Blackbody Radiation and the Planck Function

A *blackbody* is an object which absorbs all the light which hits it: hence the name "blackbody". It also emits radiation, in a very particular manner.

Total energy emitted per second, at all wavelengths

The **total** amount of energy radiated per second by a perfect blackbody depends only on its temperature **T** and area **A**:

$$\text{energy per second} = \text{sigma} * T^4 * A$$

where

$$\text{sigma is the Stefan-Boltzmann constant} = 5.67 \times 10^{-8} \frac{\text{Joules}}{s*m^2*K^4}$$

T is the temperature of the object, in Kelvin

A is the surface area of the object, in square meters

This relationship is called the Stefan-Boltzmann Law.

If we consider a patch of area exactly one square meter, then the energy radiated from it per second is

$$\text{energy per second per square meter} = \text{sigma} * T^4$$

Energy emitted per second, as a function of wavelength

A blackbody doesn't emit equal amounts of radiation at all wavelengths; instead, most of the energy is radiated within a relatively narrow band of wavelengths. The location of that band varies with the body's temperature; for example,

object	T (Kelvin)	radiates mostly
very cold gas in insterstellar space	20	radio
a live human being	310	infrared
the Sun	5,600	visible
interior of nuclear explosion	3,000,000	X-rays

The exact amount of energy emitted at a particular wavelength **lambda** is given by the Planck function:

```
               (2*h*c^2) / lambda^5
  B   (T)  =   ----------------------
   lambda            h*nu/k*T
                    e         -  1
```

Or, in beautiful typeset format,

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

In this equation,

$B_{\lambda}(T)$ is the energy (Joules) emitted per second per unit wavelength per steradian from one square meter of a perfect blackbody at temperature T

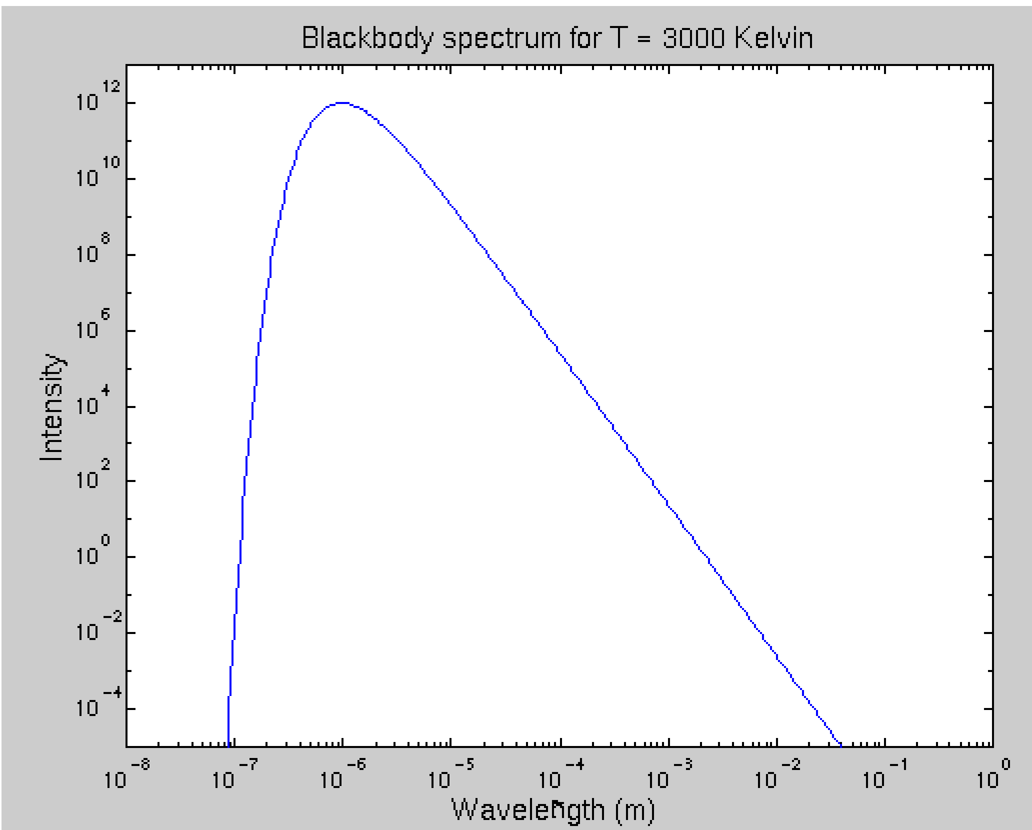
T is the temperature of the blackbody

h is Planck's constant = 6.63 x 10^(-34) J*s c is the speed of light = 3.00 x 10^(8) m/s

lambda is the wavelength

k is Boltzmann's constant = 1.38 x 10^(-23) J/K

The Planck function has a distinctive shape: it rises very sharply at short wavelengths (due to the exponential), reaches a peak at some wavelength, then falls gradually at longer wavelengths.



Integrating the Planck Function

In order to find the total energy emitted per second in some wavelength range

lambda 1 < lambda < lambda 2

from one square meter of a perfect blackbody, one can integrate the Planck function

lambda 1

```
                      /          (2*h*c^2) / lambda^5
  energy emitted  =      |         ----------------------   d(lambda)
    per second           /             h*nu/k*T
 per square meter     lambda 2       e          -  1
   per steradian
```

and then multiply by (pi) steradians (we are interested in the energy emitted at all angles from one face of a flat plate, so we integrate over solid angle to cover half the sky; that ends up being the same as multiplying by a factor of (pi) = 6.28...)

Or, in nicely typeset form,

$$\mathrm{W/m^2} = (\pi) \int_{\lambda_2}^{\lambda_1} \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda$$

Obviously, if one integrates from the shortest possible wavelength (lambda = 0) to the longest possible wavelength (lambda = infinity), and multiplies by (pi), one ought to end up with the same total energy emitted per second as given by the Stefan-Boltzmann Law,

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                           4
   energy per second  =  sigma * T
   per square meter
```

Source: http://spiff.rit.edu/classes/phys317/lectures/planck.html