CHUA'S CIRCUIT

Definition

Chua's circuit is a simple electronic circuit that exhibits classic chaos theory behavior.

Basics

Chua's circuit is a simple electronic circuit that exhibits classic chaos theory behavior. It was introduced in 1983 by Leon O. Chua, who was a visitor of Waseda University (Japan) at that time (Matsumoto 1984). The ease of construction of the circuit has made it a ubiquitous real-world example of a chaotic system, leading
some to declare it 'a paradigm for chaos' (Madan 1993).

An autonomous circuit made from standard components (resistors, capacitors, inductors) must satisfy three criteria before it can display chaotic behaviour. It must contain:

1. one or more nonlinear elements
2. one or more locally active resistors
3. three or more energy-storage elements.

Chua's circuit is the simplest electronic circuit meeting these criteria. As shown in the figure, the energy storage elements are two capacitors (labeled C1 and C2) and an inductance (labeled L1). There is an active resistor (labeled R), and a nonlinear resistor, shown in the half-right part of the figure, that is made of the ensemble of five linear resistors, two diodes and an operational amplifier in the present example.

By means of the application of the laws of electromagnetism, the dynamics of Chua's circuit can be accurately modeled by means of a system of three nonlinear ordinary differential equations in the variables x(t), y(t) and z(t), which give the voltages in the capacitors C1 and C2, and the intensity of the electrical current in
the inductance L1, respectively. These equations read:

\[
\frac{dx}{dt} = \alpha[y - x - f(x)]
\]

\[
\frac{dy}{dt} = x - y + z
\]

\[
\frac{dz}{dt} = -\beta y
\]

The function f(x) describes the electrical response of the nonlinear resistor, and its shape depends on the particular configuration of its components. The parameters \(\alpha\) and \(\beta\) are determined by the particular values of the circuit components.

A chaotic attractor, known as "The Double Scroll" because of its shape in the \((x,y,z)\) space, was first observed in a circuit containing a nonlinear element such that f(x) was a 3-segment piecewise-linear function (Matsumoto 1985).

The easy experimental implementation of the circuit, combined with the existence of a simple and accurate theoretical model, makes Chua's circuit a useful system to study many fundamental and applied issues of chaos theory. Because of this, it has been object of much study, and appears widely referenced in the literature.