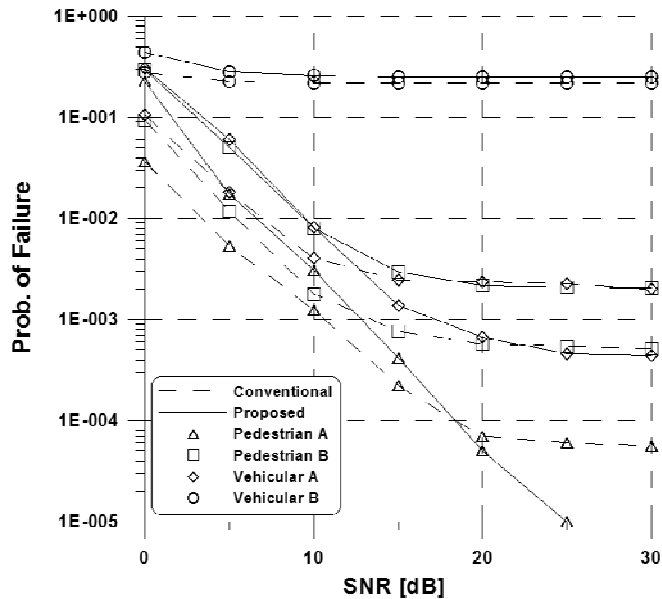


Fig. 2. The probability of failure of the IFO estimator versus SNR when  $D = 3$

Figure 2 shows the probability of failure of the conventional [4] and proposed estimator when  $D=3$ . The performance of the conventional method is quite close to that of the proposed method under Vehicular B channel due to its severe frequency selective fading. In other words, its performance is related with the channel statistics. The smaller the channel delay spread, the better detection performance is, which also can be verified by (9) and the curves of the Vehicular B channel.

VI. CONCLUSION

In this paper, we suggested low-complexity IFO synchronization based on structure of the PSS for OFDMA-based LTE Downlink System. The proposed scheme is estimated an IFO without explicit sector CID. The proposed estimator is made to keep low computational cost and reduce the implementation complexity. And the performance of the proposed simplified estimator was compared with that of the conventional jointly estimator. It has been found by extensive simulation that the proposed method maintains the performance of the conventional method.

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