Analytical Results for BER Performance of MIMO-Rake Receiver in WCDMA System

† Mr. Pravindra Kumar, * Mr. Anand Kumar,

† Department of Electronics and Communication Engineering, Roorkee Engineering & Management Technology Institute, Shamli (INDIA)
*Department of Applied Science, Krishna Institute of Management & Technology, Moradabad (INDIA)

E-mail: † ait.pravs@gmail.com, * anand12617_ibs@rediffmail.com

Abstract – The use of Rake Receiver in Wideband Code Division Multiple Access (WCDMA) system provides a unique and valuable means of combating the adverse effects of short-term multipath fading in mobile radio propagation environments. The transmitted signal bandwidth in WCDMA system is much larger than the coherent bandwidth of the channel, in this case the channel is frequency selective channel. BER performance of Rake Receiver can improve by using Spatial Multiplexing (SM) in WCDMA systems. In general, the SM increases the data rate and the diversity order as well. The approximate maximum likelihood detector is derived in this paper and it shown that the diversity order is related to the product of the numbers of receives antennas and taps (the number of Resolvable Paths or the number of Rake Fingers). Spatial Multiplexing can be realized using multiple antennas at the base station and the mobile terminal i.e. Multiple Input Multiple Output (MIMO) system. In this paper BER performance of MIMO-Rake receiver is analysed and also compared with the generalized Rake Receiver system.

Index Terms – MIMO System, WCDMA, Rake Receiver, Spatial Multiplexing.

I. INTRODUCTION:-

WCDMA [1] is a broadband technology with high potential for third generation (3G) mobile systems. Due to reflection from obstacles a radio channel can consists of many copies of originally signal having different amplitude, phase and delay. If the signal components arrive more than duration of one chip apart from each other, a Rake Receiver [2], [3] can be used to resolve and combine them.

Generalized Rake Receiver – These are simple Rake Receivers used in WCDMA systems to collect energy from multipath channels. A Rake Receiver can be seen as filter matched to the spreading code, pulse shaping filter and multipath channel. It is a combination of specified number of elementary receivers called Rake fingers. Each finger is associated with one of the multipath signals. The outputs of the Rake fingers are combined to detect the transmitted symbols. Fig.1 shows the Generalized Rake Receiver.

The BER performance of generalized Rake Receiver is shown in references- [4], [5], [6]. From these references we know that if we increase the number of fingers, BER performance of Rake Receiver is improved. The number of available fingers depends on the channel profile and chip rate, the more the resolvable paths (taps) there, and higher chip rate will cause wider bandwidth. To catch all the energy from the channel more Rake fingers are needed. A very large number of fingers lead to combining losses, practical implementation problems and design complexity. The diversity order is directly related to the number of Rake Fingers. Thus the diversity order is limited because of the limitation of the use of number of Rake Fingers.
MIMO-Rake Receiver – MIMO Rake Receiver can increase the diversity order. It can also overcome the design complexity, combining losses using large number of Rake fingers in generalized Rake Receiver. In this paper we are using $2 \times 2$ MIMO system and we are using generalized Rake Receiver at both Receiving antennas. The Block Diagram of MIMO-Rake Receiver is in fig.2.

The objective of this paper is to get higher order diversity, and to improve the BER performance of Rake Receiver. This paper is organised as follows - At first the performance is investigated with the generalized Rake receiver system, then that is investigated with MIMO-Rake Receiver systems.

II. REVERSE LINK SYSTEM MODLE :-

Block diagrams of reverse link system model is in Fig.3. The number of users are from 0 to K-1. Here the modulation technique is BPSK.

Let us assume that there are $K$ independent users transmitting signals in the DS-CDMA system. Each of them transmits the signal in the form as \([7]\):

\[
S_k(t - \tau_k) = \sqrt{2P_k} b_k(t - \tau_k) a_k(t - \tau_k) \cos(\omega_c t + \theta_k)
\]

Where - $b_k(t)$ - binary data sequence; $a_k(t)$ - pseudo random sequence; $P_k$ - power of the transmitted signal; $\omega_c$ - the carrier angular frequency; $\tau_k$ - time delay that accounts for the lack of synchronism between the transmitters, and $\theta_k$ - phase angle of the $k^{th}$ carrier. The $k^{th}$ user’s data signal is a sequence of unit amplitude rectangular pulse of duration $T_b$, taking values \{-1, +1\} with the equal probability. This sequence can be expressed as

\[
b_k(t) = \sum_{j=-\infty}^{\infty} b_j P T_b (t - j T_b)
\]

Where $PT_b = 1$, for $0 \leq t < T_b$, and $PT_b = 0$, otherwise. The spreading signal $a_k(t)$ can be expressed as:

\[
a_k(t) = \sum_{i=-\infty}^{\infty} a_i \psi (t - iT_c)
\]
\[ T_r \int_{0}^{r} y(t)dt = T \]
is the chip period, and \( a_{k}^{(i)} \) is the \( i^{th} \) chip value of \( k^{th} \) user; This chip value can be either -1 or +1.

If \( T_r \) is the chip period and there are \( N_c \) chips per bit, thus \( N_c = T_c/T_r \) is the spreading factor of user K. Let the desire user is \( K = 0 \) and all other users contribute to MAI.

If \( h_k(t) \) is the low pass impulse response of the Frequency Selective Fading channel then-

\[ h_k(t) = \sum_{l=k}^{L} \alpha_{k,l} e^{j\phi_{k,l}} \delta(t - \tau_{k,l}) \]

Where \( \phi_{k,l} \) and \( \tau_{k,l} \) are the phase and time delays introduced by the channel; they can be assumed to be random variables uniformly distributed in \((0, 2\pi)\) and \([0, \text{max}_T]\) respectively, where \( \text{max}_T \) is the maximum delay at which there can be multipath ray. \( L_k \) is the number of multipaths generated by frequency selective channel for the \( k^{th} \) transmitted signal. \( \alpha_{k,l} \) are the path gain components with Rayleigh distribution:

\[ f_{\alpha}(\alpha) = \frac{\alpha}{\sigma_{\alpha}^2} \exp \left\{ -\frac{\alpha^2}{\sigma_{\alpha}^2} \right\} \]

The received signal is:

\[ r(t) = h_k(t) \cdot S_k(t) + n(t) \]

Thus the total received signal can be written as:

\[ r(t) = \sum_{k=0}^{K} \sum_{l=1}^{L} \sqrt{P_k} \alpha_{k,l} b_k (t - \tau_{k,l}) \cos \left( t - \tau_{k,l} \right) \cos \left( \omega_k t + \phi_{k,l} \right) + n(t) \]

Where \( n(t) \) is Additive White Gaussian Noise (AWGN) with a two sided power spectral density of \( \text{No}/2 \). Where No is the noise power spectral density measured in watts/Hz (joules). Now the decision statistic is:

\[ Z_0 = \int_{0}^{T} r(t) a_{00} (t - \tau_{00}) \cos(\omega_0 t) dt = b_0 \cdot a_{00} \cdot \sqrt{P_0 T_0} + \sum_{k=0}^{K} \sum_{l=1}^{L_k} Y_{k,l} + \zeta \]

Where \( b_0 \) is the transmitted bit from user 0, \( a_{00} \) is the amplitude of the desired multipath component, \( P_0 \) is the transmitted power of the desired user, and \( \zeta = \int_{0}^{T} n(t) a_{00} (t - \tau_{00}) \cos(\omega_0 t) dt \)

is a Zero-mean Gaussian random variable with variance \( \sigma_{\zeta}^2 = \frac{N_c T_r}{4} \), we can re-write the decision statistic in (8) as:

\[ Z_0 = D_0 + Y_0 + \zeta \]

Where \( D_0 \) - desired signal component (1st term in (8)), \( Y_0 \) - the MAI (2nd term in (8)), and \( \zeta \) - the AWGN.

In case of cellular networks, it is worth to decompose the MAI term in to two distinct contributes: \( Y = Y_0 + Y_I \), where \( Y_0 \) is the interfering signal due to users within the same cell of the desired user (own-cell interference) and \( Y_I \) is the interfering signal due to presence of active users in other cells surrounding the cell of interest (inter-cell interference). Hence:

\[ Z_0 = D_0 + Y_0 + Y_I + \zeta \]

It is conceivable to suppose that \( Y_0 \) and \( Y_I \) is statistically independent.

II. GENERALIZED RAKE RECEIVER :-

The block diagram of Generalized Rake Receiver is in fig. 4. Here \( h_k'(t) \) is the estimated channel value. The channel estimated value is multiply with the each received signal and do the integration from 0 to \( T_r \) after this calculate the summation of all the estimated signal. The real part of this is known as the decision variable and then pass it through the decision device to get the decision about the bit whether it is 0 or 1. With the concept of Rake receiver in multipath fading channel the useful term in (10) becomes [8]:

\[ D_0 = \frac{P_0 T_0}{2} \sum_{l=1}^{L} \alpha_{0,l} \]

Here \( L_0 \) indicates the number of multipath rays relative to the useful signal, received with amplitude \( \sqrt{P_0} \alpha_{0,l} \).
In case of frequency-selective fading, with the hypotheses of identical mean number $L$ of multipaths for each source and identical mean number of users per cell, it is possible obtaining:

$$E\{Y^2\} = E\{\alpha^2\} \cdot \{K - 1\} \frac{A^2T^2}{3N_c} + E\{\alpha^2\} \cdot \{L - 1\} K \frac{A^2T^2}{3N_c} E\{Y^2\} = \frac{A^2T^2}{3N_c} 2\frac{\sigma^2}{3} \frac{(L - 1)}{N_c} \quad \ldots \ldots \quad (13)$$

$$E\{Y^2\} = E\{\alpha^2\} \cdot \{E\{\beta^2\}\} L K \frac{A^2T^2}{3N_c} = 2\frac{\sigma^2}{L} \frac{LK}{3N_c} \frac{A^2T^2}{3N_c} \quad \ldots \ldots \quad (14)$$

Where $\alpha_{k_0,i_0}$ are the path gains affecting signals of the reference cell, $\alpha_{k,i_0}$ are the path gains affecting signals of the surrounding cells, $\beta_{k_1} = \frac{r_1,i_0}{r_0,i_0}$ is the ratio between the distance of the $k_1$-th user of a surrounding cell from its home base station $(r_{k_0})$ and from the reference base station $(r_{0,i_0})$. If the path gains are identically Rayleigh distributed and $\beta_{k_1}$ is uniform in $(0, 1]$, we obtain:

$$E\{\alpha^2\} = E\{\alpha^2\} = \frac{2}{K} \text{ and } E\{\beta^2\} = \frac{5}{3} \text{.} \quad \ldots \ldots \quad (15)$$

If we consider that the fading is sufficiently slow to allow us to estimate the channel $h_k(t)$ perfect (with out noise). Furthermore, within any signaling interval, $h_k(t)$ is treated as a constant and denoted as $\bar{h}_k$. Thus the decision variables are represented as:

$$U_m = \text{Re} \left\{ \sum_{n=0}^{M} h_n \int_{0}^{T} r_n(t) S_m^{*} (t - k / w) dt \right\}; m = 1, 2 \quad \ldots \ldots \quad (16)$$

Where $U_m$ represent the decision variables; $S_m(t)$ is the transmitted signal transmitted signal and $r_n(t)$ is the received signal. If the transmitted signal is $S_{01}(t)$, then the received signal is:

$$r_n(t) = \sum_{n=1}^{M} h_n S_{01}(t - n / w) + n(t) \quad \ldots \ldots \quad (17)$$

Thus:

$$U_m = \text{Re} \left[ \sum_{k=1}^{M} h_n \int_{0}^{T} S_{01}(t - k / w) S_m^{*} (t - k / w) dt \right] + \text{Re} \left[ \sum_{k=1}^{M} h_n \int_{0}^{T} n(t) S_m^{*} (t - k / w) dt \right]; m = 1, 2 \quad \ldots \ldots \quad (18)$$

If we assume that our binary signals are designed to satisfy this property then,

$$U_m = \text{Re} \left[ \sum_{k=1}^{M} |h_n| \int_{0}^{T} S_{01}(t - k / w) S_m^{*} (t - k / w) dt \right] + \text{Re} \left[ \sum_{k=1}^{M} h_n \int_{0}^{T} n(t) S_m^{*} (t - k / w) dt \right]; m = 1, 2 \quad \ldots \ldots \quad (19)$$

When the binary signals are antipodal, a single decision variable suffices. In this case reduces to:

$$U_1 = \text{Re} \left\{ 2 E_b \sum_{k=1}^{M} \alpha_k^2 + \sum_{k=1}^{M} \alpha_k N_k \right\}; \text{Where } \alpha_k = |h_k| \text{ and } N_k = e^{\imath \phi} \int_{0}^{T} n(t) S_k^{*} (t - k / w) dt \quad \ldots \ldots \quad (20)$$
BIT-ERROR PROBABILITY FOR GENERALIZED RAKE RECEIVER- For coherent BPSK with Rayleigh Frequency Selective Fading channel with using convolution coding at the transmitter side the coded bit-error probability of Rake with MRC assuming identical noise in each branch is [4]:

\[
BE_{P,c} = \left(\frac{1 - \mu}{2}\right)^M \sum_{j=0}^{M-1} \left(\frac{M-j}{j}\right) \left(1 + \frac{\mu}{2}\right)^j
\]

\[\text{where } \mu = \sqrt{\frac{1}{1 + \frac{N_0}{2rE_s\sigma^2} + \frac{2}{3D_nN_c} \left(1 + \frac{M-1}{5}\right)LK - 1}}\]

Where \(M\) is the number of fingers in Rake Receiver, \(M_c\) is the interfering cells equipped by conventional correlation type receiver at the base station with perfect implementing the power control, \(L\) is the number of multipaths per signal, \(E_s\) is the energy per bit and \(N_0/2\) is the two-sided power spectral density of the thermal noise, \(D_n\) is the directivity of Base station antenna, \(r\) is the code rate of error control coding (convolutional coding), \(N_c\) is the spreading factor, \(K\) is the number of users using the channel simultaneously.

Now if we are using viterbi decoding [9] at the receiver side. The BER performance of Generalized Rake Receiver is-

\[
BER \leq \frac{d}{dN} T(D;J;N)|_{j=1;N=1;D=2\sqrt{BE_{P}(1-BE_{P})}}
\]

Where \(T(D;J;N)\) denotes the transfer function of the signal flow graph. The exponent of \(D\) on a branch describes the hamming weight of encoder output corresponding to that branch. The exponent of \(J\) is always equal to 1, since the length of each branch is one. The exponent of \(N\) denotes the number of 1’s in the information sequence for that path. (i.e. for input 0, exponent of \(N\) is 0 and for input 1, it is equal to 1).

III. MIMO-RAKE RECEIVER-

This section has two sub sections-

A. MIMO with ML Equalization-

In this paper, we will discuss another receiver structure called Maximum Likelihood (ML) decoding [10], [11] which gives us an even better performance. Now we assume that the channel is a flat fading Rayleigh multipath channel and the modulation is BPSK.

Fig.5. MIMO Structure

In a 2×2 MIMO channel with two transmit antennas if we have a transmission sequence, for example \(X_1, X_2, X_3, \ldots, X_n\). In normal transmission, we can send \(X_1\) in the first time slot, \(X_2\) in the second time slot, \(X_3\) and so on. Now as we now have 2 transmit antennas, we may group the symbols into groups of two. In the first time slot, send \(X_1\) and \(X_2\) from the first and second antenna. In second time slot, send \(X_3\) and \(X_4\) from the first and second antenna, send \(X_5\) and \(X_6\) in the third time slot and so on. From here we notice that by grouping two symbols and sending them in one time slot, we need only \(n/2\) time slots to complete the transmission i.e. data rate is doubled. This forms the simple explanation of a MIMO transmission scheme with two transmit antennas and two receive antennas.

Let us now try to understand the math for extracting the two symbols which interfered with each other. In the first time slot, the received signal on the first receive antenna is,

\[
Y_1 = h_{1,1}X_1 + h_{1,2}X_2 + n_i = \left[\begin{array}{c} h_{1,1} \\ h_{1,2} \end{array}\right] \left[\begin{array}{c} X_1 \\ X_2 \end{array}\right] + n_i
\]

(23)
The received signal on the second receive antenna is,
\[ Y_2 = h_{1,1}X_1 + h_{1,2}X_2 + n_3 = \left[ h_{1,1} \ h_{1,2} \right] \left[ X_1 \ \ X_2 \right] + n_2 \]  \quad (24)

Where- \( Y_1, Y_2 \) are the received symbol on the first and second antenna respectively, \( h_{1,1} \) is the channel from first transmit antenna to first receive antenna, \( h_{1,2} \) is the channel from second transmit antenna to first receive antenna, \( h_{2,1} \) is the channel from first transmit antenna to second receive antenna, \( h_{2,2} \) is the channel from second transmit antenna to second receive antenna, \( X_1 \) and \( X_2 \) are the transmitted symbols and \( n_1, n_2 \) is the noise on first and second receive antennas. We assume that the receiver knows \( h_{1,1}, h_{1,2}, h_{2,1}, \) and \( h_{2,2} \). The receiver also knows \( Y_1 \) and \( Y_2 \). The unknowns are \( X_1 \) and \( X_2 \).

For convenience, the above equation can be represented in matrix notation as follows:
\[
\begin{bmatrix}
Y_1 \\
Y_2
\end{bmatrix} = 
\begin{bmatrix}
h_{1,1} & h_{1,2} \\
h_{2,1} & h_{2,2}
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} +
\begin{bmatrix}
n_1 \\
n_2
\end{bmatrix}
\]  \quad (25)

Equivalently,
\[ Y = HX + n \]  \quad (26)

We have assume here that - The channel experience by each transmit antenna is independent from the channel experienced by other transmit antennas. The channel experienced between each transmit to the receive antenna is independent and randomly varying in time. The channel is known at the receiver.

**Maximum Likelihood (ML) Receiver** - The Maximum Likelihood receiver tries to find \( \hat{X} \) (estimate of the transmitted symbol \( X \)) which minimizes,
\[ J = \| Y - H\hat{X} \|^2 \]. If we are using BPSK modulation, the possible values of \( X \) is +1 or -1 similarly \( X_2 \) also take values +1 or -1. So, to find the Maximum Likelihood solution, we need to find the minimum from the all four combinations of \( X_1 \) and \( X_2 \).

\[
\hat{J} = \text{arg\,min} \left\{ \| Y - H\hat{X} \|^2 \right\} \quad (27)
\]

\[
J_{+1,+1} = \left[ \begin{array}{c} Y_1 \\ Y_2 \end{array} \right] - \left[ \begin{array}{cc} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{array} \right] \left[ \begin{array}{c} +1 \\ +1 \end{array} \right], \quad J_{+1,-1} = \left[ \begin{array}{c} Y_1 \\ Y_2 \end{array} \right] - \left[ \begin{array}{cc} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{array} \right] \left[ \begin{array}{c} +1 \\ -1 \end{array} \right], \quad J_{-1,+1} = \left[ \begin{array}{c} Y_1 \\ Y_2 \end{array} \right] - \left[ \begin{array}{cc} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{array} \right] \left[ \begin{array}{c} -1 \\ +1 \end{array} \right], \quad J_{-1,-1} = \left[ \begin{array}{c} Y_1 \\ Y_2 \end{array} \right] - \left[ \begin{array}{cc} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{array} \right] \left[ \begin{array}{c} -1 \\ -1 \end{array} \right] \quad (28)
\]

The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e. if the minimum is \( J_{+1,+1} \rightarrow [1 \ 1] \), if the minimum is \( J_{+1,-1} \rightarrow [1 \ 0] \), if the minimum is \( J_{-1,+1} \rightarrow [0 \ 1] \) and if the minimum is \( J_{-1,-1} \rightarrow [0 \ 0] \). The results for 2×2 MIMO with Maximum Likelihood (ML) equalization [11] helped us to achieve a performance closely matching the 1 transmit 2 receive antenna Maximal Ratio Combining (MRC) case.

**B. BIT-ERROR PROBABILITY FOR GENERALIZED MIMO RAKE RECEIVER**-

For coherent BPSK with Rayleigh Frequency Selective Fading channel with using convolution coding at the transmitter side the coded bit-error probability of MIMO-RAKE Receiver with ML equalizer, assuming identical noise in each branch, is given as-
\[ BEP_c = \left( \frac{1 - \mu}{2} \right) \sum_{j=0}^{d-1} \binom{d-1}{j} \left( \frac{1 + \mu}{2} \right)^j \]  \quad (30)
\[ \mu = \frac{1}{1 + \frac{N_o}{2r.E_s\sigma^2} + \frac{2}{3.D_o.N_c}} \left( 1 + \frac{M}{5} \right) LK - 1 \]

\( d = M.A \); \( M \) is the number of Rake fingers attached with one antenna in MIMO arrangement; \( A \) is the number of Antenna used in MIMO arrangement.

**IV. PERFORMANCE ANALYSIS**

In Fig. 6 the number of multipaths \( L = 8 \); the number of Users \( K = 3 \); Spreading Factor \( N_c = 32 \); Antenna Directivity \( D_a = 5\text{dB} \); Number of Interfering Cells \( M_c = 4 \); Number of Rake Fingers \( M = 3 \); Code Rate \( r = 1/2 \); Constraint length \( C_L = 3 \). From this at BER value of \( 10^{-4} \) there is 8 dB improvements in BER Performance of communication system with using MIMO Rake Receiver in comparison with Generalized Rake Receiver with 3 taps (i.e. with 3 resolvable paths).

In Fig. 7 the number of multipaths \( L = 8 \); the number of Users \( K = 1 \) to 40; Spreading Factor \( N_c = 32 \); Antenna Directivity \( D_a = 5\text{dB} \); Number of Interfering Cells \( M_c = 4 \); \( E_o/N_o = 9\text{ dB} \); number of Rake Fingers \( M = 3 \); Code Rate \( r = 1/2 \); Constraint length \( C_L = 3 \). From this at BER value of \( 10^{-4} \) there is 1 user with \( 2 \times 2 \) MIMO system with 1 tap, 3 users with generalized Rake Receiver with 3 taps, 12 users with \( 2 \times 2 \) MIMO Rake Receiver with 3 taps; Therefore there is capacity improvement in using MIMO Rake Receiver in comparison with Generalized Rake Receiver.

**REFERENCES**


[3] Chen Wei Wulinyan MengNan, “BER Analysis of Rake Receiver in Rayleigh Fading Channel”, wuhan University of Technology, China

AUTHORS BIOGRAPHIES:

Mr. Pravindra Kumar – He has done M.Tech. in Digital Communication (D.C) from Ambedkar Institute of Technology, Delhi (I.P. Univ., Delhi). He received his B.Tech Degree in Electronics and Communication Engineering from Ideal Institute of Technology, Ghaziabad (U.P) and B.Sc. degree from C. C. S. Univ., Meerut. He has 3+ years of teaching experience. His teaching and research interests are the wireless communication and digital signal processing. He has published five papers in International Journal and one in IEEE. He is working as an Assistant Professor in REMTech, Shamli (U.P).

Mr. Anand Kumar - He has done M. Phil from Dr. B.R.Ambedkar Univ.,Agra (U.P). He received his M.Sc and B.Sc degree from M. J. P. Rohilkhand Univ., Bareilly (U.P). He has 3+ years of teaching experience. His research interest area is bicomplex number. He has published two paper in international journal and also has good Gate Score-2011. He is working as Assistant Professor in KIMT, Moradabad (U.P).