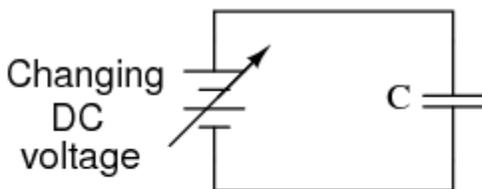


Differentiator and integrator circuits

By introducing electrical reactance into the feedback loops of op-amp amplifier circuits, we can cause the output to respond to changes in the input voltage over *time*. Drawing their names from their respective calculus functions, the *integrator* produces a voltage output proportional to the product (multiplication) of the input voltage and time; and the *differentiator* (not to be confused with *differential*) produces a voltage output proportional to the input voltage's rate of change.

Capacitance can be defined as the measure of a capacitor's opposition to changes in voltage. The greater the capacitance, the more the opposition. Capacitors oppose voltage change by creating current in the circuit: that is, they either charge or discharge in response to a change in applied voltage. So, the more capacitance a capacitor has, the greater its charge or discharge current will be for any given rate of voltage change across it. The equation for this is quite simple:



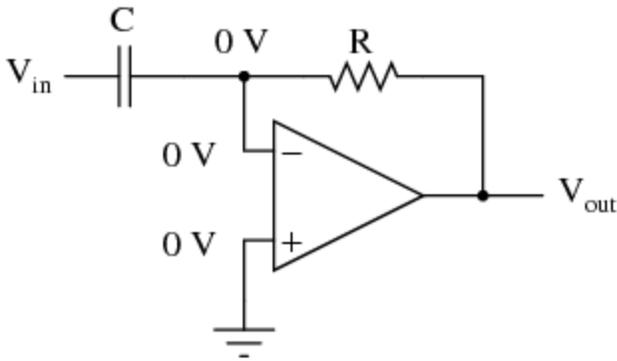
$$i = C \frac{dv}{dt}$$

The dv/dt fraction is a calculus expression representing the rate of voltage change over time. If the DC supply in the above circuit were steadily increased from a voltage of 15 volts to a voltage of 16 volts over a time span of 1 hour, the current through the capacitor would most likely be *very* small, because of the very low rate of voltage change ($dv/dt = 1 \text{ volt} / 3600 \text{ seconds}$). However, if we steadily increased the DC supply from 15 volts to 16 volts over a shorter time span of 1 second, the rate of voltage change would be much higher, and thus the charging current would be much higher (3600 times higher, to be exact). Same amount of change in voltage, but vastly different *rates* of change, resulting in vastly different amounts of current in the circuit.

To put some definite numbers to this formula, if the voltage across a $47 \mu\text{F}$ capacitor was changing at a linear rate of 3 volts per second, the current "through" the capacitor would be $(47 \mu\text{F})(3 \text{ V/s}) = 141 \mu\text{A}$.

We can build an op-amp circuit which measures change in voltage by measuring current through a capacitor, and outputs a voltage proportional to that current:

Differentiator



The right-hand side of the capacitor is held to a voltage of 0 volts, due to the "virtual ground" effect. Therefore, current "through" the capacitor is solely due to *change* in the input voltage. A steady input voltage won't cause a current through C, but a *changing* input voltage will.

Capacitor current moves through the feedback resistor, producing a drop across it, which is the same as the output voltage. A linear, positive rate of input voltage change will result in a steady negative voltage at the output of the op-amp. Conversely, a linear, negative rate of input voltage change will result in a steady positive voltage at the output of the op-amp. This polarity inversion from input to output is due to the fact that the input signal is being sent (essentially) to the inverting input of the op-amp, so it acts like the inverting amplifier mentioned previously. The faster the rate of voltage change at the input (either positive or negative), the greater the voltage at the output.

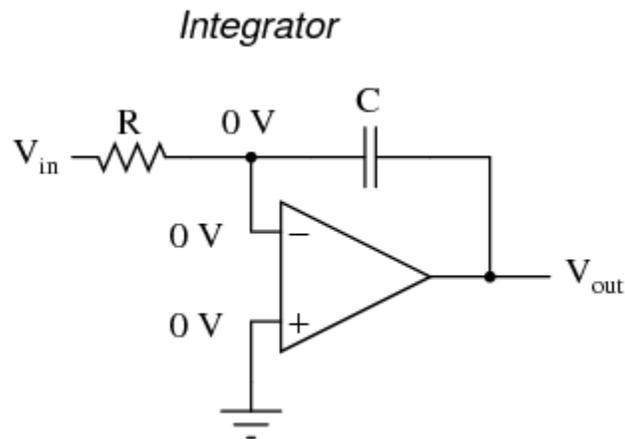
The formula for determining voltage output for the differentiator is as follows:

$$V_{\text{out}} = -RC \frac{dv_{\text{in}}}{dt}$$

Applications for this, besides representing the derivative calculus function inside of an analog computer, include rate-of-change indicators for process instrumentation. One such rate-of-change signal application might be for monitoring (or controlling) the rate of temperature change in a furnace, where too high or too low of a temperature rise rate could be detrimental. The DC voltage produced by the differentiator circuit could be used to drive a comparator, which would signal an alarm or activate a control if the rate of change exceeded a pre-set level.

In process control, the derivative function is used to make control decisions for maintaining a process at setpoint, by monitoring the rate of process change over time and taking action to prevent excessive rates of change, which can lead to an unstable condition. Analog electronic controllers use variations of this circuitry to perform the derivative function.

On the other hand, there are applications where we need precisely the opposite function, called *integration* in calculus. Here, the op-amp circuit would generate an output voltage proportional to the magnitude and duration that an input voltage signal has deviated from 0 volts. Stated differently, a constant input signal would generate a certain *rate of change* in the output voltage: differentiation in reverse. To do this, all we have to do is swap the capacitor and resistor in the previous circuit:



As before, the negative feedback of the op-amp ensures that the inverting input will be held at 0 volts (the virtual ground). If the input voltage is exactly 0 volts, there will be no current through the resistor, therefore no charging of the capacitor, and therefore the output voltage will not change. We cannot guarantee what voltage will be at the output with respect to ground in this condition, but we can say that the output voltage *will be constant*.

However, if we apply a constant, positive voltage to the input, the op-amp output will fall negative at a linear rate, in an attempt to produce the changing voltage across the capacitor necessary to maintain the current established by the voltage difference across the resistor. Conversely, a constant, negative voltage at the input results in a linear, rising (positive) voltage at the output. The output voltage rate-of-change will be proportional to the value of the input voltage.

The formula for determining voltage output for the integrator is as follows:

$$\frac{dv_{\text{out}}}{dt} = - \frac{V_{\text{in}}}{RC}$$

or

$$V_{\text{out}} = \int_0^t - \frac{V_{\text{in}}}{RC} dt + c$$

Where,

c = Output voltage at start time ($t=0$)

One application for this device would be to keep a "running total" of radiation exposure, or dosage, if the input voltage was a proportional signal supplied by an electronic radiation detector. Nuclear radiation can be just as damaging at low intensities for long periods of time as it is at high intensities for short periods of time. An integrator circuit would take both the intensity (input voltage magnitude) and time into account, generating an output voltage representing total radiation dosage.

Another application would be to integrate a signal representing water flow, producing a signal representing total quantity of water that has passed by the flowmeter. This application of an integrator is sometimes called a *totalizer* in the industrial instrumentation trade.

REVIEW:

- A *differentiator* circuit produces a constant output voltage for a steadily changing input voltage.
- An *integrator* circuit produces a steadily changing output voltage for a constant input voltage.
- Both types of devices are easily constructed, using reactive components (usually capacitors rather than inductors) in the feedback part of the circuit.

Source: http://www.allaboutcircuits.com/vol_3/chpt_8/11.html