Subproblem Approach for Thin Shell Dual Finite Element Formulations

Vuong Q. Dang1, Patrick Dular1,2, Ruth V. Sabariego1, Laurent Krähenbühl3 and Christophe Geuzaine1

1Dept. of Electrical Engineering and Computer Science, University of Liège, Belgium, Vuong.DangQuoc@student.ulg.ac.be
2Fonds de la Recherche Scientifique - F.R.S.-FNRS, Belgium
3Université de Lyon, Ampère (CNRS UMR5005), École Centrale de Lyon, F-69134 Écully Cedex, France

Abstract—A subproblem technique is applied on dual formulations to the solution of thin shell finite element models. Both the magnetic vector potential and magnetic field formulations are considered. The subproblem approach developed herein couples three problems: a simplified model with inductors alone, a thin region problem using approximate interface conditions, and a correction problem to improve the accuracy of the thin shell approximation, in particular near their edges and corners. Each problem is solved on its own independently defined geometry and finite element mesh.

I. INTRODUCTION

The solution by means of subproblems provides clear advantages in repetitive analyses and can also help in improving the overall accuracy of the solution [1], [2]. In the case of thin shell (TS) problems the method allows to benefit from previous computations instead of starting a new complete finite element (FE) solution for any variation of geometrical or physical characteristics. Furthermore, it allows separate meshes for each subproblem, which increases computational efficiency.

In this paper, a problem \( p = 1 \) involving massive or stranded inductors alone is first solved on a simplified mesh without thin regions. Its solution gives surface sources (SSs) for a TS problem \( p = 2 \) through interface conditions (ICs), based on a 1-D approximation [3], [4]. The TS solution is then considered as a volume source (VSs) of a correction problem \( p = 3 \) taking the actual field distribution of the field near edges and corners into account, which are poorly represented by the TS approximation. The method is validated on a practical test problem using a classical \textit{brute force} volume formulation.

II. DEFINITION OF THE SUBPROBLEM APPROACH

A. Canonical magnetodynamic or static problem

A canonical magnetodynamic or static problem \( p \), to be solved at step \( p \) of the subproblem approach, is defined in a domain \( \Omega \), with boundary \( \partial \Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p} \). Subscript \( p \) refers to the associated problem \( p \). The equations, material relations and boundary conditions (BCs) of the subproblems \( p = 1, 2, 3 \) are:

\[
\begin{align*}
\text{curl} \ h_p &= j_p, \ \text{div} \ b_p = 0, \ \text{curl} \ e_p = -\partial_t b_p, \\
\text{curl} \ e_p &= -\partial_t b_p, \\
\text{curl} \ e_p &= -\partial_t b_p, \\
\text{curl} \ e_p &= -\partial_t b_p, \\
\end{align*}
\]

and is the unit normal exterior to \( \Omega_p \). In what follows the notation \( [\gamma_p] = [\gamma_+^p - \gamma_-^p] \) expresses the discontinuity of a quantity through any interface \( \gamma_p \) (with sides \( \gamma_+^p \) and \( \gamma_-^p \) ) in \( \Omega_p \), defining interface conditions (ICs).

The fields \( h_{s,p} \) and \( j_{s,p} \) in (2) are VSs in the subproblem approach which can be used for expressing changes of permeability or conductivity (via \( h_{s,p} \) and \( j_{s,p} \), respectively). Indeed, changing from \( \mu_1 \) and \( \sigma_1 \) in a given subregion for problem \( p = 1 \) to \( \mu_2 \) and \( \sigma_2 \) for problem \( p = 2 \) leads to the associated VSs

\[
\begin{align*}
h_{s,2} &= (\mu_2^{-1} - \mu_1^{-1})b_1, \\
j_{s,2} &= (\sigma_2 - \sigma_1)e_1. \\
\end{align*}
\]

B. Constraints between subproblems

The constraints for the problems \( p = 2, 3 \) are respectively SSs and VSs. The TS model \( p = 2 \) [4] is written as a subproblem following the inductor source field calculation of problem \( p = 1 \). Its SSs are defined via ICs of impedance-type boundary conditions (IBC) combined with contributions from problem \( p = 1 \). The \( b \)-formulation uses a magnetic vector potential \( a = a_c + a_d \) (such that \( \text{curl} a = b \)). The \( h \)-formulation uses a similar decomposition, \( h = h_c + h_d \). Fields \( a_c, h_c \) and \( a_d, h_d \) are respectively continuous and discontinuous through the TS. The weak \( b \)- and \( h \)-formulations involve the SSs in surface integral terms, respectively

\[
\begin{align*}
\langle [n \times e_2]_{\gamma_2}, a'_c + a'_d \rangle_{\gamma_2}, \\
\langle [n \times e_2]_{\gamma_2}, h'_c + h'_d \rangle_{\gamma_2} \\
\end{align*}
\]

and

\[
\begin{align*}
\langle [n \times e_2]_{\gamma_2}, h'_c + h'_d \rangle_{\gamma_2} \\
\langle [n \times e_2]_{\gamma_2}, h'_c + h'_d \rangle_{\gamma_2} \\
\end{align*}
\]

This work has been supported by the Belgian Science Policy (IAP P6/21) and the Belgian French Community (ARC 09/14-02).
III. APPLICATION EXAMPLE

The test problem is a shielded induction heater. It comprises two inductors (stranded or massive), a plate ($\mu_{r,\text{plate}} = 100$, $\sigma_{\text{plate}} = 1 \text{ MS/m}$) in the middle, and two screens ($\mu_{r,\text{screen}} = 1$, $\sigma_{\text{screen}} = 37.7 \text{ MS/m}$) (Fig. 1). It is first considered via a stranded inductor model (Fig. 2, top left, $a_1$), adding a TS FE model (Fig. 2, bottom left, $a_2$) that does not include the inductor anymore. Finally, a correction problem replaces the TS FE with classical volume FE (Fig. 2, top right, $a_3$). The complete solution is shown as well (Fig. 2, bottom right, $a_1 + a_2 + a_3$).

Errors on the magnetic flux with the TS model between classical solution and ($p = 1 + 2$) for both $b$- and $h$-formulations are shown in (Fig. 3); they can nearly reach 65% in the end regions of the plate. Accurate local corrections can then be obtained, reducing the errors to less than 0.01% (Fig. 4). Significant TS errors are achieved for the eddy current as well (Fig. 5), up to 50% and 60% near the screen ends for ($\delta = 0.918 \text{ mm}$, $\mu_{r,\text{plate}} = 100$, $f = 3 \text{ kHz}$) and ($\delta = 0.655 \text{ mm}$, $\mu_{r,\text{plate}} = 200$, $f = 3 \text{ kHz}$) respectively, with $d = 4 \text{ mm}$ and $\sigma_{\text{plate}} = 1 \text{ MS/m}$ in both cases. The proposed technique for TS FE and correction have been presented via a subproblem approach. It leads to accurate eddy current and magnetic flux distributions at the edges and corners of thin regions. All the steps of the method will be detailed, illustrated and validated in extended paper for both $b$- and $h$-formulations in 2D and 3D cases.

Fig. 1. Shielded induction heater ($L_{pl} = 2.2 \text{ m}$, $L_s = 2 \text{ m}$, $H_s = 400 \text{ mm}$, $C_{dx} = 800 \text{ mm}$, $C_{dy} = 10 \text{ mm}$, $C_{dz} = 200 \text{ mm}$, $C_z = 50 \text{ mm}$, $d = 5 \text{ mm}$)

REFERENCES