Error Rate Probability for Coded MQAM with MRC Diversity in the Presence of Cochannel Interferers over Nakagami-Fading Channels

J.S. Ubhi, M.S. Patterh, T.S. Kamal

***Abstract*—**Exact expressions for bit-error probability (BEP) for coherent square detection of uncoded and coded M-ary quadrature amplitude modulation (MQAM) using an array of antennas with maximal ratio combining (MRC) in a flat fading channel interference limited system in a Nakagami-*m* fading environment is derived. The analysis assumes an arbitrary number of independent and identically distributed Nakagami interferers. The results for coded MQAM are computed numerically for the case of (24,12) extended Golay code and compared with uncoded MQAM by plotting error probabilities versus average signal-to-interference ratio (SIR) for various values of order of diversity *N*, number of distinct symbols *M*, in order to examine the effect of cochannel interferers on the performance of the digital communication system. The diversity gains and net gains are also presented in tabular form in order to examine the performance of digital communication system in the presence of interferers, as the order of diversity increases. The analytical results presented in this paper are expected to provide useful information needed for design and analysis of digital communication systems with space diversity in wireless fading channels.

***Keywords*—**Cochannel interference, maximal ratio combining, Nakagami-*m* fading, wireless digital communications.

I. INTRODUCTION

IN wireless communications, the presence of cochannel interference (CCI) limits the system capacity, whereas multipath fading limits the system performance. Hence, methods that address these problems are of great interest. It is well known that antenna diversity can significantly improve the capacity of many digital cellular radio systems by combating multipath fading and reducing the effect of cochannel interference. In a system with an antenna array, the signals received by several antenna elements can be properly weighted and combined to maximize the output signal-to-interference-plus-noise ratio (SINR) [1]. However, the implementation of the optimum combiner (OC) requires channel estimation for the desired signal as well as for all the interfering signals. Therefore, the implementation and performance analysis of such systems are usually quite complicated. In practice, maximal ratio combining (MRC), which maximizes the output signal-to-noise ratio (SNR) and is suboptimal in the presence of cochannel interference, is usually employed instead [2], [3]. Two performance measures that are generally considered in the performance study of digital mobile radio systems are outage probability and average probability of error. The effects of cochannel interference and multipath fading on the average bit error rate (BER) as well as outage probability of mobile radio systems employing MRC have been studied extensively for the Rayleigh fading environment, in the presence of interference and noise [1], [2], [3]. In these papers, it is assumed that both the desired and interfering signals undergo Rayleigh fading. However, it is more realistic to expect that in practice, the interferers will usually experience more severe fading than the desired signal [4]. Another channel model that is more general than the Rayleigh model that is also commonly used to characterize the urban and digital mobile radio environment is the Nakagami fading model [5]. The Nakagami distribution has the Rayleigh distribution as a special case and has the flexibility of modeling fading conditions that are more or less severe than Rayleigh fading. Studies of the effect of cochannel interference on the performance of digital mobile radio systems operating in the Nakagami fading environment have so far been limited to the calculation of outage probabilities, either for the interference-limited system [6] or for the system with both interference plus noise [4] and average probability of error restricted to binary signaling [7]. It is of interest to consider the effect of digital transmission codes [8] on the average probability of error for M-ary signaling in the presence of cochannel interference over Nakagami fading channel. The outage probability of a digital microcellular system employing MRC in an interference-limited environment has been investigated for the Rayleigh/Rayleigh [4] and the Rice/Rayleigh models [4], [9]. In the case of optimum combining, the probability density function (pdf) of the SINR as well as the outage probability for the Rayleigh/Rayleigh model in an interference-limited environment was given by Shah and Haimovich [10]. Closed-form results for the average bit-error rate (BER) of antenna array systems using MRC in a flat Rayleigh-faded environment were given for a dual diversity system with one dominant cochannel interferer, assuming that the diversity channels are correlated [3]. Also, the average BER for *N*-order diversity with an arbitrary number of identical independent interferers was derived in [3] and [7].

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In this paper, we derive expressions for the average bit-error rate (BER) of uncoded and (24, 12) extended Golay coded [8] MQAM which is coherent square modulation scheme with gray code bit mapping for an N-element antenna array system employing MRC in the presence of an arbitrary number of independent and identically distributed Nakagami fading channel with multiple Nakagami interferers having identical powers. We assume that the fading environment is such that the Nakagami fading parameter of the desired signal is limited to integer values since in practice, the measurement accuracy of the channel is typically only of integer order [11]. However, there is no such restriction on the fading severity of the interfering signals. The approach adopted in this paper results in "clean" derivations for the error probability expressions which are numerically efficient. The paper is organized as follows. In section II, we describe the communication link used. In section III, we obtain expressions for BER for uncoded and coded MQAM under MRC diversity combining over independent and identically distributed Nakagami fading channels in the presence of multiple interferers. Numerical results are discussed in section IV and finally concluding remarks are given in section V.

II. System Model

In a cellular radio environment, there is usually a large number of interfering signals at the receiver. Typically, the same interfering signals are present on each diversity branch. The number of interferers is usually much larger than the number of diversity branches, so that their effect cannot all be cancelled by the diversity combiner, especially if the powers of the interferers are close to each other. With MRC, the signals received on the N diversity branches are weighted and combined to maximize the output SNR. In an N-order MRC diversity system with independent and identical statistics on the branches, the desired instantaneous SNR at the output of the combiner in a Nakagami fading channel is a gamma distributed random variable with probability density function (pdf) given by [6]

$$p_S(x) = \left(\frac{m_S}{\Omega_S}\right)^{m_S N} \frac{x^{m_S N-1}}{\Gamma(m_S N)} \exp\left(-\frac{m_S}{\Omega_S} x\right), \quad x \geq 0 \qquad (1)$$

where Ω_S is the average desired signal-to-noise power ratio and m_S is the Nakagami fading parameter. It is assumed that L mutually independent Nakagami faded interference signals are also present at the receiver. When the interferers originate from approximately the same distance from the mobile station, they can also be assumed to have the same average power. The short-term power of the resultant interfering signal can be taken to be the sum, I, of the identical interfering signal powers. It follows then that the pdf of the total INR is given by [6]

$$p_I(y) = \left(\frac{m_I}{\Omega_I}\right)^{m_I L} \frac{y^{m_I L-1}}{\Gamma(m_I L)} \exp\left(-\frac{m_I}{\Omega_I} y\right), \quad y \geq 0 \qquad (2)$$

where Ω_I is the average INR and is the Nakagami fading parameter for the interference signals. In a typical microcellular environment in which the desired signal as well as the interfering signals undergo Nakagami fading, it is reasonable to assume that the cochannel interferers experience much deeper fading than the desired signal, that is, $m_S \geq m_I \geq ½$. In an interference-limited system the effect of noise may be ignored. In this case it can be shown that the signal-to-interference ratio (SIR), $\gamma = S/I$, has the pdf [4]

$$p_\gamma(\gamma) = \frac{\Gamma(m_S N + m_I L)}{\Gamma(m_S N)\Gamma(m_I L)} \left(\frac{m_S N}{q m_I}\right)^{m_S N} \frac{\gamma^{m_S - 1}}{\left(\frac{m_S N}{q m_I}\gamma + 1\right)^{m_S + m_I L}} \qquad (3)$$

where the ratio $q = \Omega_S / \Omega_I$ is the average-signal to average-interference power ratio (SIR), which is useful in determining the cochannel reduction factor in systems with frequency reuse.

III. Performance Analysis

The bit error probability (BEP) in multi-path fading environment is obtained by averaging the conditional bit error probability over the channel ensemble. This can be accomplished by averaging $P_b\{e|\gamma\}$ over the pdf of γ as:

$$P_b = \int_0^\infty P_b\{e|\gamma\} p_\gamma(\gamma) d\gamma \qquad (4)$$

where $P_b\{e|\gamma\}$ is the conditional BEP, conditioned on given SIR, γ for the desired MQAM in AWGN channel.

A. BER for Uncoded MQAM with MRC Diversity

The BER for MQAM in AWGN is given in [12] which on the fading channel becomes the conditional BER, i.e.,

$$P_b\{e|\gamma\} = f(M). \sum_{k=1}^{\log_2 \sqrt{M}} \sum_{j=0}^{f(k,M)} [f(j,k,M) \times \int_0^{\pi/2} \exp\{-f(j,g,\theta)\gamma\} d\theta] \qquad (5)$$

where $f(M)$, $f(k,M)$, $f(j,k,M)$ and $f(j,g,\theta)$ are defined for convenience as

$$f(M) = 2/[\pi\sqrt{M}\log_2\sqrt{M}]$$

$$f(k,M) = (1 - 2^{-k})\sqrt{M} - 1$$

$$f(j,k,M) = (-1)^{\left\lfloor \frac{j.2^{k-1}}{\sqrt{M}} \right\rfloor} \left[2^{k-1} - \left\lfloor \frac{j.2^{k-1}}{\sqrt{M}} + \frac{1}{2} \right\rfloor\right]$$

where $\lfloor x \rfloor$ denotes the largest integer to x.

$$f(j,g,\theta) = (2.j+1)^2 g / \sin^2\theta$$

$$g = 3\log_2 M / [2(M-1)]$$

Substituting (3) and (5) in (4) and then performing some mathematical manipulations yields the following expression for the average BER of MQAM under Nakagami fading environment in the presence of identical mean powers cochannel interferers

$$P_b = \frac{f(M)\Gamma(m_S N + m_I L)}{\Gamma(m_I L)} \sum_{k=1}^{\log_2 \sqrt{M}} \sum_{j=0}^{f(k,M)} [f(j,k,M) \times \int_0^{\pi/2} \psi\left(m_S N, 1 - m_I L, \frac{f(j,g,\theta)}{\beta}\right) d\theta] \tag{6}$$

where $\beta = m_S / q m_I$. The function $\psi(a,b,c)$ is the confluent hypergeometric function of second kind, defined by integral [13]

$$\psi(a,b,c) = \frac{1}{\Gamma(a)} \int_0^{\infty} t^{a-1}(1+t)^{b-a-1} e^{-xt} dt, \quad a > 0 \tag{7}$$

B. BER for Coded MQAM with MRC Diversity

Here, the discussion is restricted to the case of linear binary block codes. The average BER rate performance of a binary code of length n and of error correcting capability t on a memory-less binary symmetric channel with probability of uncoded message bit error, p is given by [8]

$$P_{b(coded)} = \frac{1}{n} \cdot \sum_{l=t+1}^{n} e_l \binom{n}{l} p^l (1-p)^{n-l} \tag{8}$$

where e_l is the average number of channel errors that remain in the corrected n tuple when the channel caused l errors. In practice, it has been fashionable [8] to use $e_l \cong l$ for $l > t$, which appears to be a good approximation for the majority of codes of greatest interest, thereby making eq. (8) easy to evaluate numerically. For MQAM system over Nakagami fading p is given by the right hand side of eq. (6) in the presence of cochannel interferers with identical power with q replaced by Rq, where R is the code rate. We choose the (24,12) extended Golay code as a specific example. This code is the only known multiple errors correcting binary perfect code that can correct any combination of three or fewer errors per 24 bit codeword.

Using (8), the BER expression for coded MQAM (when hard decision decoding is used) over frequency nonselective slow Nakagami fading channel for the case of (24,12) extended Golay code is given by

$$P_{b(coded)} = \frac{1}{24} \cdot \sum_{l=4}^{24} l \binom{24}{l} p^l (1-p)^{24-l} \tag{9}$$

IV. Numerical Results

The expressions for average probability of error are given in (6) and (9) for uncoded and (24,12) extended Golay coded MQAM respectively, under MRC diversity in the presence of identical mean power interferers. The computed results are then plotted graphically for the cases of M=16 and 64 in Figs. 1 to 3. Fig. 1 and 2 show the average BER versus the average SIR, defined as the ratio of average power of the desired signal to the average power of interferers. In Fig. 1, the average BER for uncoded and coded MQAM (M=16 and 64) is plotted against average SIR with an arbitrary number of diversity branches and six identical cochannel interferers (L=6) in a channel where Nakagami interferers (m_I=0.75) are assumed to undergo more severe fading than desired signal (m_S=1). Fig. 2 also shows the average BER for uncoded and coded MQAM (M=16 and 64) against average SIR with an arbitrary number of diversity branches in the presence of six identical cochannel interferers (L=6) in which it is assumed that interferers undergo Rayleigh fading (m_I=1) and desired signal undergo less severe fading (m_S=2)), perhaps due to the presence of a specular component. Although MRC is suboptimal in the presence of cochannel interference, increasing the order of diversity considerably improves the performance of the system which can further be enhanced by using coded MQAM. In Fig. 3, the average BER for uncoded and coded 16QAM is plotted against the number of interferers, with the SIR fixed at 10 dB. We observe from the figure that when the system is interference-limited (L>>0), the system performance still improves substantially for both uncoded and coded 16QAM although the MRC system ignores the presence of the cochannel interferers and maximizes the output SNR. The value of L beyond which the system becomes interference-limited depends on the fixed values of the SNR and INR since it is the overall average SINR that determines the performance level. It is also clear from the results that the coded MQAM system outperforms the uncoded MQAM even as the number of interferers increases.

To give more specific results, Table I and II shows the diversity gain (without coding) and net gain with diversity for 16-QAM at $P_b = 10^{-3}$ and $P_b = 10^{-5}$, for an arbitrary number of number of diversity branches, N and in the presence of six interferers (L=6). We may define the diversity gain as the difference in the average SIR required for a system with no diversity and the average SIR required for a system with diversity for an uncoded system at a fixed BER at the receiver. The net gain may be defined as the difference in the average SIR required for an uncoded system with no diversity and the average SIR required for a coded system with diversity for a fixed BER at the receiver. From figures and tables, it is clear that the net gain increases as order of diversity branches increase in the presence of cochannel interferers.

TABLE I DIVERSITY GAIN (dB) AT $P_B = 10^{-3}$ & 10^{-5} FOR 16-QAM WITHOUT CODING IN THE PRESENCE OF 6 INTERFERERS IN RAYLEIGH FADED ENVIRONMENT (m_S=1, m_I=1)

P_b	N=2	N=3	N=4
10^{-3}	12.24	16.46	18.81
10^{-5}	21.91	29.48	32.46

TABLE II NET GAIN (dB) AT $P_B = 10^{-3}$ & 10^{-5} FOR 16-QAM AFTER DIVERSITY IN THE PRESENCE OF 6 INTERFERERS IN RAYLEIGH FADED ENVIRONMENT (m_S=1, m_I=1)

P_b	N=2	N=3	N=4
10^{-3}	21.68	24.35	26.01
10^{-5}	37.84	40.99	42.87

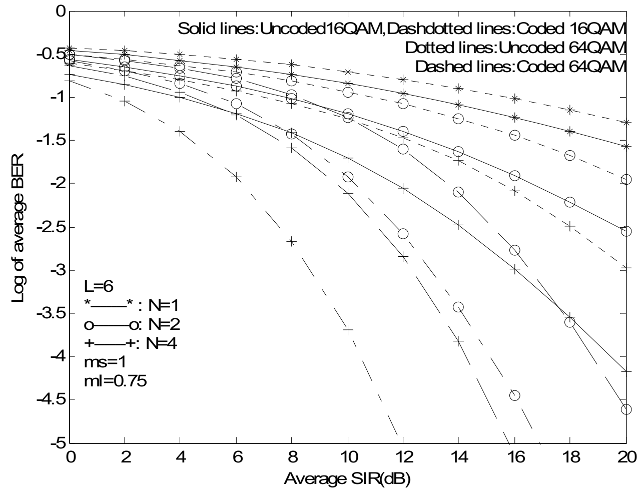

Fig. 1 The average BER versus average SIR of uncoded and coded MQAM under N-branch MRC diversity in the presence of six distinct mean power interferers in a Nakagami fading environment (m_S=1, m_I=0.75).

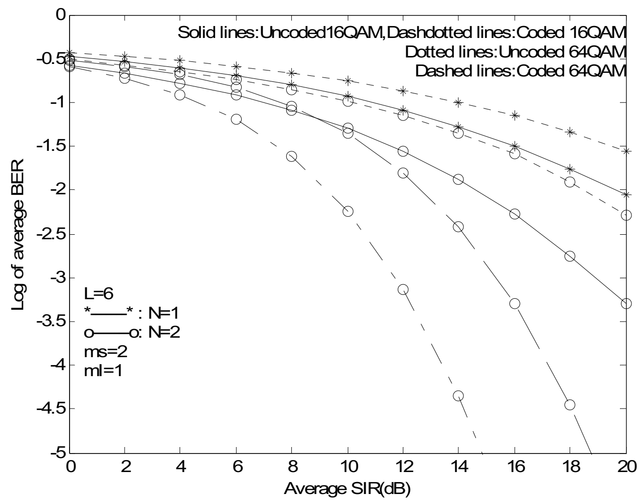

Fig. 2 The average BER versus average SIR of uncoded and coded MQAM under N-branch MRC diversity in the presence of six identical mean power interferers in a Nakagami fading environment (m_S=2, m_I=1)

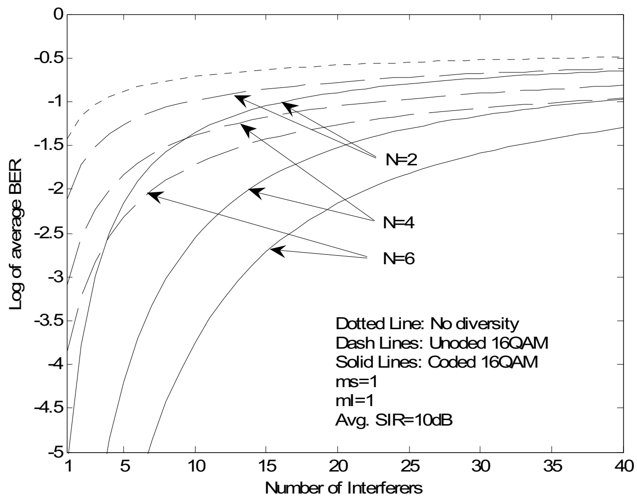

Fig. 3 The average BER versus number of interferers, L for uncoded and coded 16-QAM under N-branch MRC diversity with identical mean power interferers in a Nakagami fading environment (m_S=1, m_I=1, Avg. SIR q=10dB).

V. CONCLUSION

In this paper we have investigated the effect of cochannel interference on the performance of digital cellular mobile radio systems. We have studied the improvement in the performance of 16- and 64-QAM system under MRC diversity in frequency non-selective slow Nakagami fading environment in the presence of cochannel interferers due to the use of linear binary block codes. The desired signal ,as well as the interferers were assumed to be subject to Nakagami fading, but the cochannel interferers experience deeper fading, which characterizes the microcellular environment. The performance of the maximal ratio combiner that maximizes the output SNR in the presence of cochannel interference and noise is analyzed. It is assumed that there is an arbitrary number of independent Nakagami interferers, each having the same power and undergoing the same amount of fading. The results show that since the MRC system is optimized without regards to the presence of cochannel interference, its performance deteriorates when interferers are present. In general, the use of MRC diversity is beneficial even in an interference-limited environment since the use of linear binary block codes and increasing the order of diversity improves system performance considerably. The closed-form expressions presented in this paper are sufficiently simple and can be computed without any approximations. It is expected that the analytical results presented in this paper will provide a convenient tool for design and analysis of a radio communication system with space diversity reception in the presence of identical cochannel interferers over Nakagami fading environment.

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