

LTI SYSTEM MODELS FOR RANDOM SIGNALS – AR, MA AND ARMA MODELS

Signal models are used to analyze stationary univariate time series. The goal of signal modeling is to estimate the process from which the desired signal is generated. Though the concept described here is related to the topic of “system identification”, they are quite different.

A signal model is an unique combination of a filter and a source input that may fall into any of the following categories

- **Filter:** state-space model, AR, MA, ARMA (see below)
- **Source:** pulse, pulse train, white noise,...

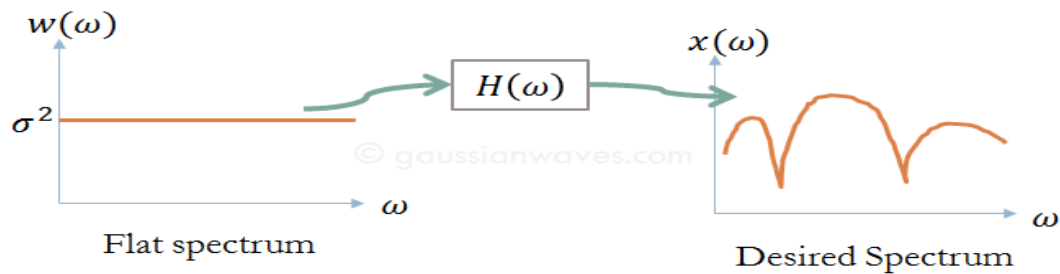
Motivation

Let's say we observe a real world signal $x[n]$ that has a spectrum $x[\omega]$ (the spectrum can be arbitrary – bandpass, baseband etc...). We would like to describe the long sequence of $x[n]$ using very few parameters (application : Linear Predictive Coding (LPC)). The modelling approach, described here, tries to answer the following two questions.

- Is it possible to model the first order (mean/variance) and second order (correlations, spectrum) statistics of the signal just by shaping a white noise spectrum using a transfer function ? (see image below)

- Does this produce the same statistics (spectrum, correlations, mean and variance) for a white noise input ?

If the answer is “yes” to the above two questions, we can simply set the modeled parameters of the system and excite the system with white noise to produce the desired real world signal. This reduces the amount to data we wish to transmit in a communication system application.



LTI system model

In the model given below, the random signal $x[n]$ is observed. Given the observed signal $x[n]$, the goal here is to find a model that best describes the spectral properties of $x[n]$ under the following assumptions

- $x[n]$ is WSS (Wide Sense Stationary) and ergodic
- The input signal to the LTI system is white noise following Gaussian distribution – zero mean and variance σ^2
- The LTI system is BIBO (Bounded Input Bounded Output) stable.

Source: <http://www.gaussianwaves.com/2014/05/lti-system-models-for-random-signals-ar-ma-and-arma-models/>