Study of Optimal Design of Low Pass Block Digital Filter

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***Abstract*—In this paper design of low pass optimal block digital filter is compared with traditional low pass overlap-save block digital filter. Simulation results show that global error obtained by optimal method is lower than that obtained by traditional overlap-save method.**

***Index Terms* — Block digital filters, optimum design, overlap-save method, frobenius norm, aliasing, time-varying system.**

1. INTRODUCTION

In some real time signal processing applications concerned with signal monitoring and analysis, filtering of long duration signal is a familiar problem. Block digital filtering is a powerful tool to reduce the computational complexity of digital filtering system where, fast digital filtering of signal is required [1][2]. In block digital filtering long signals are broken into smaller segments for easier processing by blocks using FFT or other fast transform [3].

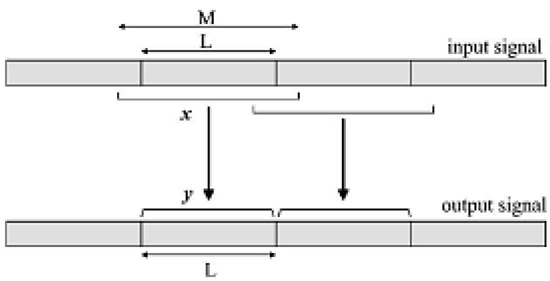


Figure 1: Block digital filtering

As shown in figure 1, the input signal is divided into overlapping blocks of M samples. The amount of overlapping is $M - L$. Then each block is processed and provides L samples of the output signal ($L \leq M$). The concatenation of the L points kept in each filtered block is the filtered signal. Many approaches to block digital filters (BDFs) design exist. Overlap-save and Overlap-add are two techniques which are widely used for block digital filtering [4]. In traditional OLS and OLA implementation, the system is generally compelled to be time invariant i.e. aliasing error is null. This ensures equality between the linear convolution (direct filtering) and the circular convolution (block filtering). Traditional synthesis methods, such as different window design, the least square method design or others are often used for designing the block filter.

In some other approaches, no such constraint on the BDF is imposed so that the BDF can be time-variant . For that, a simple efficient matrix-oriented approach to optimize the BDF coefficients has been proposed in [5]. The criterion defined for the optimal BDF design consists in minimizing the global quadratic distortion between the desired filtering and the block filtering. Although the aliasing distortion is not zero, the global distortion error associated to this optimal approach is lower than that associated to the other traditional approaches.

In this paper, based on the work cited above, optimal design method will be compared to the traditional overlap-save method. Here example of low pass filtering will be taken and comparison of time-invariant, aliasing and global error obtained by optimal method and overlap-save method will be done.

After brief introduction given in section I, overlap-save scheme is given in section II. In section III the optimal method for BDF parameters is given and results are provided in section IV. Conclusion is drawn in section V followed by some of the references given in section VI.

II. OVERLAP-SAVE SCHEME

Block digital filtering using the OLS scheme is illustrated in Fig. 2. The input signal is divided into overlapping blocks x_i of M samples. The amount of overlapping is $M-L$. Then, for each block, the following computations are performed.

- The block x_i is transformed through an M-point DFT.
- The transformed block vector coefficients are multiplied term by term by an M-length vector g_s. In traditional methods, this vector g_s corresponds to the M-point DFT of the filter impulse response h designed by one of the standard filter design methods (such as different window methods, the least square method design or others).

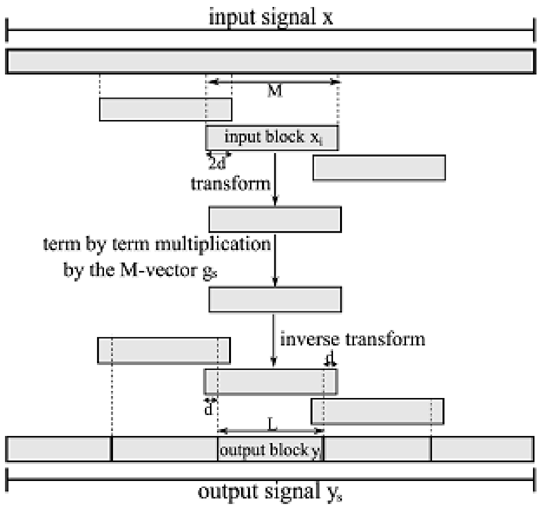


Figure 2: Overlap-save diagram

- An M-point inverse DFT (IDFT) is performed.
- Only the L central points of the resulting block are kept. Then, each M length input block x_i provides an L–length output block y_i ($L \leq M$ and $M-L$ is even to preserve symmetry).
- The concatenation of the output blocks forms the filtered output signal.

III. OPTIMAL BDF DESIGN

From the fig. 1, the input signal x_i and the output y_i are related as [5]:

$$y_i = Ax_i \tag{1}$$

Where A is the L × M matrix below (∗ stands for the conjugate transpose):

$$A = SW_M^* GW_M \tag{2}$$

Where W_M a matrix is corresponds to M point FFT, S is a selection matrix and is defined as:

$$S = [0_{L\times d} \ \ I_{L\times L} \ 0_{L\times d}] \tag{3}$$

Let us define K, the number of points we want to consider on the frequency response , such that K=bL, where b is an integer. Due to the well known properties of the DFT, this allows considering periodical signals x of period K (indeed, with a K-points frequency resolution, considering signals of period K or non-periodical signals provide the same result). The filtered signal is:

$$y = Hx \tag{4}$$

where x and y are K-dimensional vectors and H is a K×K matrix containing b copies of matrix A, as shown on figure 3.

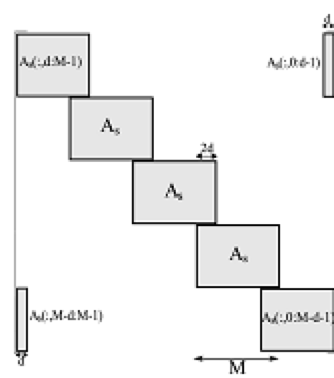


Figure 3: structure of matrix H (for b=5)

Let us note $\bar{y}$ and $\bar{x}$ the K point DFTs of y and x, then:

$$\bar{y} = \bar{\bar{H}}\bar{x} \tag{5}$$

Where:

(6)

Let us note the K-dimensional vector containing the desired frequency response, and a diagonal matrix, the diagonal of which is . The quadratic error is a good measure of the quality of the obtained frequency response:

(7)

where stands for the Frobenius norm. the algorithm [5] has been verified on lowpass and it can be extended to the design of other filters also. Simulation verification of the algorithm has been given in next section.

IV. RESULTS

Simulation results are obtained for and K K is taken to obtain a frequency resolution . These are relatively low values but provide results that are easier to visualize.

Fig. 4 shows the desired frequency response . The horizontal axis is the frequency index (k represents the normalized frequency). The desired filter is a lowpass filter. It is equal to zero between indexes 21 and 171 and 1 elsewhere.

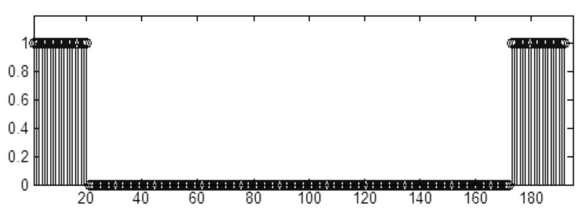


Figure 4: Desired frequency response

The optimal matrix G is obtained by using the method described above. The matrix G obtained by overlap-save using hamming window is also obtained. Fig. 6 shows the values obtained on the diagonal of G.

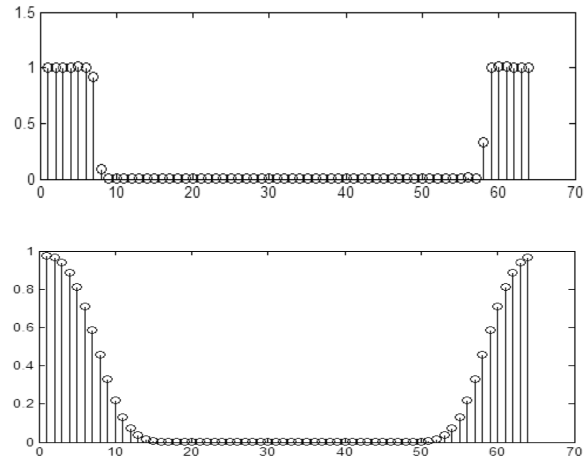


Figure 5: Diagonal of matrix G (top: optimal method, bottom: overlap-save using hamming window design)

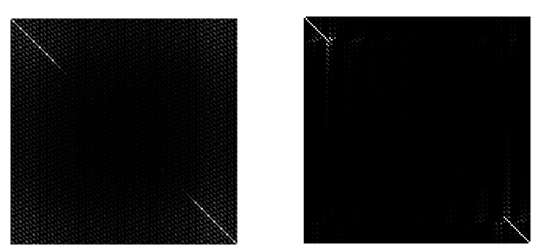
From matrix G, matrix A, H and are also obtained. Fig. 6, 7 and 8 shows matrices A, H and .

(a) (b)

Figure 6: Matrices A ((a): optimal, (b): overlap-save using hamming window)

(a) (b)

Figure 7: Matrices H ((a): optimal, (b): overlap-save using hamming window)



(a) (b)

Figure 8: Matrices A ((a): optimal, (b): overlap-save using hamming window)

From matrix , we can say that overlap-save provides a diagonal matrix (hence, there is no aliasing), while the optimal method provides a non-diagonal one (hence, aliasing is present).

The table 1. below shows the time-invariant and aliasing error (between the obtained and desired frequency response) provided by optimal and using some of the traditional windows. Fig. 9 shows the global error (sum of time-invariant and aliasing error) in the form of bar-graph.

TABLE 1

Method	time-invariant	Aliasing
Hamming window design	8.8737	0
Blackman window design	8.6564	0
Hanning window design	8.8149	0
Gaussian window design	8.8268	0
Kaiser window design	9.5876	0
Optimal method	0.8164	0.1708

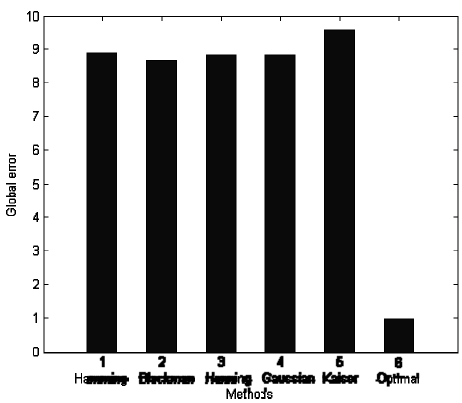


Figure 9:Global error

We see from the above table that optimal design approach provides lower distortion by tolerating a slight aliasing which provides a considerable gain on the overall quality of the filter.

V. CONCLUSION

In this paper we have shown that designing of lowpass BDF by optimal method decreases the Global error as compared to the other traditional methods. So we can say that tolerating a small amount of aliasing yields a large improvement of obtained frequencies responses.

VI. REFERENCES

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