Module
5
Carrier Modulation

Lesson 26

Differential Encoding and Decoding

After reading this lesson, you will learn about

- ***Differential Encoding of BPSK Modulation (DEBPSK);***
- ***Differential Coding for QPSK;***

Let us consider the processes of quadrature carrier modulation and demodulation and express the output of a quadrature modulator as,

$$s(t) = \text{Re}\ \{A.\tilde{u}(t).e^{jw_c t}\} = \text{Re}\ \{[u_i(t) + j\,u_q(t)].\ A.\,e^{jw_c t}\}$$
$$= A.\{u_i(t)\cos w_c t - u_q(t)\sin w_c t\} \qquad 5.26.1$$

'A' is a scalar quantity. The two orthogonal carriers which act on $u_i(t)$ and $u_q(t)$ are 'A $\cos w_c t$' and '-A$\sin w_c t$'. The appropriate carriers in the receiver are as shown in **Fig. 5.25.3** which result in correct estimates in absence of noise $\hat{u}_i(t) = u_i(t)$ and $\hat{u}_q(t) = u_q(t)$. Now think what may happen if, say, '-A$\sin w_c t$' multiplies r(t) in the I-arm and 'A $\cos w_c t$' multiplies r(t) in the Q-arm? One can easily see that, the quadrature demodulation structure produces $\hat{u}_i(t) = u_q(t)$ and $\hat{u}_q(t) = u_i(t)$, i.e. the expected outputs have swapped their places. Do we lose information if it happens? No, provided (i) either we are able to recognize that swapping of I-path signal with Q-path signal has occurred (so that we relocate the signals appropriately before delivering to the next stage) or (ii) we devise a scheme which will extract proper signal even when such anomalies occur.

In a practical coherent demodulation scheme, the phase of the carrier is assessed almost continuously against background noise. This issue of phase synchronization is treated separately at some length in **Lesson #31**.

Summarily, precise phase synchronization is a complex process and it increases the cost of a receiver. In any case, sudden change in phase in the transmit oscillator by multiples of 90^0 is never completely ruled out. In view of several such reasons, the approach of differential encoding is followed in practice. Differential encoding and decoding also aid the process of differential demodulation. In the following, we briefly discuss the issue of differential encoding and differential decoding for PSK modulations.

Differential Encoding of BPSK Modulation (DEBPSK)

Let us assume that for an ordinary BPSK modulation scheme, the carrier phase is 0^c when the message bit, 'm_k' is logic '1' and it is π^c if the message bit m_k' is logic '0'.

When we apply differential encoding, the encoded binary '1' will be transmitted by adding 0^c to the current phase of the carrier and an encoded binary '0' will be transmitted by adding π^c to the current phase of the carrier. Thus, relation of the current message bit to the absolute phase of the received signal is modified by differential encoding. The current carrier phase is dependent on the previous phase and the current message bit. For BPSK modulation format, the differential encoder generates an encoded binary logic sequence $\{d_k\}$ such that, $d_k = 1$ if d_{k-1} and m_k are similar and $d_k = 0$ if d_{k-1} and m_k are not similar.

For completeness, let us assume that the first encoded bit, say, d_0 is '1' while the index 'k' takes values 1, 2, ….**Fig. 5.26.1(a)** shows a block schematic diagram for differential encoding and BPSK modulation. For clarity, we will refer the modulated signal as 'Differentially Encoded and BPSK modulated (DEBPSK)' signal. The level shifter converts the logic symbols to binary antipodal signals of ±1. Note that the encoding logic is simple to implement:

$$d_k = d_{k-1} m_k + \overline{d_{k-1}}.\overline{m_k} \qquad 5.26.2$$

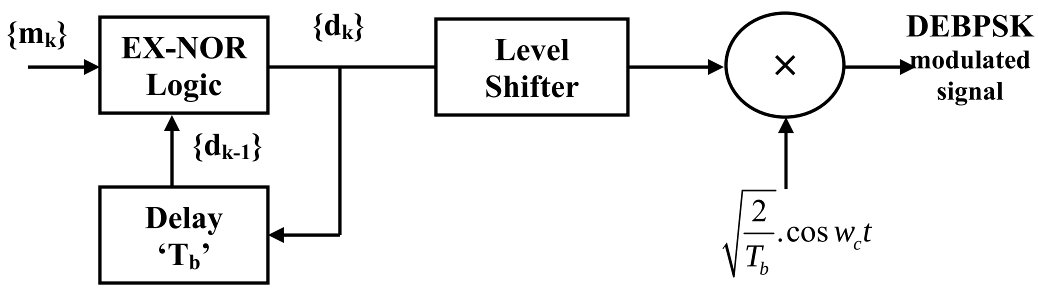
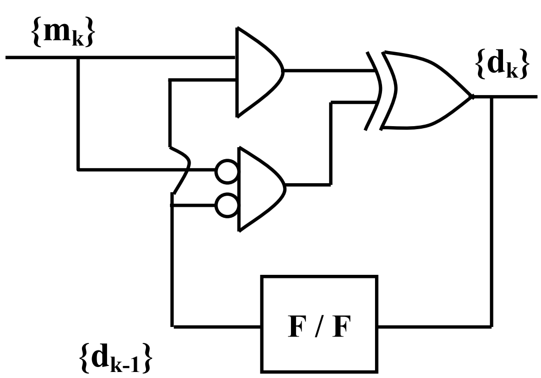


Fig. 5.26.1(a) *Block schematic diagram showing differential encoding for BPSK modulation*

Fig. 5.26.1(b) shows a possible realization of the differential encoder. It also explains the encoding operation for a sample message sequence {1,0,1,1,0,1,0,0,…} highlighting the phase of the modulated carrier.



$\{m_k\}$	:		1	0	1	1	0	1	0	0
$\{d_k\}$	:	1	1	0	0	0	1	1	0	1
Tx.Phase	:	0	0	π	π	π	0	0	π	0
Demod. Data	:		1	0	1	1	0	1	0	0

Fig. 5.26.1(b) *A realization of the differential encoder for DEBPSK showing the encoding operation for a sample message sequence*

Now, demodulation of a DEBPSK modulated signal can be carried out following the concept of correlation receiver as we have explained earlier in **Lesson #24** (**Fig. 5.24.3**), followed by a differential decoding operation. This ensures optimum (i.e., best achievable) error performance while not requiring a very precise carrier phase recovery scheme. We will refer this combination of correlation receiver with differential encoding-decoding also as the DEBPSK modulation-demodulation scheme.

This is to avoid confusion with another possible scheme of demodulation, which uses a concept of direct differential demodulation. **Fig.5.26.2** explains the differential demodulation scheme for BPSK when differential encoding has been used for BPSK modulation. We refer this demodulator as 'Differential Binary PSK (DBPSK) demodulator'. This is an example of 'non-coherent' demodulation scheme, as it does not require the regenerated carrier for demodulation. So, it is simpler to implement. With reference to the diagram, note that the output x(t) of the multiplier can be expressed without considering the noise component as:

$$x(t) = r(t)\times r(t-T_b) = A_c^{\,2}\left\{\cos\left[\varpi_c t+\theta+\bar{\mathrm{d}}_k.\pi\right]\times\cos\left[\varpi_c\left(t-T_b\right)+\theta+\bar{\mathrm{d}}_{k-1}.\pi\right]\right\}$$

$$= \frac{A_c^{\,2}}{2}\left\{\cos\left[\left(\bar{d}_k-\bar{d}_{k-1}\right)\pi\right]+\cos\left[2.\varpi_c t+2\theta+\left(\bar{d}_k+\bar{d}_{k-1}\right)\pi\right]\right\} \qquad 5.26.3$$

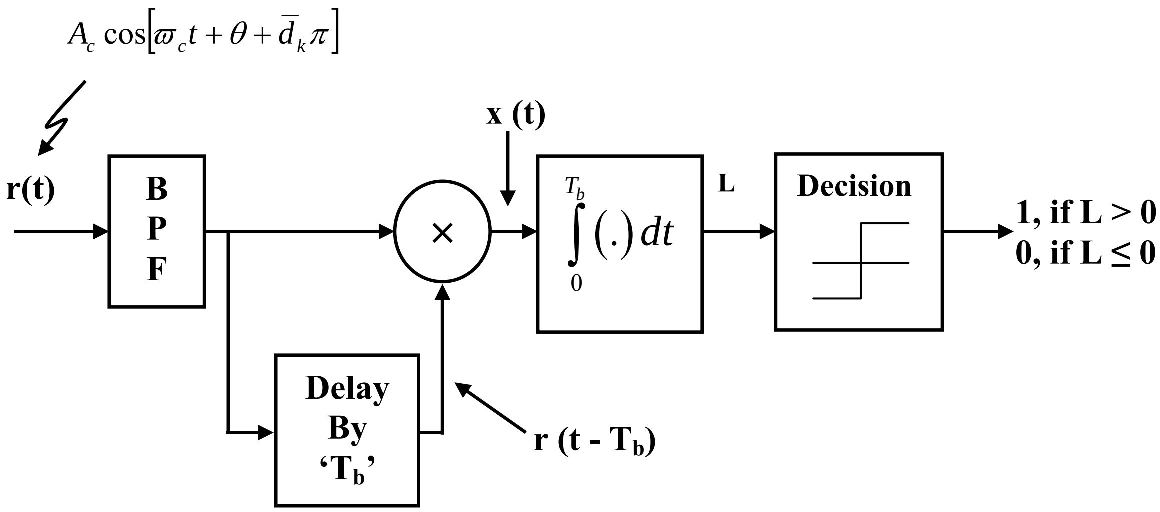


Fig.5.26.2 *Differential demodulation of differentially encoded BPSK modulated signal*

Here, the received signal r(t) is:

$$r(t) = A_c\cos\left[\varpi_c t+\theta+\bar{d}_k\pi\right] \qquad 5.26.4$$

The integrator, acting as a low pass filter, removes the second term of x(t), which is centered around $2\omega_c$ and as a result, the output 'L' of the integrator is $\pm\, A_c^{\,2}/2$ which is used by the threshold detector to determine estimates of the message bit 'm_k' directly. Unlike the DEBPSK demodulation scheme, no separate differential decoding operation is necessary. However, the DBPSK demodulator scheme requires that the IF modulated signal (or equivalently, its time samples) is delayed precisely by 'T_b', the duration of one message bit, and fed to the multiplier. Error performance of the DBPSK demodulation scheme is somewhat inferior to that of ordinary BPSK (or DEBPSK) as one decision error in the demodulator may cause two bits to be in error in quick succession. However, the penalty in error performance is not huge for many applications where lower cost or complexity is preferred more. DBPSK scheme needs about 0.94 dB of additional E_b/N_0 to ensure a BER of 10^{-05}, compared to the optimum and coherent BPSK demodulation scheme.

Differential Coding for QPSK

The four possibilities that are to be considered for designing differential encoder and decoder for QPSK are shown in **Table 5.26.1**, assuming that the I-path carrier in the modulator is A cos w_c t and the Q-path carrier is -Asin w_c t. In any of the four possibilities listed in **Table 5.26.1**, we wish to extract u_i(t) in the I-arm and u_q(t) in the Q-arm using differential encoding and decoding

I-Path Regenerated Carrier	**Q-Path Regenerated carrier**	$\hat{u}_i(t)$	$\hat{u}_q(t)$	**Remarks**
A cos w_c t	-Asin w_c t	u_i(t)	u_q(t)	Correctly derived
-A cos w_c t	Asin w_c t	- u_i(t)	-u_q(t)	Inverted
Asin w_c t	-A cos w_c t	-u_q(t)	- u_i(t)	Swapped and inverted
-Asin w_c t	A cos w_c t	u_q(t)	u_i(t)	Swapped

Table 5.26.1 *The outputs of the I- and Q- correlators in the demodulator*

One can easily verify from the truth table (**Table 5.26.2**) that,

$$d_{ik} = \overline{u_{ik}} \,.\, \overline{u_{qk}} \,.\, d_{qk-1} + u_{ik} \,.\, \overline{u_{qk}} \,.\, d_{qk-1} + u_{ik} \,.\, u_{qk} \,.\, d_{ik-1} + u_{qk} \,.\, u_{ik} \,.\, d_{qk-1} \qquad 5.26.4$$

$$d_{qk} = \overline{u_{ik}} \,.\, \overline{u_{qk}} \,.\, d_{qk-1} + u_{ik} \,.\, \overline{u_{qk}} \,.\, d_{ik-1} + u_{ik} \,.\, u_{qk} \,.\, \overline{d_{qk-1}} + \overline{u_{ik}} \,.\, u_{qk} \,.\, d_{ik-1} \qquad 5.26.5$$

u_{ik}	u_{qk}	d_{ik-1}	d_{qk-1}	d_{ik}	d_{qk}
0	0	0	0	0	0
0	0	0	0	0	1
0	0	1	0	1	0
0	0	1	1	1	1
0	1	0	0	0	1
0	1	0	1	1	1
0	1	1	0	0	0
0	1	1	1	1	0
1	0	0	0	1	0
1	0	0	1	0	0
1	0	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	1	0	0

Table 5.26.2 *Truth table for differential encoder for QPSK*

A feed forward logic circuit is used in a differential decoder in a DEQPSK scheme, which considers the output from the quadrature demodulator to recover u_{ik} and u_{qk} in the correct arms.

Let us consider a situation, represented by the 11th row of the encoder truth table, i.e., u_{qk} =0, u_{ik} =1, d_{ik-1} =1, d_{qk-1} =0, d_{ik} = 0 and d_{qk} =1.

Now, let us consider the four possible phase combination of the quadrature demodulator at the receiver to write the values of e_i 's and e_q 's (**Table 5.26.3).**

I-Path carrier	**Q-path carrier**	d_{ik-1} d_{qk-1}	d_{ik} d_{qk}	e_{ik-1} e_{qk-1}	e_{ik} e_{qk}	u_{ik} u_{qk}	**Remarks**
A cos w_c t	-A sin w_c t	1 0	1 1	1 0	1 1	1 0	Phase OK
-A cos w_c t	A sin w_c t	1 0	1 1	0 1	0 0	1 0	Phase inverted
A sin w_c t	-A cos w_c t	1 0	1 1	1 0	0 0	1 0	Data swapped and inverted
-A sin w_c t	A cos w_c t	1 0	1 1	0 1	0 1	1 0	Data swapped

Table 5.26.3 *Four phase combinations of the quadrature demodulator at the receiver*

Related to the values of e_i ’s and e_q ’s

The last three columns showing e_i ’s, e_q ’s and the desired outputs partially indicate the necessary logic for designing a differential decoder. Continuing in a similar fashion, one can construct the complete truth table of a differential decoder (**Table 5.26.4**).

e_{ik-1}	e_{qk-1}	e_{ik}	e_{qk}	$\hat{u}_{ik}$	$\hat{u}_{qk}$
0	0	0	0	0	0
		0	1	0	1
		1	0	1	0
		1	1	1	1
0	1	0	0	1	0
		0	1	0	0
		1	0	1	1
		1	1	0	1
1	0	0	0	0	1
		0	1	1	1
		1	0	0	0
		1	1	1	0
1	1	0	0	1	1
		0	1	1	0
		1	0	0	1
		1	1	0	0

Table 5.26.4 *Truth Table of the differential decoder for QPSK*

It is easy to deduce that,

$$\hat{u}_{ik} = \overline{e_{ik}} \,.\, \overline{e_{qk}} \,.\, e_{qk-1} + e_{ik} \,.\, \overline{e_{qk}} \,.\, \overline{e_{ik-1}} + e_{ik} \,.\, e_{qk} \,.\, \overline{e_{qk-1}} + \overline{e_{ik}} \,.\, e_{qk} \,.\, e_{ik-1}$$

$$\hat{u}_{qk} = \overline{e_{ik}} \,.\, \overline{e_{qk}} \,.\, e_{ik-1} + e_{ik} \,.\, \overline{e_{qk}} \,.\, e_{qk-1} + e_{ik} \,.\, e_{qk} \,.\, \overline{e_{ik-1}} + \overline{e_{ik}} \,.\, e_{qk} \,.\, \overline{e_{qk-1}}$$

Somewhat analogous to DBPSK, one can design a QPSK modulation-demodulation scheme-using differential encoding in the modulator and employing noncoherent differential demodulation at the receiver. The resultant scheme may be referred as DQPSK. The complexity of such a scheme is less compared to a coherent QPSK scheme because precise recovery of carrier phase is not necessary in the receiver. However, analysis shows that the error performance of DQPSK scheme is considerably poorer compared to the coherent DEQPSK or ordinary coherent QPSK receiver. The differential demodulation approach requires more than 2dB extra E_b/N_o to ensure a BER of 10^{-05} when compared to ordinary uncoded QPSK with correlation receiver structure.

Problems

Q5.26.1) Justify the need for differential encoding.

Q5.26.2) Re-design the circuit of Fig. 5.26.1(b). Considering negative logic for the binary digits.

Q5.26.3) Mention two merits and two demerits of QPSK modem compare to a BPSK modem.