

# Module 5

## Carrier Modulation

# Lesson 24

## Binary Phase Shift Keying (BPSK) Modulation

## After reading this lesson, you will learn about

- **Binary Phase Shift Keying (BPSK);**
- **Power Spectrum for BPSK Modulated Signal;**

## Binary Phase Shift Keying (BPSK)

BPSK is a simple but significant carrier modulation scheme. The two time-limited energy signals  $s_1(t)$  and  $s_2(t)$  are defined based on a single basis function  $\phi_1(t)$  as:

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}} \cdot \cos 2\pi f_c t \quad \text{and} \quad s_2(t) = \sqrt{\frac{2E_b}{T_b}} \cdot \cos[2\pi f_c t + \pi] = -\sqrt{\frac{2E_b}{T_b}} \cdot \cos 2\pi f_c t \quad 5.24.1$$

The basis function, evidently, is,  $\phi_1(t) = \sqrt{\frac{2}{T_b}} \cdot \cos 2\pi f_c t$ ;  $0 \leq t < T_b$ . So, BPSK may be described as a one-dimensional digital carrier modulation scheme. Note that the general form of the basis function is,  $\phi_1(t) = \sqrt{\frac{2}{T_b}} \cdot \cos(2\pi f_c t + \phi)$ , where ‘ $\Phi$ ’ indicates an arbitrary but fixed initial phase offset. For convenience, let us set  $\Phi = 0$ .

As we know, for narrowband transmission,  $f_c \gg \frac{1}{T_b}$ . That is, there will be multiple cycles of the carrier sinusoid within one bit duration ( $T_b$ ). For convenience in description, let us set,  $f_c = n \times \frac{1}{T_b}$  (though this is not a condition to be satisfied theoretically).

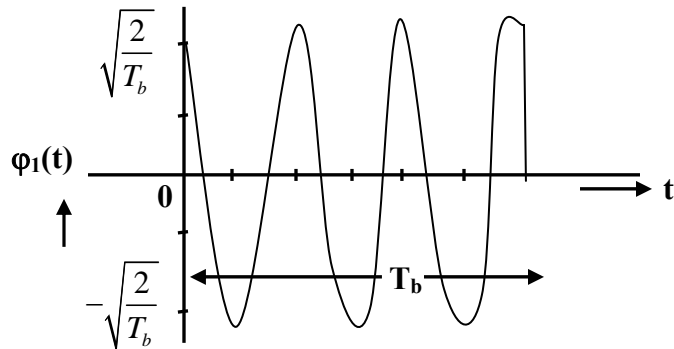
Now, we see,

$$s_1(t) = \sqrt{E_b} \cdot \phi_1(t) \quad \text{and} \quad s_2(t) = -\sqrt{E_b} \cdot \phi_1(t), \quad 5.24.2$$

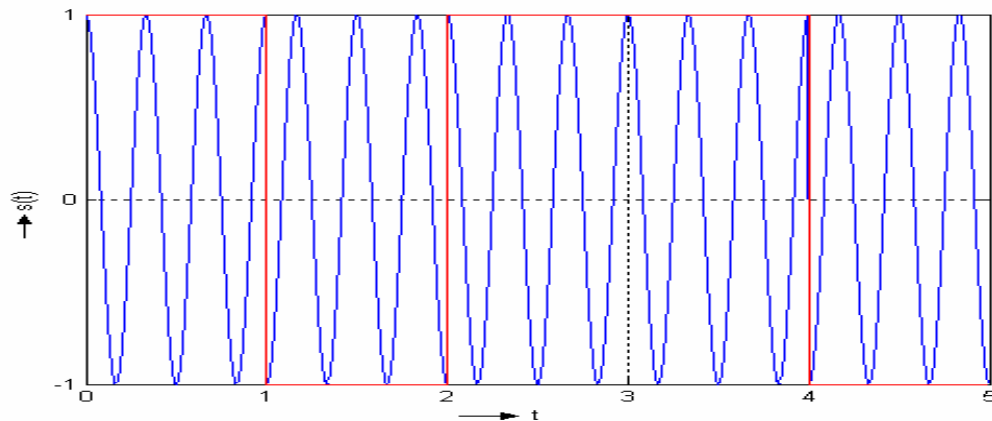
The two associated scalars are:

$$s_{11}(t) = \int_0^{T_b} s_1(t) \cdot \phi_1(t) dt = +\sqrt{E_b} \quad \text{and} \quad s_{21}(t) = \int_0^{T_b} s_2(t) \cdot \phi_2(t) dt = -\sqrt{E_b} \quad 5.24.3$$

**Fig. 5.24.1 (a)** presents a sketch of the basis function  $\phi_1(t)$  and **Fig. 5.24.1 (b)** shows the BPSK modulated waveform for a binary sequence. Note the abrupt phase transitions in the modulated waveform when there is change in the modulating sequence. On every occasion the phase has changed by  $180^\circ$ . Also note that, in the diagram, we have chosen to set  $\sqrt{\frac{2E_b}{T_b}} = 1$ , i.e.  $\frac{E_b}{T_b} = \frac{1}{2} = 0.5$ , which is the power associated with an unmodulated carrier sinusoid of unit peak amplitude.

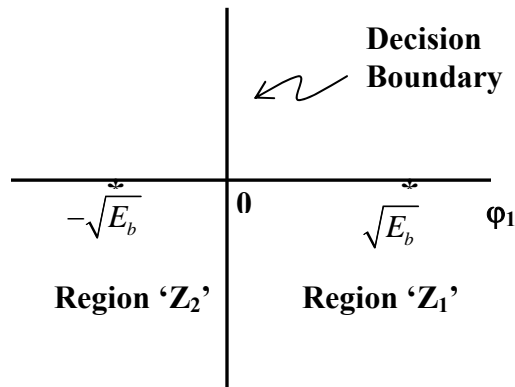


**Fig. 5.24.1: (a)** Sketch of the basis function  $\phi_1(t)$  for BPSK modulation



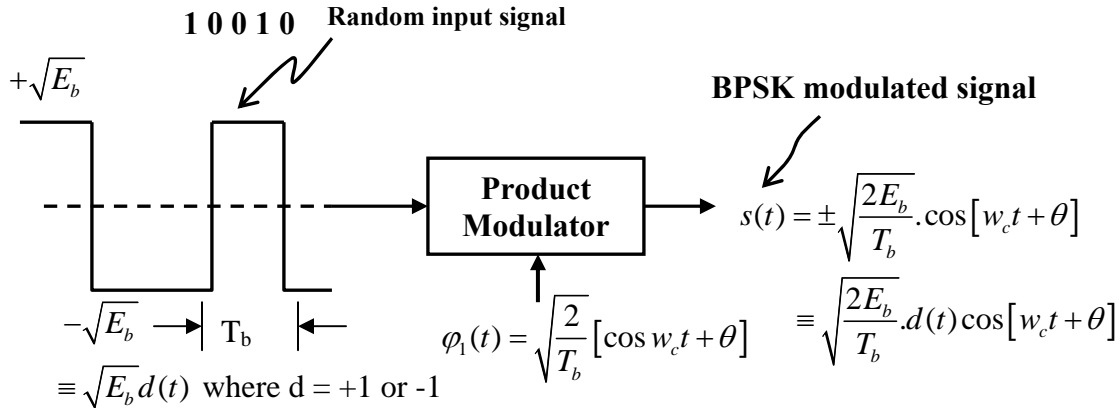
**Fig. 5.24.1: (b)** BPSK modulated waveform for the binary sequence 10110. Note that the amplitude has been normalized to  $\pm 1$ , as is a common practice.

**Fig. 5.24.1: (c)** shows the signal constellation for binary PSK modulation. The two points are equidistant from the origin, signifying that the two signals carry same energy.



**Fig. 5.24.1: (c)** Signal constellation for binary PSK modulation. The diagram also shows the optimum decision boundary followed by a correlation receiver

**Fig. 5.24.2** shows a simple scheme for generating BPSK modulated signal without pulse shaping. A commonly available balanced modulator (such as IC 1496) may be used as the product modulator to actually generate the modulated signal. The basis function  $\varphi_1(t)$ , shown as the second input to the product modulator, can be generated by an oscillator. Note that the oscillator may work independent of the data clock in general.



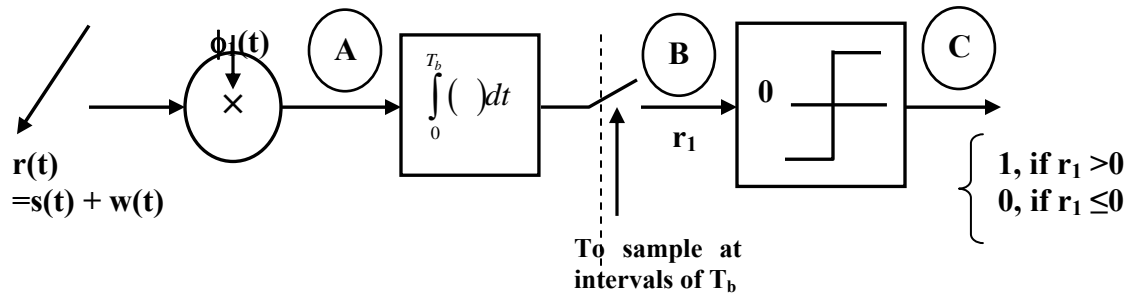
**Fig. 5.24.2** A simple scheme for generating BPSK modulated signal. No pulse-shaping filter has been used.

**Fig. 5.24.3** presents a scheme for coherent demodulation of BPSK modulated signal following the concept of optimum correlation receiver. The input signal  $r(t)$  to the demodulator is assumed to be centered at an intermediate frequency (IF). This real narrowband signal consists of the desired modulated signal  $s(t)$  and narrowband Gaussian noise  $w(t)$ . As is obvious, the correlation detector consists of the product modulator, shown as an encircled multiplier, and the integrator. The vector receiver is a simple binary decision device, such as a comparator. For simplicity, we assumed that the basis function phase reference is perfectly known at the demodulator and hence the  $\varphi_1(t)$ , shown as an input to the product demodulator, is phase-synchronized to that of the modulator. Now it is straightforward to note that the signal at (A) in **Fig. 5.24.3** is:

$$r_A(t) = [s(t) + w(t)] \cdot \sqrt{\frac{2}{T_b}} \cdot \cos(w_c t + \theta) \quad 5.24.4$$

The signal at (B) is:

$$\begin{aligned} r_1 &= \sqrt{\frac{2}{T_b}} \int_0^{T_b} \left[ d(t) \cdot \sqrt{\frac{2E_b}{T_b}} \cdot \cos(w_c t + \theta) + w(t) \right] \cos(w_c t + \theta) dt \\ &= \sqrt{E_b} \cdot d(t) + \sqrt{\frac{2}{T_b}} \int_0^{T_b} w(t) \cdot \cos(w_c t + \theta) dt \end{aligned} \quad 5.24.5$$



**Fig. 5.24.3** A scheme for coherent demodulation of BPSK modulated signal following the concept of optimum correlation receiver

Note that the first term in the above expression is the desired term while the second term represents the effect of additive noise. We have discussed about similar noise component earlier in Module #4 and we know that this term is a Gaussian distributed random variable with zero mean. Its variance is proportional to the noise power spectral density. It should be easy to follow that, if  $d(t) = +1$  and the second term in Eq. 5.24.5 (i.e. the noise sample voltage) is not less than  $-1.0$ , the threshold detector will properly decide the received signal as a logic '1'. Similarly, if  $d(t) = -1$  and the noise sample voltage is not greater than  $+1.0$ , the comparator will properly decide the received signal as a logic '0'. These observations are based on 'positive binary logic'.

## Power Spectrum for BPSK Modulated Signal

Continuing with our simplifying assumption of zero initial phase of the carrier and with no pulse shaping filtering, we can express a BPSK modulated signal as:

$$s(t) = \sqrt{\frac{E_b \cdot 2}{T_b}} \cdot d(t) \cos \omega_c t, \text{ where } d(t) = \pm 1 \quad 5.24.6$$

The baseband equivalent of  $s(t)$  is,

$$\tilde{u}(t) = u_I(t) = \sqrt{\frac{2E_b}{T_b}} \cdot d(t) = \pm g(t), \quad 5.24.7$$

$$\text{where } g(t) = \sqrt{\frac{2E_b}{T_b}} \text{ and } u_Q(t) = 0.$$

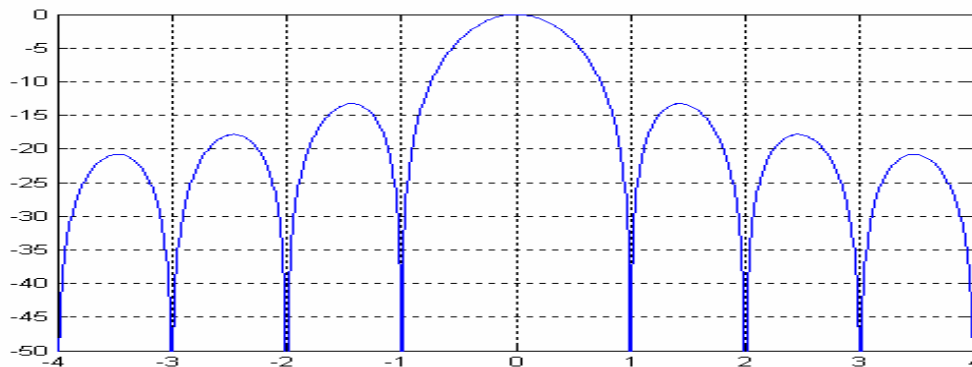
Now,  $u_I(t)$  is a random sequence of  $+\sqrt{\frac{2E_b}{T_b}}$  and  $-\sqrt{\frac{2E_b}{T_b}}$  which are equi-probable. So, the power spectrum of the base band signal is:

$$\rightarrow U_B(f) = \frac{2E_b \cdot \sin^2(\pi T_b f)}{(\pi T_b f)^2} = 2 \cdot E_b \cdot \text{sinc}^2(T_b f) \quad 5.24.8$$

Now, the power spectrum  $S(f)$  of the modulated signal can be expressed in terms of  $U_B(f)$  as:

$$S(f) = \frac{1}{4} [U_B(f - f_c) + U_B(f + f_c)] \quad 5.24.9$$

*'Fig.5.24.4 shows the normalized base band power spectrum of BPSK modulated signal. The spectrum remains the same for arbitrary non-zero initial phase of carrier oscillator.'*



**Fig.5.24.4:** Normalized base band power spectrum of BPSK modulated signal

## Problems

- Q5.24.1) Does BPSK modulated signal have constant envelope?
- Q5.24.2) Why coherent demodulation is preferred for BPSK modulation?
- Q5.24.3) Do you think the knowledge of an optimum correlation receiver is useful for understanding the demodulation of BPSK signal?
- Q5.24.4) Sketch the spectrum of the signal at the output of a BPSK modulator when the modulating sequence is 1, 1, 1, .....

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