APPLICATIONS OF RECIPROCITY THEOREM

1. Equality of Directional Patterns-

Statement: 'The directional pattern of an antenna as a receiving antenna is identical to that when used as a transmitting antenna."

Proof: The above mentioned antenna theorem is the outcome of the application of the reciprocity theorem used in the linear and bilateral networks. Basically a directional pattern of a transmitting antenna is represented as a polar characteristic because it indicates the strength (amplitude) of the radiated field at a fixed distance in several directions in the space. (As the amplitude and direction, both are involved, it is called polar characteristics). Similar to this the directional pattern of a receiving antenna is also a polar characteristic which indicates the response of the antenna for unit field strength (amplitude) from different directions. To measure the directional pattern of an antenna as a transmitting antenna, the test antenna is kept at the centre of very large sphere and the small dipole antenna is moved along the surface of this sphere as shown in the Fig. 14.

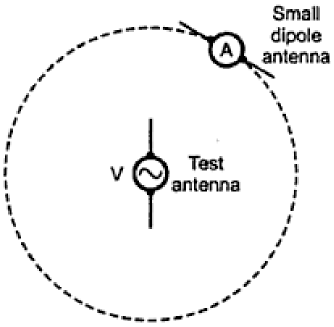

Fig 14 Directional pattern measurement for a transmitting antenna

A voltage V is connected to the test antenna placed at the centre of the imaginary sphere and the current I flowing in short dipole antenna is measured using ammeter at different positions. This current is the measure of the electric field at different positions of the dipole antenna. Now using the concept of the reciprocity theorem, the positions of the voltage excitation and the current measurement are interchanged. Now the same voltage V is applied to the terminals of the small dipole antenna which is moved along the surface of the sphere and the current I is measured in the test antenna located at the centre. Thus the receiving pattern for the test antenna can be obtained. But according to the reciprocity theorem, for every location of the dipole antenna, the ratio of V to I is same as before obtained for the test antenna as a transmitting antenna. Thus the radiation pattern i.e. directional pattern of a receiving antenna is identical to that of the transmitting antenna.

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Instead of the circular polarization, if the linear polarization is considered, then under such condition, the small dipole exploring antenna is oriented in such a way that direction is perpendicular to the radius vector and parallel to the electric vector.

2. Equivalence of Transmitting and Receiving Antenna Impedances-

Statement: "The impedance of an isolated antenna used for transmitting as well as receiving purposes is identical."

Proof: Consider two antennas, namely A_1 and A_2, are widely separated. As the antenna A_2 is located far away from A_1, the self impedance of the antenna A_1 can be written as,

$$\text{Self impedance of } A_1 = Z_{s1} = V_1 / I_1 = Z_{11} \quad \text{... (1)}$$

Note that when two antennas are separated widely, the mutual impedance Z_{12} of the antenna A_1 can be neglected if the antenna A_1 is used as a transmitting antenna. But if the same antenna A_1, is used as a receiving antenna the mutual impedance Z_{12} cannot be neglected as it is the only parameter indicating coupling between two antennas. So consider that load Z_L is connected to the antenna A_1 used as receiving antenna. Similarly the coupling between A_1 and A_2 is represented with the help of a mutual voltage Z_{12} I_2 which is due to the mutual impedance Z_{12} and current I_2 in the antenna A_2. The equivalent circuit of antenna used as a receiving antenna is as shown in the Fig. 15 (a). Since the two antennas are separated with a large distance, the variation in the load impedance Z_L connected to the antenna A_1 will not change the current I_2, in the antenna A_2. Thus the generator of value Z_{12} I_2 can be considered as an ideal generator with zero internal impedance providing constant voltage at its output terminals.

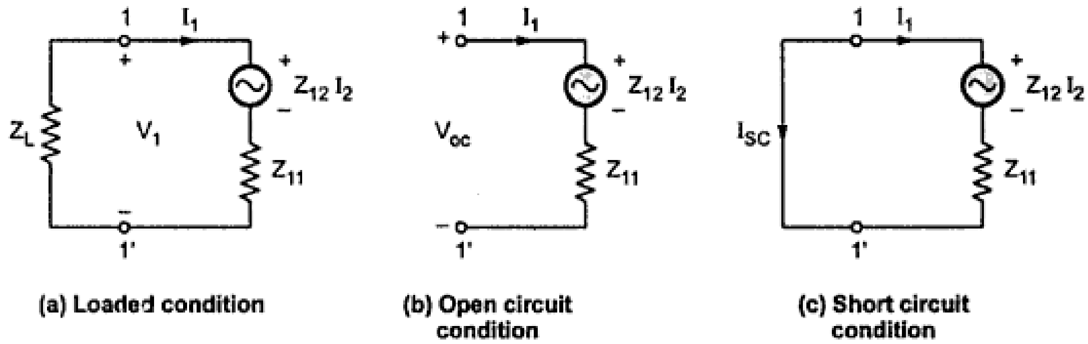

Fig. 15 Equivalent of the receiving antenna

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Under the open circuit condition, the voltage measured across terminals 1-1' is given by,

$$V_{oc} = Z_{12}\ I_2 \qquad \text{... (2)}$$

Under the short circuit condition, the short circuit current flowing from terminal 1 to 1' is given by,

$$I_{sc} = \frac{(Z_{12}\ I_2)}{Z_{11}} \qquad \text{... (3)}$$

Hence the ratio of V_{oc} & I_{sc} is called as the transfer impedance and it is given by,

$$\frac{V_{oc}}{I_{sc}} = \frac{(Z_{12}\ I_2)}{\left(\frac{Z_{12}\ I_2}{Z_{11}}\right)} = Z_{11} \qquad \text{... (4)}$$

Under all the three conditions mentioned above, the generator of value (Z_{12} 1_2) acts as a generator with internal impedance Z_{11}. Hence from equation (4) it is clear that the receiving antenna impedance is equal to the transmitting antenna impedance.

In the discussion of the impedance measurement of an antenna, we consider two antennas which are widely separated. But we can use the same concept for several antennas assuming all are widely separated from the antenna whose impedance is to be measured. Now assume that the other antennas are placed very close to the antenna being considered for impedance measurement. Under such condition, the mutual impedance cannot be ignored as it is comparable with the self impedance. Thus the receiving impedance will be the addition of the antenna self impedance and the impedance due to the presence of other antennas. But even under such condition, the transmitting impedance is equal to the receiving impedance if for the transmitting condition, other antennas are connected to the impedances equal to the impedances of the generators that used to excite them.