Carrier Phase Measurement in Ultra-tight Integration Tracking Loops Based on EKF

Gaoshun Song, Changming Wang, Fanghua Xi, Aijun Zhang

Abstract—There is a problem that the accuracy and stability of the carrier phase measurements cannot meet the requirement of the short baseline attitude determination, ultra-tight integration tracking loop for the carrier phase measurement is raised. In this paper, the mathematical model for carrier phase measurements is derived. The factors affecting the accuracy and stability of carrier phase measurements are analyzed. The EKF is proposed as the filter of the ultra-tightly integration carrier tracking loop and to obtain the carrier phase measurement. The filter model is designed. These experiment results show that, the EKF-based ultra-tight integration tracking loop can inhibit the noise of the carrier phase tracking, improve the tracking loop stability, and enhance the accuracy and reliability of carrier phase measurement. The carrier phase measurement noises are inhibited about 30%.

Keywords—carrier phases measurement, carrier tracking loop, EKF, ultra-tight integration.

I. INTRODUCTION

Carrier phase measurement is a measurement of the phase difference between the satellite carrier signals with Doppler shifted received by the receiver and the reference carrier signals generated by the receiver [1]. As we all know, the carrier wavelength is much shorter than code width. C/A code element width is 293m, while the L1 carrier wavelength is 19.03cm, and L2 carrier wavelength of 24.42cm. With the same resolution (such as 1%), the observation error of L1 carrier is about 2.0mm, the observation error of L2 carrier is about 2.5mm, but the observation error of C/A code is nearly 2.9m. So the carrier phase measurement is the most precise method for observation currently. Carrier phase measurement normally used in static high-precision single-point positioning, RTK positioning, GPS orientation, attitude determination and so on.

Carrier phase observation has high requirement of the accuracy and stability for the GPS carrier tracking loops, it also has high requirement of the work environment for the receiver, such as signal strength, dynamic, oscillator stability, and these factors will all affect the accuracy of carrier phase measurement. Therefore, the carrier phase measurement of conventional receiver is limited by many factors. Moreover, the carrier phase measurement contains an unknown number of integer circles. Ambiguity resolution also limits the carrier phase measurement.

Ambiguity resolution will not be discussed in this paper as it is not the main contest. In recent years, a number of scholars began to study ultra-tight coupling of the Inertial Navigation System (INS) and Global Navigation Satellite Systems (GNSS) [2]-[6]. In order to improve the performance of carrier tracking loops, INS is used to assist the GNSS carrier tracking. INS-assisted-based carrier tracking loop not only eliminate the offset generated by the dynamic, but also reduce the loops bandwidth to enhance the accuracy and stability of the tracking loops. However, most scholars focus on the improvement of measurement performance for code phase and carrier frequency that based on the ultra-tight coupling, and then, they use the pseudorange and pseudorange rate for the field of navigation and positioning. Few of them were paid attention on the advantages of ultra-tight in carrier phase measurement, and use it in directional area [7].

In current market, carrier phase measurement accuracy of the receiver can reach 0.01 circles [8]. It meets the requirement of most high-precision positioning. However, the attitude determination needs high-precision angle information with short baseline, a rough estimate of 0.01 circles will lead to 0.1° angle errors in 2m baseline. Taking other factors, such as difference correlation [9], into consideration, the errors will increase. This paper mainly studies the influence on carrier phase measurements with the ultra-tight integration carrier tracking loops performance improved.

II. PHASE DIFFERENCE CORRELATION

There are many unknown elements in carrier phase observation equation while using carrier phase measurement for position calculation and attitude determination. And some of them are necessary, such as station coordinates, the others are unnecessary, such as receiver clock sent and satellite clock sent. The unnecessary elements are much more than necessary numbers. If calculating blindly, workload will be increased. In order to leave the unnecessary numbers from the phase measurement and decrease the numbers of equations, phase difference is used. Combining different receivers, different satellites and original phase measurements of different epochs appropriately, single differential, double differential, three differential equations were adopt. From the point of computing
workload, differential is a wise method. But correlation between combined measurements will produce by differential, and will increase the measurement errors.

III. CARRIER TRACKING LOOP DESIGN

A. Conventional Carrier Tracking Loop

Carrier tracking loop of GPS receiver usually consists four components: carrier pre-detection integrator, carrier loop phase detector, carrier loop filter and numerically controlled oscillator, and it is shown in Fig. 1. The integration time of pre-detection integrator can be set, and the time determines the tracking loop dynamic performance and noise suppression performance of the loop [10]. Carrier loop phase detector implements the errors between the estimation and received value of the receiver. Different phase detector will make up different carrier tracking loop, such as phase-locked loop (PLL), frequency-locked loop (FLL). Carrier phase loop filter filtering the output of the detector, carrier loop filter is usually the low-pass filter of first order or second-order, different order of the filters determine the performance of tracking dynamic signals. NCO adjusts its output frequency according to the output of the filter, in order to adjust the estimation of signal, and correct the estimated frequency to track the received signal.

B. Ultra-tight Coupling Carrier Tracking Loop

Ultra-tight integration brings the measurement of the INS into the tracking loop of GNSS, and re-schedules the structure of the tracking loop, in order to assist the tracking loop and improve the performance of tracking loop. Ultra-tight integration system works in the conventional tracking loop first, when the receiver obtains a stable position and velocity information and completed the initial state of INS, the receiver switches to ultra-tight integration mode. In the ultra-tight integration loop, code phase tracking error is usually provided by the main filter of navigation, but the navigation accuracy cannot meet the requirements of the carrier tracking accuracy, so the carrier tracking is completed by the inner loop of the receiver. Therefore, the ultra-tightly Integration systems generally work in the federal filter mode, the main filter is used to generate navigation error and measurement error of the INS system, the sub-filter is mainly used in carrier tracking loop, and improving tracking accuracy and reliability [12]. Ultra-tight integration carrier tracking loop is shown in Fig. 2. Compared with the conventional carrier tracking loop, non-linear Kalman filter take place of carrier loop phase detector and carrier loop filter, and auxiliary compensation is added to the loop for acceleration compensation between satellite and receiver baseline. Non-linear Kalman filter can estimate the output of pre-detection integrator better, and enhance the measurement accuracy; the INS will greatly increase the tracking performance for dynamic signals, and enhance system stability.

IV. CARRIER PHASE MEASUREMENT PRINCIPLE

Carrier phase observations are the phase of IF signals after mixing in actual measurement. The GPS satellite phase signals that GPS receiver received are a modulated signal, because the GPS satellites have modulated the ranging code signals and data signals (navigation messages) to the carrier when sending the carrier signals. Thus the phase of the received carrier is no longer continuous, so before carrier phase measuring, demodulation should be done first. Ranging code and navigation messages should be remove from the carrier signal, and then extract the carrier. Phase can be measured after the receiver getting pure carrier. In other words, the carrier phase measurement is based on the carrier tracking loop is locked.

A. Carrier Phase Measurements

Once the receiver locked a satellite signals, the carrier tracking loop will track the satellite signals continuously, and acquire the carrier Doppler deviation, carrier phase offset and other information. Measurement of carrier phase rate during an epoch is to integral the Doppler frequency shifted \( f_d \) of this epoch. The frequency \( f_d \) is the time rate of carrier phase, so integral one epoch will get carrier phase rate during this epoch. Carrier phase measurements take cycle as a unit, which mean that the carrier phase changes \( 2\pi \) radians or a wavelength. After each epoch, the receiver will measure the decimal part. The value is exported by the carrier tracking loop of the receiver. The relationship of carrier phase is given by [8]:

![Fig. 1 Conventional carrier tracking loop architecture](image1)

![Fig. 2 Ultra-tight integration carrier tracking loop architecture](image2)
\[ \Phi_n = \Phi_{n-1} + \int_{T_{n-1}}^{T_n} f_D(\tau) d\tau + \delta \phi_n. \]  

(1)

In (1), \( \Phi \) is the accumulated phase in the epoch, \( \Phi_0 = \delta \theta_0 \). Which contains the ambiguity \( N \); \( f_D \) is time-varying Doppler frequency shifted; \( \delta \theta_0 \) is the decimal part of measured phase in the epoch, that is, the carrier phase deviation measured by carrier phase tracking loop.

In a 1ms epoch, \( f_D \) is seen as constant in the epoch period. Equation (1) can be rewritten as:

\[ \Phi_n = \Phi_{n-1} + f_D \cdot \Delta T + \delta \phi_n. \]  

(2)

\( f_D \) is the Doppler frequency shifted offset in the n epoch cycle, \( f_D \) is the Doppler frequency offset at the beginning of the carrier phase measurement, it is a given value for carrier phase measurement. So, the carrier phase measurements are related to the carrier frequency offset and carrier phase offset from the carrier tracking loop. And the accuracy and stability of \( \delta f \) and \( \delta \phi \) will have a direct impact on the accuracy and stability of carrier phase measurements \( \Phi \).

B. Measurement Error and Stability

The GPS signal received contains original carrier phase, Doppler frequency shifted generated by the motion and a variety of errors, such as ionosphere noises, atmospheric noises, satellite oscillator noises, multi-path noises and RF interference, these errors will lead to phase deviations, and errors when the carrier phase is used to navigation positioning. But these deviations are all generated by the receiver, and it has nothing to do with the carrier phase itself. Carrier phase measurement accuracy is the inconsistency extent of carrier phase measurement and the phase of the received signal. It is mainly influenced by the performance of the tracking loop within the receiver, such as thermal noises, oscillator noises and measurement noises, what’s more, for the hardware receiver, the resolution of itself is also have impact on this problem. Thus \( \delta f \) and \( \delta \phi \) acquired from the carrier tracking loop are not only influenced by the Doppler shift generated by the relative motion of the satellite and receiver, but also influenced by the measurement error due to the thermal noises, oscillator noises and other factors. Measurement error of carrier phase within an epoch can be expressed as:

\[ \sigma_\phi = \sqrt{\sigma_{PLL}^2 + \sigma_r^2 + \theta_A^2 + \epsilon_\phi}. \]  

(4)

Where, \( \sigma_\phi \) is the carrier phase measurement errors, \( \sigma_{PLL} \) is \( 1 \sigma \) thermal noises; \( \sigma_r \) is the vibration of the oscillator generated by \( 1 \sigma \) vibration, \( \theta_A \) is the vibration of the oscillator generated by the Allen variances, \( \epsilon_\phi \), is measurement noises.

Meanwhile, the carrier phase tracking loop should be locked when measuring the carrier phase, so the GPS tracking loop measurement errors must be less than a certain threshold. Otherwise it will lead to losing lock, and have a direct impact on the stability of the carrier phase measurement. Typically, the relationship between the tracking error and the threshold of GPS tracking loop is given by [8]:

\[ 3\sigma_{PLL} = 3\sqrt{\sigma_r^2 + \theta_A^2 + \epsilon_\phi} \leq \text{PLL}_{\text{threshold}}. \]  

(5)

Where, \( \sigma_{PLL} \), \( \sigma_r \), \( \theta_A \), has the same as in (4); \( \sigma_{PLL} \), is the carrier loop tracking error, \( \epsilon_\phi \), is the dynamic stress error; \( \text{PLL}_{\text{threshold}} \), is the tracking loop error threshold.

Thus, for conventional carrier tracking loop, these errors will affect the stability of tracking loop, even will lead to losing lock, and directly affect the stability of the carrier phase measurement.

V. CARRIER PHASE MEASUREMENT BASED ON EKF

A. State Equation

In the ultra-tight integration carrier tracking loop, the output of pre-detection integrator is sent to the Kalman filter, the filter estimates the tracking carrier phase errors \( \delta \phi \) and frequency errors \( \delta f \) to drive the NCO and generate the carrier phase rate in the epoch. So the selection of the Kalman filter has an important impact on the performance of the ultra-tight integration receiver. Based on these studies [13]-[18], this paper chooses Extended Kalman Filter (EKF) as the loop filter. The state vector of channel filter can be expressed as:

\[ X = [A \ \Phi \ f_D \ \delta \phi \ \delta f \ \delta a]^T. \]  

(6)

Where, \( A \) is the signal amplitude, \( \Phi \) is the measurement of carrier phase, \( f_D \) is the Doppler shifted, \( \delta \phi \) is the phase deviation, \( \delta f \) is the frequency deviation, \( \delta a \) is the auxiliary bias of acceleration on the baseline of satellite and receiver.

The state equation of channel filter can be expressed as:

\[ X_{k+1} = B_{k+1/k} X_k + \Gamma_{k+1/k} W_k. \]  

(7)

In (7), State transition matrix \( B \) is:

\[ B = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & \Delta T & 1 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & \Delta T \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}. \]  

(8)

Noise-driven matrix \( \Gamma \) is:

\[ \Gamma = \begin{bmatrix}
\Delta T & 0 & 0 & 0 & 0 \\
0 & \Delta T & 0 & 0 & 0 \\
0 & 0 & \Delta T & 0 & 0 \\
0 & 0 & 0 & f \Delta T & 0 \\
0 & 0 & 0 & 0 & f \Delta T/\lambda \\
\end{bmatrix}. \]  

(9)

State noise matrix \( W \) is:
\[
W = \begin{bmatrix}
w_A & w_\phi & w_{\tilde{f}_0} & w_{\tilde{a}} & w_{\delta a} & w_{\delta \phi}
\end{bmatrix}^T.
\] (10)

In (8), (9), (10), \( \Delta T \) is the pre-integration time; \( f \) is the carrier frequency; \( \lambda \) is the carrier wavelength of GPS signal; \( w_A \) is drive noise of signal amplitude; \( w_\phi \) is drive noise of carrier phase measurement; \( w_{\tilde{f}_0} \) is drive noise of Doppler frequency shifted; \( w_{\delta a} \) is the drive noise of clock offset; \( w_{\delta \phi} \) is the drive noise of clock drift; \( w_{\delta a} \) is the drive noise of the acceleration on the baseline of satellite and receiver.

**B. Observation equation**

Observations of the filter are the outputs of tracking loop pre-detection integrator:

\[
Z = \begin{bmatrix} I \\ Q \end{bmatrix}.
\] (11)

Where \( I \) and \( Q \) are as follows:

\[
I = A \cdot N \cdot R(\delta \tau) \sin c(\pi \cdot \delta f \cdot \Delta T) \cos(\overline{\delta \phi}) ,
\] (12)

\[
Q = A \cdot N \cdot R(\delta \tau) \sin c(\pi \cdot \delta f \cdot \Delta T) \sin(\overline{\delta \phi}).
\] (13)

Where \( A, \delta f, \Delta T \) are all the contents talked above; \( N \) is the amplitude of the navigation data, \( R \) is the C/A code autocorrelation function, \( \delta \tau \) is the code phase error. When tracking loop work in the ultra-tight coupling mode, \( N \cdot R(\delta \tau) \) only affects observations. The sign bit of \( I \) and \( Q \) is known, so it need not to consider them. \( \overline{\delta \phi} \) is the accumulated mean of carrier phase deviation, and it can be expressed as:

\[
\overline{\delta \phi} = \delta \phi - \frac{1}{2} \delta f \cdot \Delta T + \frac{1}{6} \delta \tilde{a} \cdot \Delta T^2.
\] (14)

Where \( \delta \phi, \delta f, \delta \tilde{a} \) are the estimated carrier phase, frequency and deviation of frequency rate, they are same with state quantity in state vector.

System uses ultra-tight coupling mode with the updated epoch of 1ms, \( sinc(\pi \cdot \delta f \cdot \Delta T) \rightarrow 1 \), do differential with \( I \), the results are as follows:

\[
\frac{\partial I}{\partial A} = \cos(\overline{\delta \phi})
\]
\[
\frac{\partial I}{\partial \delta \phi} = -\dot{\lambda} \sin(\overline{\delta \phi})
\]
\[
\frac{\partial I}{\partial \delta f} = -\frac{1}{2} \Delta T \dot{\lambda} \sin(\overline{\delta \phi})
\]
\[
\frac{\partial I}{\partial \delta \tilde{a}} = -\frac{1}{6} \Delta T^2 \dot{\lambda} \sin(\overline{\delta \phi})
\] (15)

By the same way:

\[
\frac{\partial Q}{\partial A} = \sin(\overline{\delta \phi})
\]
\[
\frac{\partial Q}{\partial \delta \phi} = \dot{\lambda} \cos(\overline{\delta \phi})
\]
\[
\frac{\partial Q}{\partial \delta f} = \frac{1}{2} \dot{T} \dot{\lambda} \cos(\overline{\delta \phi})
\]
\[
\frac{\partial Q}{\partial \delta \tilde{a}} = -\frac{1}{6} \Delta T^2 \dot{\lambda} \cos(\overline{\delta \phi})
\] (16)

Observation equation is:

\[
Z_k = H_k X_k + V_k.
\] (17)

Where, \( H \) is the observation matrix, \( V_k \) is the measurement noises,

\[
H = \begin{bmatrix}
\frac{\partial I}{\partial A} & 0 & 0 & \frac{\partial I}{\partial \delta \phi} & \frac{\partial I}{\partial \delta f} & \frac{\partial I}{\partial \delta \tilde{a}} \\
0 & 0 & 0 & \frac{\partial Q}{\partial \delta \phi} & \frac{\partial Q}{\partial \delta f} & \frac{\partial Q}{\partial \delta \tilde{a}}
\end{bmatrix}
\] (18)

\[
V_k = \begin{bmatrix} v_I \\ v_Q \end{bmatrix}.
\] (19)

Where, \( v_I, v_Q \) is the measurement noise of pre-detection integrator, and

\[
\sigma_{v_I}^2 = \sigma_{v_Q}^2 = \frac{1}{2 \cdot 10^{9 \cdot C/N_0} \cdot \Delta T}.
\] (20)

Where, \( C/N_0 \) is the carrier to noise ratio of measured signal.

**VI. SEMI-PHYSICAL SIMULATION**

In order to verify the carrier phase measurement performance of the ultra-tight Integration loop, semi-physical simulation is used, and static test is used to verify the analysis. IMU system uses simulation results as input, as here is only related to the acceleration signal, take 100ng as the acceleration auxiliary deviation and 10μg as white noise variance in order to simplify the conversion process. Use IF collector to acquire GPS signal [19], the IF frequency is 9.55MHz, sampling frequency is 38.192MHz, pre-detection integration time is 1ms. For performance comparison, take the two methods, routine processing and ultra-tight coupling processing, to deal with the IF data. And then compare the results to draw the conclusions discussed above. Preferences in normal mode are: second-order PLL, the bandwidth of carrier loop noise is 10Hz, damping factor is 0.707, and carrier loop gain is 1. In ultra-tight coupling mode, the carrier tracking loop using EKF filter, and the state noise is Gaussian white noise.

After processing the IF data, can get the satellite signal acquisitions, and they are shown in Table I.

| TABLE I
SATELLITE ACQUISITIONS |
<table>
<thead>
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<th></th>
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<tbody>
<tr>
<td>Satellite number</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>
The lowest C/N$_0$ NO.3 and highest C/N$_0$ NO.21 are chosen from the available satellite to processing, and the results shown in Fig. 3-5.

<table>
<thead>
<tr>
<th>Satellite Number</th>
<th>C/N$_0$ 1st</th>
<th>C/N$_0$ 2nd</th>
</tr>
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<tbody>
<tr>
<td>6</td>
<td>-3688</td>
<td>39.8</td>
</tr>
<tr>
<td>9</td>
<td>2832</td>
<td>38.4</td>
</tr>
<tr>
<td>15</td>
<td>1921</td>
<td>47.8</td>
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<td>18</td>
<td>246</td>
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<td>-574</td>
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<td>22</td>
<td>1694</td>
<td>48.5</td>
</tr>
<tr>
<td>26</td>
<td>-2987</td>
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</tr>
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</table>

Fig. 3 Pre-detection integrator output of NO.3 satellite

(a) Fig. 3 Pre-detection integrator output of NO.3 satellite

(b) Fig. 4 Pre-detection integrator output of NO.21 satellite

(a) Fig. 4 Pre-detection integrator output of NO.21 satellite

(b) Fig. 5 Tracking phase tracking error of 2 order PLL

(a) Fig. 5 Tracking phase tracking error of 2 order PLL

(b) Fig. 6 Tracking phase tracking error of EKF filter loop

(a) Fig. 6 Tracking phase tracking error of EKF filter loop

(b) Tracking phase tracking noise rates before and after filtering are shown in Table II:

<table>
<thead>
<tr>
<th>Satellite Number</th>
<th>2 order PLL (cycle)</th>
<th>EKF loop (cycle)</th>
<th>Improved proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.027</td>
<td>0.018</td>
<td>31%</td>
</tr>
<tr>
<td>21</td>
<td>0.011</td>
<td>0.008</td>
<td>29%</td>
</tr>
</tbody>
</table>

The error due to thermal noise of the carrier phase tracking loop is related to the C/N$_0$ of the received signal. It can be found that the tracking noises of NO.3 satellite are larger than NO.21 satellite. By using the EKF tracking loop in ultra-tight coupling
loops, signal phase tracking noise is decreased significantly, and phase tracking accuracy is improved. Meanwhile, for the NO.3 satellite, it means that the C/N0 of NO.3 satellite is improved. Ultra-tight coupling loop can enhance weak signal tracking capability for receiver, and increase the tracking loop stability.

The measurements of carrier phase tracking errors are well inhibited, and the measurement noises of carrier phase observations are also inhibited, so the measurement accuracy is improved. EKF-based ultra-tight integration loop enhances the measurement accuracy of the carrier phase observations by about 30%. Carrier phase measurements generated in the ultra-tight integration tracking loop are shown in Fig. 7.

![Fig. 7 Carrier phase measurements of ultra-tight integration loop](image)

VII. CONCLUSION

In this paper, the errors of carrier phase measurement are analyzed, the method that ultra-tight integration tracking loop to inhibit noise in carrier phase measurement is raised, and the loop filter of the carrier tracking loop is designed. Through the simulation, in the ultra-tight integration tracking loop, the carrier phase measurement noises are inhibited about 30%, and at the same time, the low C/N0 signal tracking capability is improved, carrier tracking loop stability is enhanced. The improved accuracy and stability of the carrier phase measurement has an important meaning for high-precision positioning and attitude determination. Test and analysis of the tracking loop in dynamic circumstance and interference circumstance is the emphasis in future study.

REFERENCES
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