Blackbody Radiation and the Planck Function

A blackbody is an object which absorbs all the light which hits it: hence the name "blackbody". It also emits radiation, in a very particular manner.

Total energy emitted per second, at all wavelengths

The total amount of energy radiated per second by a perfect blackbody depends only on its temperature $T$ and area $A$:

$$\text{energy per second} = \sigma \times T \times A$$

where

- $\sigma$ is the Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ Joules $\text{s} \cdot \text{m}^2 \cdot \text{K}^4$
- $T$ is the temperature of the object, in Kelvin
- $A$ is the surface area of the object, in square meters

This relationship is called the Stefan–Boltzmann Law.

If we consider a patch of area exactly one square meter, then the energy radiated from it per second is

$$\text{energy per second} = \sigma \times T$$

per square meter

Energy emitted per second, as a function of wavelength
A blackbody doesn’t emit equal amounts of radiation at all wavelengths; instead, most of the energy is radiated within a relatively narrow band of wavelengths. The location of that band varies with the body’s temperature; for example,

<table>
<thead>
<tr>
<th>object</th>
<th>T (Kelvin)</th>
<th>radiates mostly</th>
</tr>
</thead>
<tbody>
<tr>
<td>very cold gas in interstellar space</td>
<td>20</td>
<td>radio</td>
</tr>
<tr>
<td>a live human being</td>
<td>310</td>
<td>infrared</td>
</tr>
<tr>
<td>the Sun</td>
<td>5,600</td>
<td>visible</td>
</tr>
<tr>
<td>interior of nuclear explosion</td>
<td>3,000,000</td>
<td>X-rays</td>
</tr>
</tbody>
</table>

The exact amount of energy emitted at a particular wavelength \( \lambda \) is given by the Planck function:

\[
B(\lambda, T) = \frac{(2\pi c^2)}{\lambda^5} \frac{\nu^3}{e^{\nu/kT} - 1}
\]

Or, in beautiful typeset format,

\[
B_{\lambda}(T) = \frac{2\pi c^2/\lambda^5}{e^{h\nu/kT} - 1}
\]

In this equation,

\( B(\lambda, T) \) is the energy (Joules) emitted per second per unit wavelength per steradian from one square meter of a perfect blackbody at temperature \( T \)

\( T \) is the temperature of the blackbody

\( h \) is Planck’s constant \( = 6.63 \times 10^{(-34)} \) J*s
c is the speed of light \( = 3.00 \times 10^8 \) m/s

\( \lambda \) is the wavelength

\( k \) is Boltzmann’s constant \( = 1.38 \times 10^{-23} \) J/K

The Planck function has a distinctive shape: it rises very sharply at short wavelengths (due to the exponential), reaches a peak at some wavelength, then falls gradually at longer wavelengths.

---

**Integrating the Planck Function**

In order to find the total energy emitted per second in some wavelength range

\[ \lambda_1 < \lambda < \lambda_2 \]

from one square meter of a perfect blackbody, one can integrate the Planck function \( \lambda_1 \)
Energy emitted per second per square meter per steradian is given by:

\[
\frac{(2h^*c^2)}{\lambda^5} \cdot \frac{\lambda^2}{h^*nu/k^*T} \cdot e^{-1}
\]

and then multiply by \((\pi)\) steradians (we are interested in the energy emitted at all angles from one face of a flat plate, so we integrate over solid angle to cover half the sky; that ends up being the same as multiplying by a factor of \((\pi) = 6.28\ldots\))

Or, in nicely typeset form,

\[
W/\text{m}^2 = (\pi) \int_{\lambda_2}^{\lambda_1} \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda
\]

Obviously, if one integrates from the shortest possible wavelength \((\lambda = 0)\) to the longest possible wavelength \((\lambda = \infty)\), and multiplies by \((\pi)\), one ought to end up with the same total energy emitted per second as given by the Stefan–Boltzmann Law,

\[
\text{energy per second} = \sigma T
\]

per square meter

Source: http://spiff.rit.edu/classes/phys317/lectures/planck.html