Module 3

LOSSY IMAGE COMPRESSION SYSTEMS

Lesson 7

Delta Modulation and DPCM

Instructional Objectives

At the end of this lesson, the students should be able to:

1. Describe a lossy predictive coding scheme.
2. Define Delta Modulation.
3. Encode and decode a sequence of pixels through Delta Modulation.
4. Define granular noise and slope overload.
5. State the effects of granular noise and slope overload on the reconstructed image.
6. State the basic principle of Differential Pulse Code Modulation (DPCM).
7. Design an optimal predictor for DPCM.
8. Design a Lloyd-Max quantizer for DPCM.
9. Design an adaptive quantizer for DPCM.

7.0 Introduction

We have already learnt the basic theory of quantization and seen how to design an optimum quantizer in the mean square error sense for a fixed number of levels in lesson-6. We are going to find use of this theory in every lossy compression system, since quantizer is an essential block in all such systems.

In this lesson, we begin our discussions with the concept of lossy predictive coding scheme, which calls for modifications of the lossless predictive coding scheme discussed in lesson-5. Not only is a quantizer introduced on the error signal, it must be ensured that the encoder and the decoder follow the same predictions. We thereafter present a very simple form of lossy predictive scheme that is Delta Modulation, which uses a simple past-sample predictor and a two-level quantizer. Such lossy predictive coding encodes every pixel at the rate of only 1-bit/pixel but fails to provide an acceptable reconstruction quality due to presence of two prominent effects, namely granular noise and slope overload, that affect uniform intensity region and sharp intensity gradients at the edges significantly. A more refined lossy predictive compression scheme is the Differential Pulse Code Modulation (DPCM) scheme, which uses more than two levels of quantization. DPCM can use optimal Lloyd-Max quantizer or an adaptive quantizer that can select the quantization based on the local statistics of the error signal.

7.1 Lossy Predictive Coding

Like lossless predictive coding schemes, the basic principle of lossy predictive coding is also the prediction of current sample, based on the past samples, usually picked up from the neighborhood of the current pixels for images. The error in prediction, given by

$$e(n) = s(n) - \hat{s}(n) \qquad (7.1)$$

is quantized and further compressed through one of the lossless compression schemes. Fig.7.1 shows the block diagram of the lossy predictive coding encoder and Fig.7.2 shows the corresponding decoder to generate the reconstructed sample. At the decoder, the quantized error signal $\dot{e}(n)$ is added to the predicted sample $\hat{s}(n)$ to generate the reconstructed sample $\dot{s}(n)$, which is not equal to the original sample $s(n)$ because of the introduction of the quantizer. The reconstructed sample is given by

$$\dot{s}(n) = \dot{e}(n) + \hat{s}(n) \quad \text{...(7.2)}$$

The reconstructed sample (and a set of past reconstructed samples) is used to generate the next predicted sample. Identical predictors should exist at both encoder and the decoder to prevent error accumulation and hence, the encoder should also derive the reconstructed sample, in accordance with equation (7.2). The encoder thus contains a feedback path to derive the predicted sample, as shown in Fig.7.1.

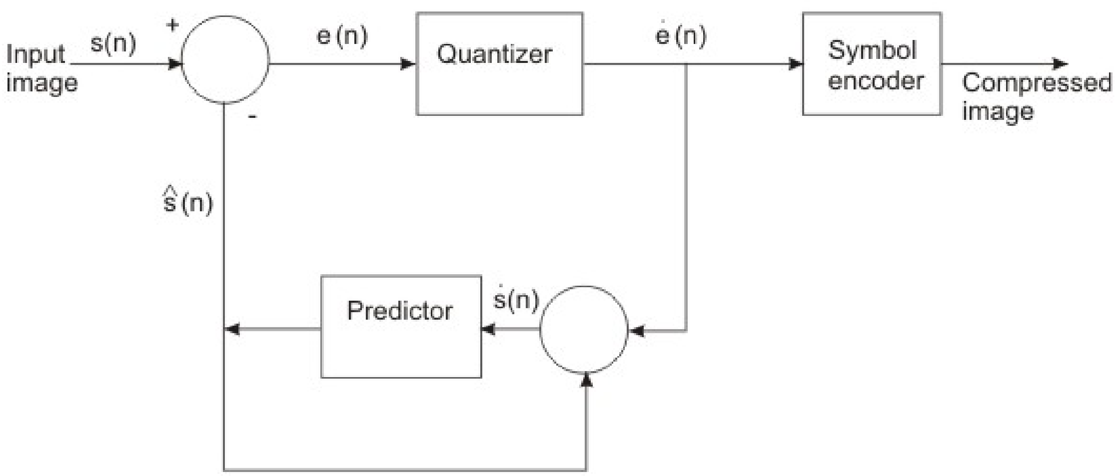

Fig 7.1 Lossy predictive coding scheme

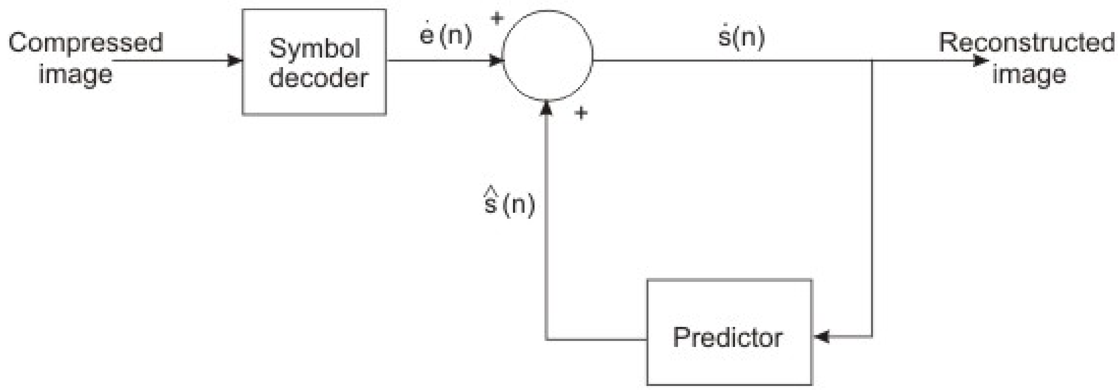

Fig 7.2 Decoder for lossy predictive coding

7.2 Delta Modulation

One of the simplest lossy predictive schemes is *Delta Modulation*, in which the present sample is predicted as $\hat{s}(n)$ in terms of the immediate past reconstructed sample $\dot{s}(n-1)$
and is given by

$$\hat{s}(n) = \alpha_{n-1}\,\dot{s}(n-1)$$

where, α_{n-1} is the coefficient of the prediction ($\alpha_{n-1} \le 1$). The error is quantized to two levels, depending upon whether the error is positive or negative and is given by

$$\dot{e}(n) = \begin{cases} \zeta & \text{if } e(n) \ge 0 \\ -\zeta & \text{otherwise} \end{cases}$$

The two-level quantizer is shown in Fig.7.3.

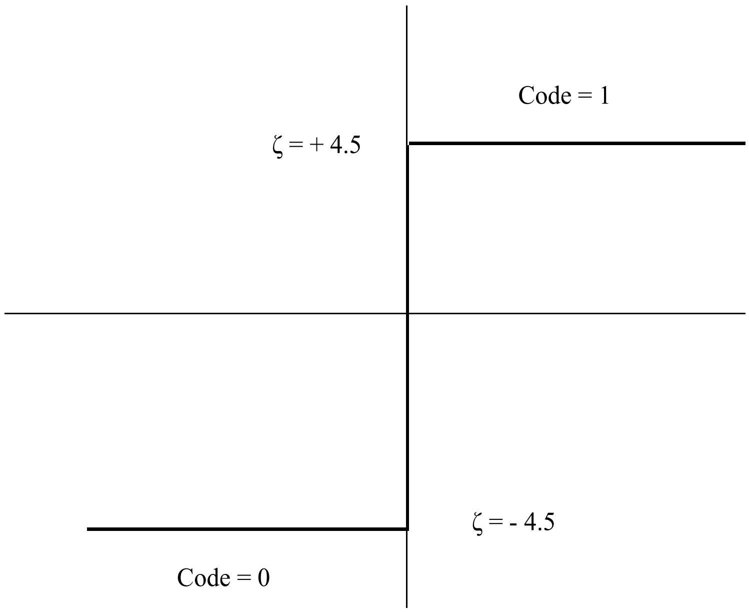

Fig 7.3 Delta-modulation-two level quantization

7.2.1 An example sequence :

Let us consider a row of pixels, whose intensity values are given as follows:

33 35 34 36 35 34 35 35 38 44 50 59 73 81 82 82 81 81

A plot of intensity vs. pixel (s(n) vs. n) is plotted in Fig.7.4.

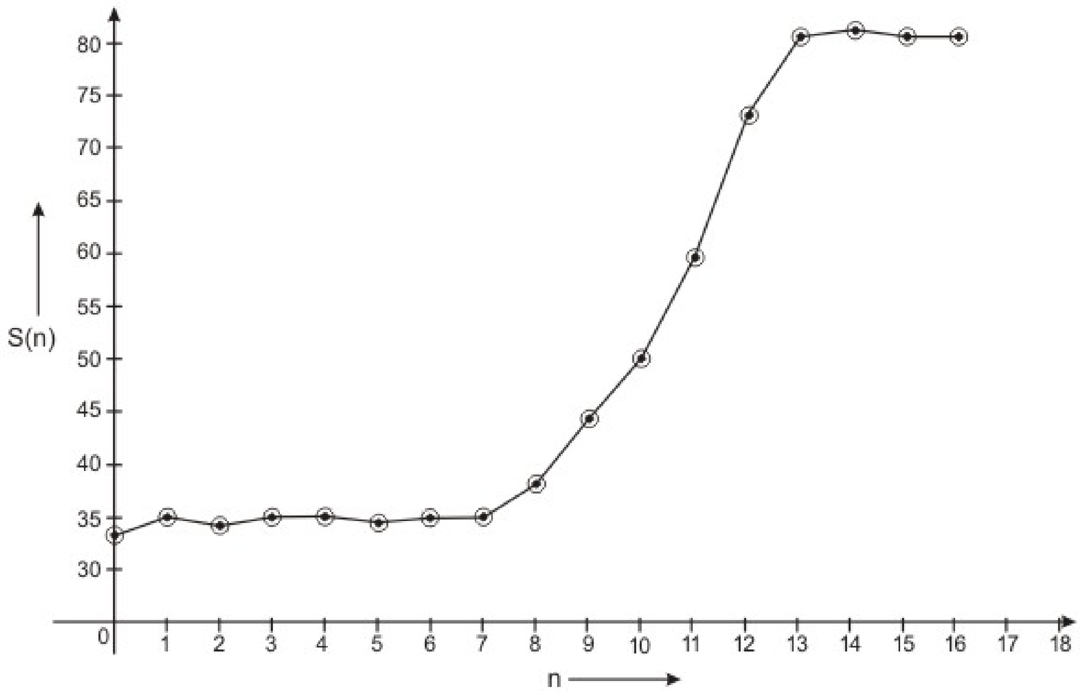

Fig.7.4 a: *Example of Delta Modulation*

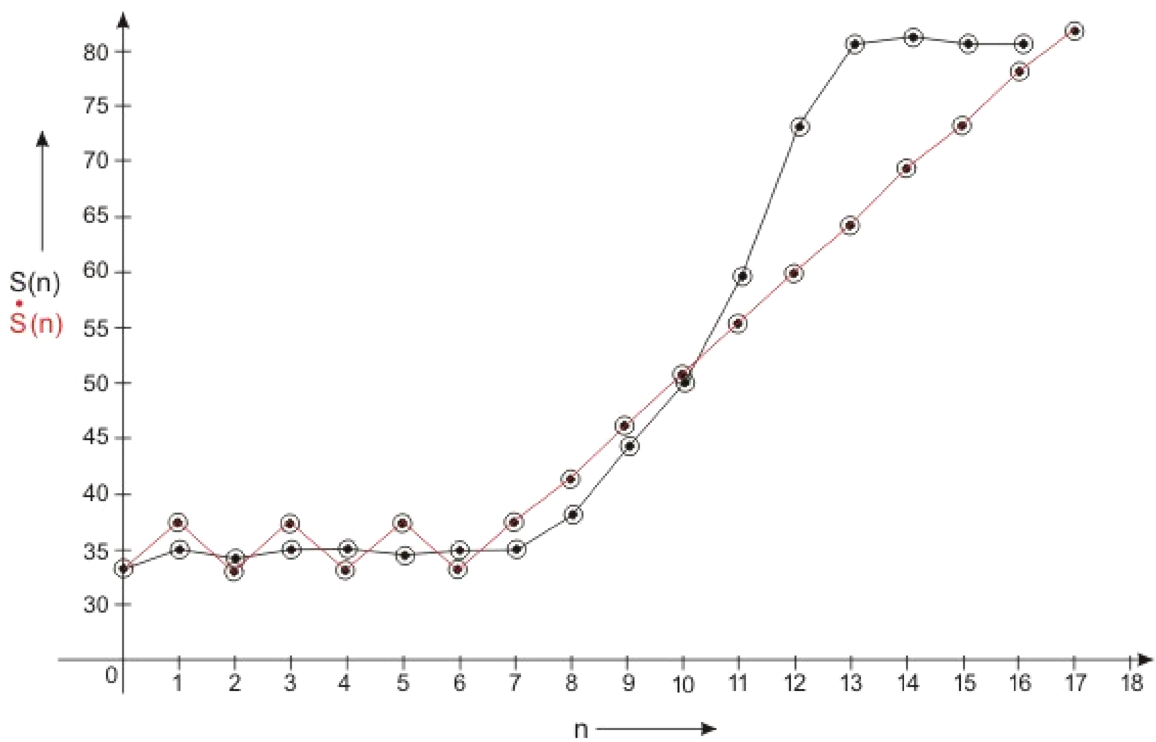

Fig.7.4 b: *Example of Delta Modulation*

Let us apply Delta modulation on these pixels, with $\alpha_i = 1$ for all pixels and $\zeta = 4.5$. For *n* =1, i.e. for the first pixel, no previous pixel is available for prediction and this has to be encoded as it is. For *n* =2, we have $\hat{s}(2) = 33, \quad e(2) = 2, \quad \dot{e}(2) = 4.5 \quad \dot{s}(2) = 37.5$. Again, $\dot{s}(2)$ will be used as $\hat{s}(3)$ and the next error, quantized error and reconstructed sample will be generated. Table-7.1 shows the predicted sample, the error and the reconstructed values.

It may be noted that between *n* = 1 to 8, the pixel values are slowly changing, but the reconstructed values are rapidly fluctuating relatively, giving rise to a distortion, which is known as *granular noise*. Thus, granular noise results when the actual variations are much less than the quantization step-size $\pm\zeta$. This noise causes false variations in uniform intensity regions, where actual variations are minimal. Also observe that between *n* = 13 to 18, the step-size becomes much smaller as compared to very large change in the intensity values. This gives rise to large distortions and are defined as *slope overload*, which means that the slope of the reconstructed pixel change can not cope up with the actual slope of the pixel change. This results in blurring of the edges in images. Thus, small values of ζ often gives rise to *slope overload*, whereas large values of ζ often causes *granular noise*. If ζ is changed in accordance with the change in image statistics, the distortions described above may be reduced. Such Delta modulators are known as *Adaptive Delta Modulator*.

Input		Encoder				Decoder		Error
n	*s*	$\hat{s}$	*e*	$\dot{e}$	$\dot{s}$	$\hat{s}$	$\dot{s}$	$s-\dot{s}$
1	33	-	-	-	33.0	-	33.0	0.0
2	35	33.0	2.0	4.5	37.5	33.0	37.5	-2.5
3	34	37.5	-3.5	-4.5	33.0	37.5	33.0	1.0
4	36	33.0	3.0	4.5	37.5	33.0	37.5	-1.5
5	35	37.5	-2.5	-4.5	33.0	37.5	33.0	2.0
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
12	59	51.0	8.0	4.5	55.5	51.0	55.5	3.5
13	73	55.5	17.5	4.5	60.0	55.5	60.0	13.0
14	81	60.0	21.0	4.5	64.5	60.0	64.5	16.5
15	82	64.5	17.5	4.5	69.0	64.5	69.0	13.0
16	82	69.0	13.0	4.5	73.5	69.0	73.5	8.5
17	81	73.5	7.5	4.5	78.0	73.5	78.0	3.0
18	81	78.0	3.0	4.5	82.5	78.0	82.5	-1.5

Table-7.1 Delta modulation example

7.3 Differential Pulse Code Modulation (DPCM)

In any digital image, the intensity values of pixels are represented by the Pulse Code Modulation (PCM) of its quantized values. PCM only considers the current pixel for representation, without any reference to its spatial correlation. A modified form of PCM, called the Differential Pulse Code Modulation (DPCM) exploits the spatial correlation by predicting the current pixel from its past neighbors and quantizing the error in prediction. It is thus a PCM applied over the quantized value of the differential, i.e. the error signal and hence the name. The only difference between the DPCM and lossless linear predictive coding is the presence of quantizer at the encoder.

DPCM is simple to implement, but its compression ratio is rather limited. Although, quantization is related to the prediction error, the relationship is hard to determine. Usually, the prediction error is minimized without any reference to the quantizer and the quantizer is designed without any reference to the prediction. We are now going to present a predictor design, that optimal in linear minimum mean square error (LMMSE) sense.

7.3.1 Optimal Predictor Design

We consider the linear predictor that we had used in Section-5.3 to predict the current pixel value $s(n_1, n_2)$ and reproduce below the general fourth-order prediction equation stated in equation (5.4):

$$\hat{s}(n_1, n_2) = a_1 s(n_1 - 1, n_2 - 1) + a_2 s(n_1 - 1, n_2) + a_3 s(n_1 - 1, n_2 + 1) + a_4 s(n_1, n_2 - 1) \ldots\ldots(7.3)$$

where a_1, a_2, a_3, a_4 are the coefficients of prediction. The problem is to design a_1, a_2, a_3, a_4 such that the prediction error is optimal in linear minimum mean square error (LMMSE) sense. The mean squared error (MSE) in DPCM is given by

$$D = E\left[(\mathbf{s} - \dot{\mathbf{s}})^T (\mathbf{s} - \dot{\mathbf{s}})\right]$$

where s is a vector containing lexicographically ordered pixel intensities in the original and $\dot{\mathbf{s}}$ is the corresponding vector for the reconstructed image. Neglecting the quantization error, $\hat{\mathbf{s}} = \dot{\mathbf{s}}$, where $\hat{\mathbf{s}}$ is the vector for the predicted image. Using equation (7.3), it is possible to solve for the coefficient vector $\mathbf{a} = (a_1 \quad a_2 \quad a_3 \quad a_4)^T$ from the equation

$$\phi = \varphi \ \mathbf{a} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7.4)$$

where φ is defined as the autocorrelation matrix

$$\varphi = \begin{bmatrix} R(0,0) & R(0,1) & R(0,2) & R(1,0) \\ R(0,-1) & R(0,0) & R(0,1) & R(1,-1) \\ R(0,-2) & R(0,-1) & R(0,0) & R(1,-2) \\ R(-1,0) & R(-1,1) & R(-1,2) & R(0,0) \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(7.5)$$

and ϕ is the vector $[R(1,1) \quad R(1,0) \quad R(1,-1) \quad R(0,1)]^T$.

The autocorrelation values $R(i,j)$ are given by

$$R(i,j) = E[s(n_1,n_2)s(n_1-i,n_2-j)]$$

and these can be measured from the image as

$$R(i,j) = \frac{1}{N}\sum_{n,n_{21}} s(n_1,n_2)s(n_1-i,n_2-j)$$

where *N* is the number of pixels in the image.
The φ matrix defined in equation (7.5) is mostly invertible and the solution of a is given by
$\mathbf{a} = \varphi^{-1}\phi$
The solution of a provides optimal prediction in terms of LMMSE, but not in terms of entropy.

7.3.2 Quantization of Prediction Error:
The prediction error in DPCM is scalar quantized to *K* bits/sample, where *K* must be less than 8 in monochrome images to achieve compression. It is empirically shown that the prediction error in typical images can be modeled by Laplacian distribution with variance σ^2. Since, this distribution is heavily peaked about zero, a non-uniform Lloyd-Max quantization is employed in order to achieve the best image quality. For Laplacian distribution with unit variance, quantizer design is shown in Table-7.2 for 2,4 and 8 reconstruction levels. For $\sigma^2 \neq 1$, the decision and the reconstruction levels are accordingly scaled.

7.4 Adaptive Quantization

The above quantizer design approach is based on the global statistics of the error distribution over the entire image. However, σ is not likely to remain constant over the entire image and the local statistics of error distribution may vary from one part to another. It therefore makes sense in designing a quantizer that should be made adaptive to local statistics. An image can be divided into a number of non-overlapping blocks and based on the local σ, the quantizer can be adapted on a block-by-block basis. However, this would require transmission of σ for every block as overhead information.

An alternative strategy is to use fixed number of quantizers for different σ's as shown in Fig.7.5.

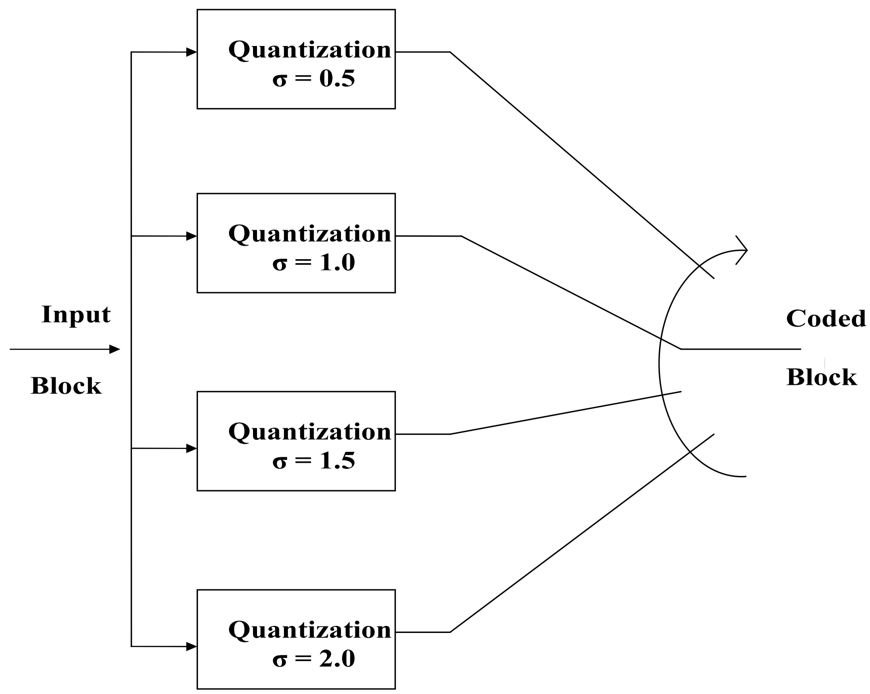

Fig 7.5: *Block diagram of an adaptive quantizer*

The example shown in the figure uses four quantizers. Thus, every block requires a selection of one out of four quantizers and hence, the overhead is only two bits per block. This nominal overhead may be marginal as compared to the improvement in the reconstruction quality.

Questions

NOTE: The students are advised to thoroughly read this lesson first and then answer the following questions. Only after attempting all the questions, they should click to the solution button and verify their answers.

PART-A

A.1. State the basic principles of lossy predictive scheme.

A.2. Write down the expressions for prediction and error quantization in Delta modulation.

A.3. Define: (a) slope overload and (b) granular noise.

A.4. What should be the design consideration for quantization level in Delta modulation?

A.5. State the basic principles of Differential Pulse Code Modulation (DPCM).

A.6. Distinguish between DPCM and PCM.

PART-B: Multiple Choice

In the following questions, click the best out of the four choices.

B.1 An input sequence $\{14, 17, 15, 16, \cdots\cdots\}$ is Delta modulated with unity prediction coefficient and quantization levels of ± 5.5. The reconstruction error for the second sample at the decoder is

(A) -4.5

(B) -2.5

(C) 2.5

(D) 3.0

B.2 Small step-size in Delta modulation may lead to

(A) granular noise.

(B) slope overload.

(C) poor compression ratio.

(D) high compression ratio.

B.3 A fourth order linear predictor with prediction coefficients $a_1 = 0.1,\ a_2 = 0.4, a_3 = 0.1, a_4 = 0.4$ is used to predict the pixel marked "X", as shown below:

58	49	53
57	X	

If the "X" marked pixel has an intensity value of 52, the prediction error is:

(A) -5.0

(B) -2.25

(C) -1.5

(D) +1.5

B.4 A 2-bits/sample DPCM employing uniform quantizer is used to encode the following array of prediction errors within their maximum and values:

$$\begin{bmatrix} 0.5 & -3 & 1 \\ 4 & -1 & -0.5 \\ -2.5 & 3 & -4 \end{bmatrix}$$

The quantized prediction error array is given by

(A) $\begin{bmatrix} 0 & -4 & 0 \\ 4 & 0 & 0 \\ -4 & 4 & -4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -2 & 2 \\ 4 & 0 & 0 \\ -2 & 2 & -4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & -3 & 1 \\ 3 & -1 & 0 \\ -3 & 3 & -3 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -3 & 1 \\ 3 & -1 & -1 \\ -3 & 3 & -3 \end{bmatrix}$

B.5 Adaptive quantizers used in DPCM adjust their decision and reconstruction levels in accordance with

(A) local statistics of the prediction error.

(B) local statistics of the intensity values of the predicted image.

(C) local statistics of the intensity values of the original image.

(D) variation in channel characteristics.

B.6 Which of the following statements is wrong?

(A) Prediction error is likely to increase whenever image intensity values have discontinuities.

(B) Entropy of the prediction error is lower than the entropy of the original image.

(C) Standard deviation of the prediction error is higher than the standard deviation of the original intensity values.

(D) Prediction error is often modeled as a Laplacian distribution.

B.7 Adaptive quantizers used in DPCM suffer from

(A) additional overhead of bits.

(B) local variations in reconstruction quality.

(C) poor prediction capability.

(D) all of the above.

B.8 An optimal predictor is used to determine

(A) the variance of the prediction error.

(B) the elements of the correlation matrix.

(C) the prediction coefficients in minimizing the entropy of the prediction error.

(D) the prediction coefficients in LMMSE sense.

B.9 In the given image, the pixels lying outside the given boundary may be assumed to have the nearest boundary pixel value.

$$\begin{bmatrix} 3 & 7 & 5 & 2 \\ 4 & 5 & 2 & 7 \\ 3 & 5 & 4 & 5 \\ 2 & 5 & 3 & 4 \end{bmatrix}$$

The estimated value of the autocorrelation R[0,1] is

(A) 12.67

(B) 15

(C) 19.4

(D) 22.4

PART-C: Computer Assignments

C-1.
(a) Write a computer program to implement a delta modulation based encoder and decoder.
(b) Apply it on a monochrome image from the image archive. Use the following values: (i) $\varsigma = \pm 5$ (ii) $\varsigma = \pm 10$ (iii) $\varsigma = \pm 20$. Observe the reconstructed image in each case and comment on the granular noise and slope overload observed, if any.
(c) In each case, compute the PSNR of the reconstructed image.

C-2.
(a) Apply equation (7.3) on a monochrome image with $a_1 = 0.1,\ a_2 = 0.4,\ a_3 = 0.1,\ a_4 = 0.4$ and obtain the predicted image. Consider all pixels beyond the boundary to have the nearest boundary pixel value.
(b) Obtain the error image in (a) and plot the error image histogram.
(c) Determine the error image variance, model its distribution as Laplacian and design Lloyd-Max quantizers for (i) 2, (ii) 4 and (iii) 8 levels, based on the Lloyd-Max quantization table for unit variance Laplacian.
(d) Write a computer program to implement a DPCM encoder and decoder.
(e) For each of the quantizers designed in (c), obtain the reconstructed images and compute their PSNR values.

SOLUTIONS

A.1

A.2

A.3

A.4

A.5

A.6

B.1 (B) B.2 (B) B.3 (C) B.4 (D)

B.5 (A) B.6 (C) B.7 (A) B.8 (D) B.9 (B)

C.1 C.2

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