WAVES ON A STRING

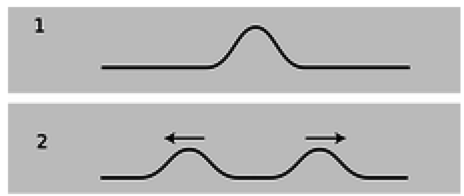
k / Hitting a key on a piano causes a hammer to come up from underneath and hit a string (actually a set of three). The result is a pair of pulses moving away from the point of impact.

So far you've learned some counterintuitive things about the behavior of waves, but intuition can be trained. The first half of this subsection aims to build your intuition by investigating a simple, one-dimensional type of wave: a wave on a string. If you have ever stretched a string between the bottoms of two open-mouthed cans to talk to a friend, you were putting this type of wave to work. Stringed instruments are another good example. Although we usually think of a piano wire simply as vibrating, the hammer actually strikes it quickly and makes a dent in it, which then ripples out in both directions. Since this chapter is about free waves, not bounded ones, we pretend that our string is infinitely long.

After the qualitative discussion, we will use simple approximations to investigate the speed of a wave pulse on a string.

This quick and dirty treatment is then followed by a rigorous attack using the methods of calculus, which turns out to be both simpler and more general.

Intuitive ideas



l / A pulse on a string splits in two and heads off in both directions.

Consider a string that has been struck, l/1, resulting in the creation of two wave pulses, l/2, one traveling to the left and one to the right. This is analogous to the way ripples spread out in all directions from a splash in water, but on a one-dimensional string, "all directions" becomes "both directions."

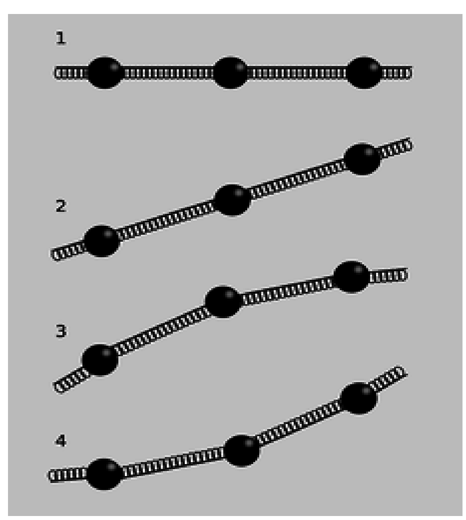
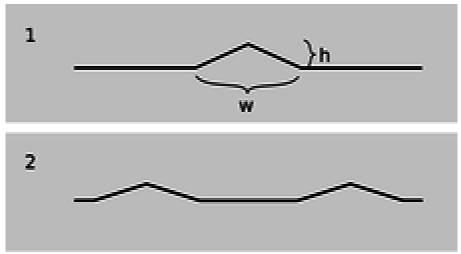


Figure m: Modeling a string as a series of masses connected by springs.

We can gain insight by modeling the string as a series of masses connected by springs, m. (In the actual string the mass and the springiness are both contributed by the molecules themselves.) If we look at various microscopic portions of the string, there will be some areas that are flat, 1, some that are sloping but not curved, 2, and some that are curved, 3 and 4. In example 1 it is clear that both the forces on the central mass cancel out, so it will not accelerate. The same is true of 2, however. Only in curved regions such as 3 and 4 is an acceleration produced. In these examples, the vector sum of the two forces acting on the central mass is not zero. The important concept is that curvature makes force: the curved areas of a wave tend to experience forces resulting in an acceleration toward the mouth of the curve. Note, however, that an uncurved portion of the string need not remain motionless. It may move at constant velocity to either side.

Approximate treatment

We now carry out an approximate treatment of the speed at which two pulses will spread out from an initial indentation on a string. For simplicity, we imagine a hammer blow that creates a triangular dent, n/1. We will estimate the amount of time, t, required until each of the pulses has traveled a distance equal to the width of the pulse itself. The velocity of the pulses is then $\pm w/t$.



n / A triangular pulse spreads out.

As always, the velocity of a wave depends on the properties of the medium, in this case the string. The properties of the string can be summarized by two variables: the tension, T, and the mass per unit length, μ (Greek letter mu).

Source: http://physwiki.ucdavis.edu/Fundamentals/06._Waves/6.1_Free_Waves