TIME RESPONSE ANALYSIS OF CONTROL SYSTEMS

Introduction:

Time is used as an independent variable in most of the control systems. It is important to analyse the response given by the system for the applied excitation, which is function of time. Analysis of response means to see the variation of out put with respect to time. The output behavior with respect to time should be within these specified limits to have satisfactory performance of the systems. The stability analysis lies in the time response analysis that is when the system is stable out put is finite

The system stability, system accuracy and complete evaluation is based on the time response analysis on corresponding results.

DEFINITION AND CLASSIFICATION OF TIME RESPONSE

Time Response:

The response given by the system which is function of the time, to the applied excitation is called time response of a control system.

Practically, output of the system takes some finite time to reach to its final value.
This time varies from system to system and is dependent on different factors.
The factors like friction mass or inertia of moving elements some nonlinearities present etc.
Example: Measuring instruments like Voltmeter, Ammeter.

Classification:

The time response of a control system is divided into two parts.

1 Transient response $c_t(t)$
2 Steady state response $c_{ss}(t)$

$$\therefore c(t)=c_t(t)+c_{SS}(t)$$

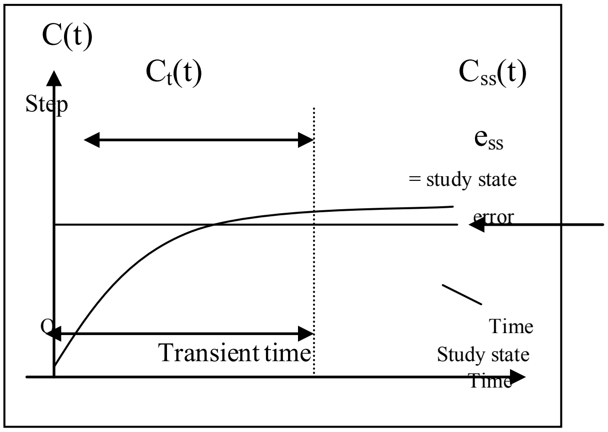
Where c(t)= Time Response
Total Response=Zero State Response +Zero Input Response

Transient Response:

It is defined as the part of the response that goes to zero as time becomes very large. i,e,

$$\lim_{t \to \infty} c_t(t)=0$$

A system in which the transient response do not decay as time progresses is an **Unstable system.**



The transient response may be experimental or oscillatory in nature.

2. <u>Steady State Response</u>:

It is defined the part of the response which remains after complete transient response vanishes from the system output.

. i,e, $\lim_{t \to \infty} c_t(t) = c_{ss}(t)$

The time domain analysis essentially involves the evaluation of the transient and Steady state response of the control system.

Standard Test Input Signals

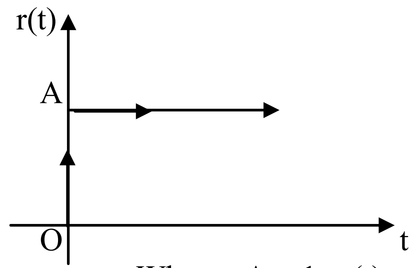
For the analysis point of view, the signals, which are most commonly used as reference inputs, are defined as **standard test inputs.**

- The performance of a system can be evaluated with respect to these test signals.
- Based on the information obtained the design of control system is carried out.
- The commonly used test signals are
  1. Step Input signals.
  2. Ramp Input Signals.
  3. Parabolic Input Signals.
  4. Impulse input signal.

<u>Details of standard test signals</u>

1. Step input signal (position function)
   It is the sudden application of the input at a specified time as usual in the figure or instant any us change in the reference input
   Example :-
   a. If the input is an angular position of a mechanical shaft a step input represent the sudden rotation of a shaft.
   b. Switching on a constant voltage in an electrical circuit.

c. Sudden opening or closing a valve.



When, $A = 1$, $r(t) = u(t) = 1$

The step is a signal who's value changes from 1 value (usually 0) to another level A in Zero time.

In the Laplace Transform form $R(s) = A / S$

Mathematically $r(t) = u(t)$

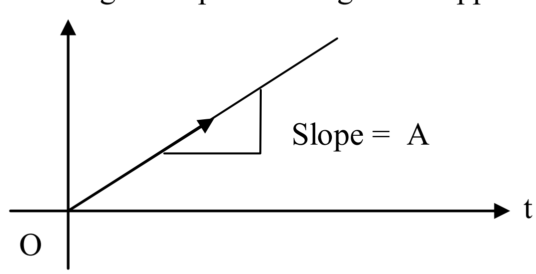
$= 1$ for $t \geq 0$

$= 0$ for $t < 0$

2. Ramp Input Signal (Velocity Functions):

It is constant rate of change in input that is gradual application of input as shown in fig (2 b).

Ex:- Altitude Control of a Missile



The ramp is a signal, which starts at a value of zero and increases linearly with time.

Mathematically $r(t) = At$ for $t \geq 0$

$= 0$ for $t \leq 0$.

In LT form $R(S) = \frac{A}{S^2}$

If A=1, it is called **Unit Ramp Input**

Mathematically

$r(t) = t\, u(t)$

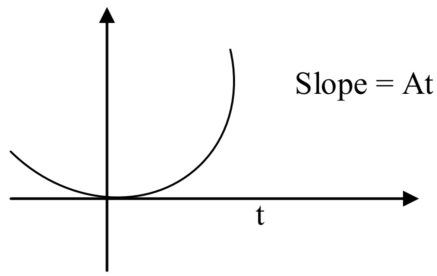
In LT form $= \{$ t for $t \geq 0$, 0 for $t \leq 0$

3. **Parabolic Input Signal (Acceleration function):**

- The input which is one degree faster than a ramp type of input as shown in fig (2 c) or it is an integral of a ramp .
- Mathematically a parabolic signal of magnitude

A is given by $r(t) = \frac{A t^2}{2} u(t)$

$$r(t) = \begin{cases} \frac{At^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t \leq 0 \end{cases}$$



In LT form $R(S) = \frac{A}{S^3}$

- If A = 1, a unit parabolic function is defined as $r(t) = \frac{t^2}{2} u(t)$

ie., $r(t) = \begin{cases} \frac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t \leq 0 \end{cases}$

In LT for $R(S) = \frac{1}{S}$

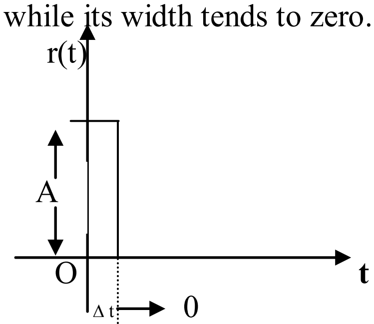
4. **Impulse Input Signal :**

It is the input applied instantaneously (for short duration of time) of very high amplitude as shown in fig 2(d)

Eg: Sudden shocks i e, HV due lightening or short circuit.

It is the pulse whose magnitude is infinite while its width tends to zero.

ie., t ⟶ 0 (zero) applied momentarily



Area of impulse = Its magnitude

If area is unity, it is called **Unit Impulse Input** denoted as $\delta(t)$

Mathematically it can be expressed as

$r(t) = A$ for $t = 0$

$= 0$ for $t \neq 0$

In LT form $R(S) = 1$ if $A = 1$

Standard test Input Signals and its Laplace Transforms.

r(t)	R(S)
Unit Step	$1/S$
Unit ramp	$1/S^2$
Unit Parabolic	$1/S^3$
Unit Impulse	1