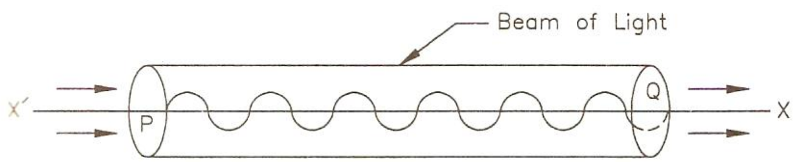
COHERENCE

Coherence – The waves maintaining constant and predictable phase difference are called coherent waves. Coherence requires that there is a connection between the amplitude and phase of the light at one point and time, and the amplitude and phase of the light at another point and time. If there are many waves of different frequencies, then the waves do not interfere and are said to be incoherent. It can be said that interference is a sure test of coherency.

When complete coherence is not achieved but some interference still exists then it refers to a condition of partial coherence.

Two types of coherence –

Temporal coherence – Temporal coherence (also called as longitudinal coherence or time coherence) is associated with the frequency spread of a wave pulse or difference in frequencies of waves. This type of coherence refers to the correlation between the wave field at a point and the field at the same point at a later time. If the phase difference measured at a single point in space at beginning and end of a time interval Δt remain predictable and constant, then waves are said to have temporal coherence in the time interval. The maximum value of the time interval in which waves maintain constant and have predictable phase difference is known as coherent time t_c. Greater is t_c, higher are the waves temporally coherent.



The beam is said to possess temporal coherence if the phase difference of the waves crossing P and Q at any instant remains constant

The temporal coherence also refers to the relative phase or coherence of waves at two separate locations along the direction of propagation of the waves. The maximum separation between two points along the direction of propagation of waves which maintain constant phase difference is known as temporal coherence length l_c.

If coherent time is t_c, coherence length l_c and bandwidth $\Delta\omega$, then

$$\Delta\omega = \frac{2\pi}{t_c}$$

If c is velocity of light, then

$$l_c = ct_c$$

Since $\Delta\omega = 2\pi\Delta v$

Therefore, $2\pi\Delta v = \frac{2\pi}{t_c}$

Or $\Delta v = \frac{1}{t_c}$

Also, $v = \frac{c}{\lambda}$

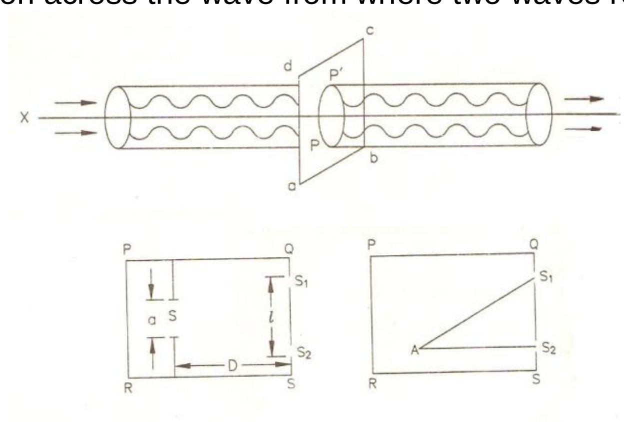
Therefore, $dv = \frac{c}{\lambda^2} d\lambda$

Therefore $dv = \frac{c}{l_c}$

Comparing the above two equations, $d\lambda = \frac{\lambda^2}{l_c}$

Here, $d\lambda$ is called the natural line width. Hence, temporal coherence depends upon the value of coherent length and coherent time.

Spatial coherence – Light is said to possess spatial coherence if the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of propagation of the beam is time-independent. This type of coherence is also called as transverse or lateral coherence. It is measure of minimum separation across the wave from where two waves remain coherent.



The beam is said to possess spatial coherence if the phase difference of the waves crossing P and P' at any instant remains constant

Spatial coherence describes the ability for two points in space, in the extent of a wave to interfere, when averaged over time. More precisely, the spatial coherence is the cross correlation between two points in a wave for all times. If a wave has only one value of amplitude over an infinite length, it is perfectly spatially coherent.

In the figure S is a source and S_1 and S_2 are two slits of separation l. If interference occurs for $l_{\max} = l_\omega$ then this dimension is called the spatial coherence length of the source.

The condition for coherence of S_1 and S_2 is

$$\frac{\lambda}{a} > \frac{l}{D} \quad \text{or} \quad l < \frac{\lambda}{a/D}$$

Since, $\frac{a}{D} = \theta$, angle subtended by width of slit S. Thus, we get

$$l_\omega = \frac{\lambda}{\theta}$$

Q factor for light – The monochromaticity can be expressed in terms of spectral purity factor Q defined as

$$Q = \frac{\lambda}{d\lambda} = \frac{\lambda}{\Delta\lambda} = \frac{v}{\Delta v}$$

For absolutely monochromatic light $\Delta v = 0$ and the purity factor is infinite which is practically impossible. For partially coherent light, coherence length l_c is given by

$$l_c = \frac{\lambda}{\Delta\lambda}\lambda = Q\lambda$$

Visibility as a measure of coherence:

Absolute coherence is impossible and therefore, one should talk of partial coherence or rather degree of coherence.

We know that interference takes place when waves are coherent. Consider two wave trains of light, each of length l_c, overlapping to their full extent. If this is so, interference pattern will have distinct maxima and minima of the highest degree of contrast. But when wave trains overlap only partially, there will be interference

but degree of contrast is less depending on amount of overlap. If the wave trains do not overlap, there will be no interference. Thus, contrast or visibility is a measure of degree of coherence.

Visibility, V is defined as the ratio of difference between maximum intensity $I_{\max}$ and minimum intensity $I_{\min}$ to the sum of these intensities.

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Generally, contrast or visibility of 0.8 is considered high, and at 0.2 fringes are barely visible, that is, visibility must be related to degree of coherence.

Consider two interfering waves each of intensity I_o. Each of them consists of coherent part say I_c and incoherent part I_{inc}. If C be the degree of coherence, then

$I_c = CI_o$ and $I_{inc} = (1-C)I_o$

When superimposed, the coherent part shall interfere adding there by their amplitudes whereas for the incoherent parts the intensities are simply added. Thus,

$$I_{\max} = 4CI_o + 2(1-C)I_o$$
$$I_{\min} = 0 + 2(1-C)I_o$$

Therefore visibility,

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{4CI_o + 2(1-C)I_o - 2(1-C)I_o}{4CI_o + 2(1-C)I_o + 2(1-C)I_o}$$

Or $V = \dfrac{4CI_o}{4I_o} = C$

Thus, degree of contrast is a measure of degree of coherence between waves of equal intensities.

If the intensities of waves are unequal, the visibility is given by

$$V = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2} C$$

Visibility in interference measures degree of coherence but the two, interference and coherence, should not be mistaken for one.
Thus, coherence is the property of light and interference is the process of interaction whose result depends on the above property.

Source : http://msk1986.files.wordpress.com/2011/04/sem-ii-coherence-laser.pdf