Solving the Economic Dispatch Problem using Novel Particle Swarm Optimization

S. Khamsawang and S. Jiriwibhakorn

Abstract**—**This paper proposes an improved approach based on conventional particle swarm optimization (PSO) for solving an economic dispatch(ED) problem with considering the generator constraints. The mutation operators of the differential evolution (DE) are used for improving diversity exploration of PSO, which called particle swarm optimization with mutation operators (PSOM). The mutation operators are activated if velocity values of PSO nearly to zero or violated from the boundaries. Four scenarios of mutation operators are implemented for PSOM. The simulation results of all scenarios of the PSOM outperform over the PSO and other existing approaches which appeared in literatures.

Keywords**—**Novel particle swarm optimization, Economic dispatch problem, Mutation operator, Prohibited operating zones, Differential Evolution.

I. Introduction

THE main objective of ED problem is to decrease the fuel cost of generators, satisfying many equality and inequality constraints. In the past, classical ED problem is solved using classical mathematical optimization methods, such as lambda method, gradient method and Newton method [1].

Many researchers exert to improve many optimization techniques for solving ED problem such as PSO [3-6], GA [3], [10-11] chaotic particle swarm optimization (CPSO) [7] and clocal algorithm (AIS) [8] and multiples tabu search (MTS) [9]. PSO was introduced by J. Kenedy and R. Eberhart in 1995 [12]. PSO is a type of modern optimization techniques and a kind of swarm intelligence. PSO has been tested and seen to be high efficiency in solving continuous nonlinear optimization problems [12-13].

This paper proposed the techniques are based on particle swarm optimization and mutation operators of the differential evolution algorithm for guarantee the global optimal solution and reduced the computational time. Four scenarios of mutation operators are introduced, which can enhance the exploration performance of the PSO.

The paper is organized as follows: Section II formulates the ED problem. Section III describes detail of particle swarm optimization. Section IV proposes the mutation operators to improve the PSO. Section V describes the details of the proposed method apply for solving the economic dispatch problem. Section VI shows the simulation results. Lastly, conclusion is given in Section VII.

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II. Formulation of ED Problems

The economic dispatch problem is one of the important problems in the power system planning and the operation. Therefore, the ED problem can be formulated mathematically as an optimization problem. Minimizing the fuel cost function of all generating units in the power system subjected to power system balanced constraint, power losses and generating unit operation is the main purpose of the economic dispatch problem. and represented as following

$$\text{Minimize } F_T = \sum_{i=1}^{n} F_i(P_i) \quad (1)$$

where F_T is total fuel cost, n is number of online generating unit and $F_i(P_i)$ is operating fuel cost of generating unit *i.*

The simplified fuel cost function of the generators in the economic dispatch problem is most represented as quadratic function [1] as given in (2)

$$F_i(P_i) = a_i + b_i P_i + c_i P_i^2 \quad (2)$$

where a_i , b_i , c_i are cost coefficients of generating unit i , P_i is the real power output of unit i .

The minimization of the ED problem is subjected to the following constraints

1. Generator constraint:

$$P_{i,\min} \le P_i \le P_{i,\max} \quad (3)$$

2. Power balance constraint:

$$\sum_{i=1}^{n} P_i = D + P_L \quad (4)$$

with

$$P_L = \sum_{i=1}^{n}\sum_{i=1}^{n} P_i B_{ij} P_j + \sum_{i=1}^{n} P_i B_{oi} + B_{oo} \quad (5)$$

where D is total load demand, P_L is total transmission line loss, $P_{i,\min}$ and $P_{i,\max}$ are minimum and maximum power output of unit i and B_{ij} , B_{oi} and B_{oo} are transmission line loss coefficients.

A. Economic Dispatch Problem with Prohibited Operating Zones (POZ)

The economic dispatch problem which includes the effect of prohibited zones is called "economic dispatch problem with prohibited operating zones" [1], [3]. The fuel cost function of the POZ with two prohibited operating zones is illustrated in Fig 1. The possible operating zones of the generators can be expressed as follows:

$$\begin{aligned} &P_{i,\min} \le P_i \le P_{i,1}^L \\ &P_{i,k-1}^U \le P_i \le P_{i,k}^L \qquad k = 2,3,\ldots,n_i,\, n_i = l,\ldots,m \\ &P_{i,n_i}^U \le P_i \le P_{i,\max} \end{aligned} \tag{6}$$

where k is the number of prohibited operating zones of generating unit i, $P_{i,k}^L$ and $P_{i,k}^U$ are lower and upper limits of the k^{th} prohibited zone of generating unit i, respectively.

The economic dispatch problem with prohibited operating zones the ramp rate limit constraints, prohibited operating zones constraints and transmission line losses are included and can be expressed as follows:

Ramp Rate Limit Constraints: According to the operating increases and operating decreases of the generators are ramp rate limit constraints illustrated in Fig 2 and can be described as follow

1*) as generation increases:*

$$P_{i(t)} + P_{i(t-1)} \le UR_i \tag{7}$$

2) as generation decreases:

$$P_{i(t-1)} - P_{i(t)} \ge DR_i \tag{8}$$

where $P_{i(t)}$ is output power of generating unit i at current and $P_{i(t-1)}$ is output power at previous. UR_i is upramp limit of generating unit i (MW/time−period) and DR_i is downrampt limit of generating unit i (MW/time−period)

Generator Operating Constraint:

$$\begin{aligned} &\max(P_{i,\min}, P_{i(t-1)} - DR_i) \le P_{i(t)} \\ &\le \min(P_{i,\max}, P_{i(t-1)} - UR_i) \end{aligned} \tag{9}$$

where

$$P_{i(t)} = \begin{cases} P_{i,\min} \le P_i \le P_{i,1}^L \\ P_{i,k-1}^U \le P_i \le P_{i,k}^L & k = 2,3,\ldots,n_i,\, n_i = l,\ldots,m \\ P_{i,n_i}^U \le P_i \le P_{i,\max} \end{cases} \tag{10}$$

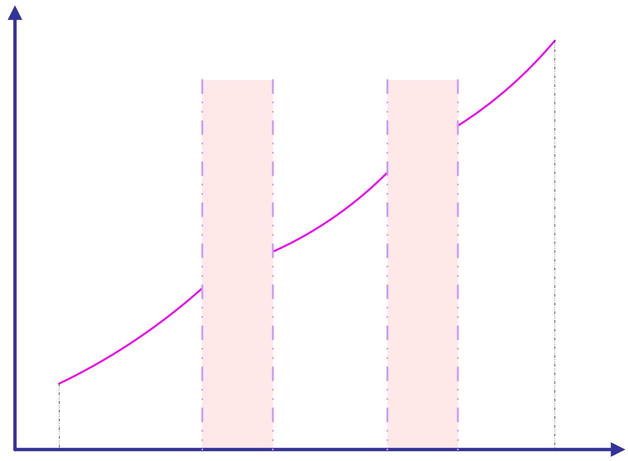

Fig.1 Two prohibited operating zones function cost curve.

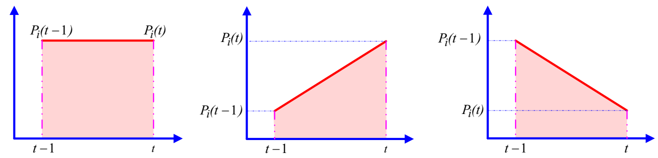

Fig.2 Three feasible conditions of generating unit i .

III. Particle Swarm Optimization (PSO)

Kenedy and Eberhart proposed a particle swarm optimization in 1955. The basic idea of PSO based on food searching of a swarm of animals, such as fish flocking or birds swarm as depicted in Fig 3. Calculating the new velocity and new position of particles can use these below equations.

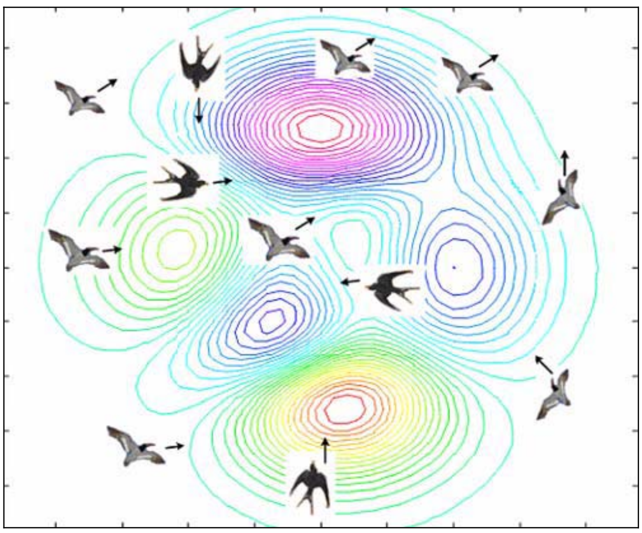

Fig. 3 Food searching of a swarm of birds mimetic the PSO.

- velocities are calculated by using equation below:

$$V_i^{(t+1)} = K \times \begin{pmatrix} \omega \times V_i^{(t)} + \\ c_1 \times r_1 \times (pbest_i - x_i^{(t)} + \\ c_2 \times r_2 \times (gbest - x_i^{(t)}) \end{pmatrix} \quad (11)$$

$$K = \left| \frac{2}{2 - c - \sqrt{c^2 - 4 \times c}} \right| \quad (12)$$

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} \times t \quad (13)$$

- new particles position is calculated using equation below:

$$x_i^{(t+1)} = x_i^{(t)} + V_i^{(t+1)}, \; i = 1,2,..,n \quad (14)$$

where V_i^t is velocity of particle i at iteration t, α is constriction factor, t is number of iterations, ω is inertia or weighting factor, c_1 and c_2 are accelerating factor, r_1 and r_2 are positive random number between 0 and 1, $pbest_i$ is the best position of particle i, $gbest$ is the best position of the group, $\omega_{\max}$ and $\omega_{\min}$ are minimum and maximum of inertia weight factor, $iter_{\max}$ is maximum iteration, $V_i^{\max}$ and $V_i^{\min}$ are minimum and maximum velocity of particle i, n is number of particles.

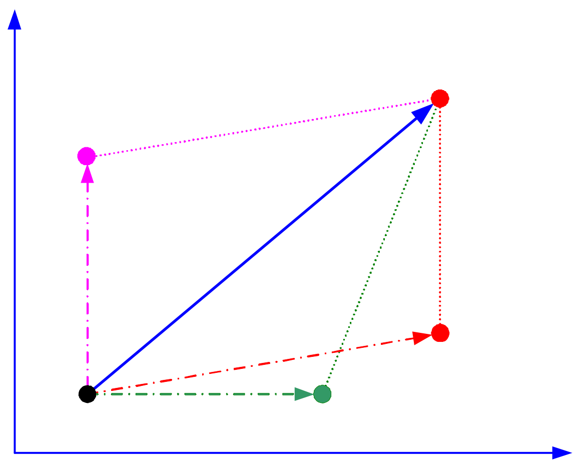

Fig.4 Updating the position mechanism of PSO.

Fig 4 shows the position mechanism of standard PSO in two dimensions.

IV. Mutation Operators

In this section, four scenarios of mutation operators for improving diversity exploration of the standard PSO are proposed. The mutation operators are the one important process of the DE algorithm, which used for generates a mutant vector for crossover scheme. The mutant operators are the distance between the difference populations that multiplied by the constant factor. The scenarios of mutation operators are used for standard PSO express as following.

1) Scenario 1 (PSOM1)

$$V_i^{(t+1)} = SC \times \left((x_k^{(t)} - x_i^{(t)}) - (x_q^{(t)} - x_i^{(t)}) \right) \quad (15)$$

2) Scenario 2 (PSOM2)

$$V_i^{(t+1)} = SC \times \left((x_k^{(t-\beta)} - x_i^{(t)}) - (x_q^{(t-\beta)} - x_i^{(t)}) \right) \quad (16)$$

3) Scenario 3 (PSOM3)

$$V_i^{(t+1)} = SC \times \begin{pmatrix} (x_k^{(t)} - x_i^{(t)}) - (x_q^{(t)} - x_i^{(t)}) \\ - (x_r^{(t)} - x_i^{(t)}) \end{pmatrix} \quad (17)$$

4) Scenario 4 (PSOM4)

$$V_i^{(t+1)} = SC \times \begin{pmatrix} (x_k^{(t-\beta)} - x_i^{(t)}) - (x_q^{(t-\beta)} - x_i^{(t)}) \\ - (x_r^{(t-\beta)} - x_i^{(t)}) \end{pmatrix} \quad (18)$$

where SC is a real number between 0.1 and 2 and called scaling factor, which controls the amplification of differences populations for escape the local solutions, β is previous iteration that user defined, k, q and r are random index of particles, randomly chose from population set and $k \neq q \neq r$.

The mutation operators of scenario 1 and scenario 2 are calculated by using two difference populations and the index of iteration. The scenario 1 does use the current iteration index, the scenario 2 does use the previous iteration index for selection the pair of the two difference populations. Scenario 3 and 4 calculates the mutation operator by using the three difference populations. Like scenario 1 and 2, scenario 3 and 4 are utilizing the current iteration and previous iteration index.

V. The PSOM Solve the ED Problem

The computational processes of PSOM apply for solving the ED problem describes as follow.

Step1) Set iteration = 0, initialized particles and must be satisfied all constraints, the objective function is calculated, the best particle i is set as $pbest_i$ and the best particle of all particles is set as $gbest$.

Step 2) Calculate the velocities by using equations (6-8) these velocities must be according to all constraints. Ones velocity is out of boundary or closely to zero, a mutation operator is activated, recalculate the velocity of this particle by using mutation operator scenarios (PSOM1, PSOM2, PSOM3 or

PSOM4). Adjust the position of particles by using equation (14).

Step 3) Calculate the objective function of adjusted position of all particles in step 2, particle i which has yield the best generation cost than previous position is set as $pbest_i$, the particle which has yield the best generation cost of all searching iterations is set as $gbest$.

Step 4) Increased the iteration, Iteration = Iteration + 1.

Step 5) If the stopping criteria is true, go to Step 6. Otherwise go to Step 2. Generally, the stopping criterions are the objective function value can not improve for along time and the iteration reach to the maximum iteration.

Step 6) The $gbest$ at maximum iteration is the best particle which has yield the optimum objective function value and satisfying all the constraints.

VI. Simulation results and comparisons

In this section, to demonstrate the effectiveness of the proposed method, the PSOMs are applied to solve the six thermal units with considers the prohibited operating zones of the ED problem. The simulation results are compared with various methods reported in literatures, such as the PSO [3], GA [3], CPSO [7], AIS [8], MTS [9] and the bees algorithm (BA) [18]. The PSOM, PSO, TSA, GA and BA are implemented in MATLAB language and executed on an Intel(R) Core2 Duo 3.0 GHz personal computer with a 4.0 GB of RAM. The parameters of the PSOMs such as c_1 and c_2 are set as $2.05, K = 0.729$, $\omega_{\min}$=0.4, $\omega_{\max}$=0.9, PSOM1 has SC= 1.0, PSOM2 has SC= 0.2, PSOM3 and PSOM4 have SC= 0.3.

TABLE I
The ramp rate limits and prohibited operating zones of the six generator system

Unit	$P_{\max}$ (MW)	$P_{\min}$ (MW)	P_0	UR	DR	Prohibited zones (MW)	
1	500	100	440	80	120	[210-240]	[350-380]
2	200	50	170	50	90	[90-110]	[140-160]
3	300	80	200	65	100	[150-170]	[210-240]
4	150	50	150	50	90	[80-90]	[110-120]
5	200	50	190	50	90	[90-110]	[140-150]
6	120	50	150	50	90	[75-85]	[100-105]

$$B_{ij} = 1\times10^{-1} \times \begin{bmatrix} 0.017 & 0.012 & 0.007 & -0.001 & -0.005 & -0.002 \\ 0.012 & 0.014 & 0.009 & 0.001 & -0.006 & -0.001 \\ 0.007 & 0.009 & 0.031 & 0 & -0.010 & -0.006 \\ -0.001 & 0.001 & 0 & 0.024 & -0.006 & -0.008 \\ -0.005 & -0.006 & -0.010 & -0.006 & 0.129 & -0.002 \\ -0.002 & -0.001 & -0.006 & -0.008 & -0.002 & 0.150 \end{bmatrix} \tag{19}$$

$$B_{oi} = [-0.00391 \quad -0.0013 \quad 0.007047 \quad 0.000591 \quad 0.002161 \quad -0.00664] \tag{20}$$

$$Boo = 0.056 \tag{21}$$

TABLE II
Comparison of the best results

Methods	P1 (MW)	P2 (MW)	P3 (MW)	P4 (MW)	P5 (MW)	P6 (MW)	Total cost ($/h)
GA [3]	474.81	178.64	262.21	134.28	151.90	74.18	15 459.0
PSO [3]	447.50	173.32	263.47	139.06	165.48	87.13	15 450.0
CPSO [7]	434.43	173.32	274.47	128.06	179.48	85.93	15 446.0
AIS [8]	458.29	168.05	262.52	139.06	178.39	69.34	15 448.0
MTS [9]	449.37	182.25	254.29	143.45	161.97	86.02	15 451.6
TSA	451.73	185.23	260.93	133.10	171.08	73.51	15 449.2
BA	438.65	167.90	262.82	136.77	171.76	97.67	15 445.9
PSO	444.24	170.83	254.68	141.32	173.04	91.36	15 446.1
GA	438.42	178.99	270.88	131.59	166.55	89.20	15 446.6
PSOM1	451.36	174.21	257.36	137.05	165.15	90.36	15 444.8
PSOM2	444.72	172.37	260.50	144.86	167.71	85.23	15 444.5
PSOM3	450.08	170.83	270.00	129.01	166.99	88.76	15 444.9
PSOM4	447.77	178.19	256.46	134.75	171.63	86.80	15 444.9

TABLE III
PERFORMANCE PARAMETERS COMPARISON

Methods	Cost ($/h)			Average	Standard
	Min.	Average.	Max.	CPU time (s)	deviation
GA [3]	15 459.00	15 469.00	15 469.00	41.58	-
PSO [3]	15 450.00	15 454.00	15 492.00	14.86	-
CPSO [7]	15 446.00	15 449.00	15 490.00	8.13	-
AIS [8]	15 448.00	15 459.70	15 472.00	NA	-
MTS [9]	15 450.06	15 451.17	15 453.64	5.98	0.93
TSA	15 449.20	15 495.82	15 632.14	18.97	35.10
BA	15 445.87	15 448.83	15 452.92	5.64	1.56
PSO	15 446.06	15 450.35	15 463.19	2.06	2.88
GA	15 446.55	15 451.55	15 480.94	25.31	5.47
PSOM1	15 444.79	15 448.07	15 453.78	1.26	1.57
PSOM2	15 444.45	15 448.07	15 449.98	0.99	1.45
PSOM3	15 444.93	15 447.93	15 453.32	0.84	1.43
PSOM4	15 444.88	15 448.03	15 449.94	0.78	1.47

Table I shows the data of the test system, equation (19)-(21) are the loss coefficients of this case. The best results are obtained from the PSOMs' and others method compared in Table II. The results show that the proposed approaches have high solution quality than others method as depicted.

Table III shows the effectiveness in term of the solution quality among 100 trials of proposed methods. The solutions of the proposed methods higher quality than the rest methods in term of minimum cost, average cost, maximum cost, computational time and solution deviation. Fig. 5 shows the profiles of the solutions obtained from running of 100 different trials of the proposed approaches. This paper demonstrates the tuning of scaling factors. Fig. 6 shows the variation of scaling factors from 0.1 to 1.0 versus generation cost. Fig. 7 shows the effect of scaling factor to standard deviation of generation cost. Fig. 8 demonstrates the computation time depend on the scaling factors.

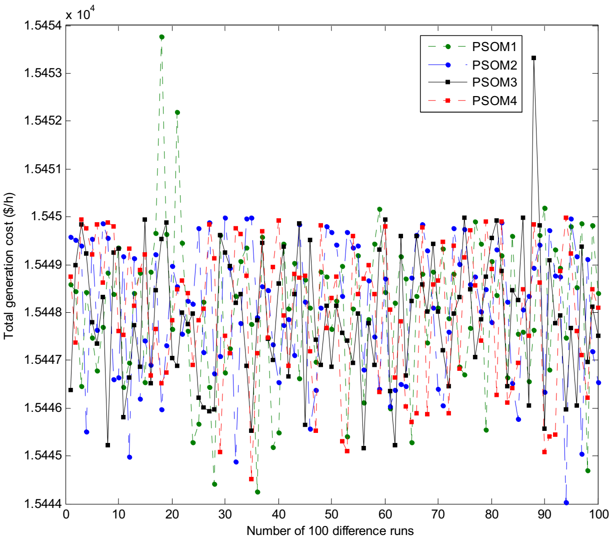

Fig.5 100 solutions profile of PSOM, PSO, TSA, GA and BA.

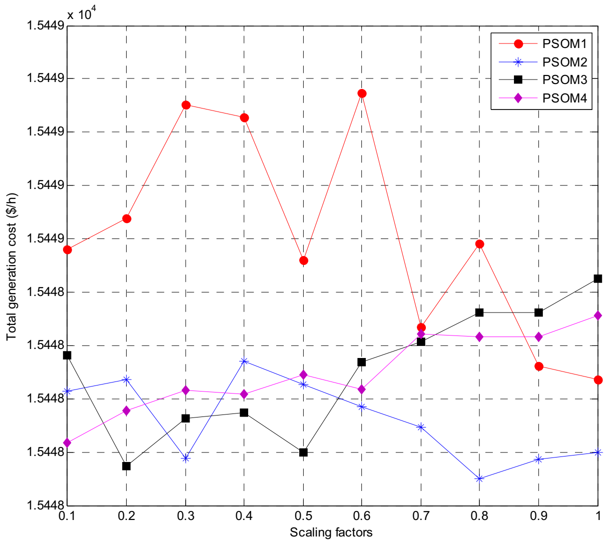

Fig.6 variation of scaling factors versus generation cost.

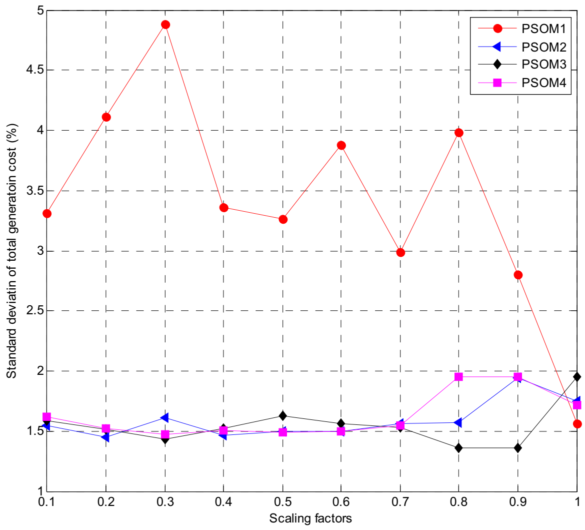

Fig.7 variation of scaling factors versus standard deviation.

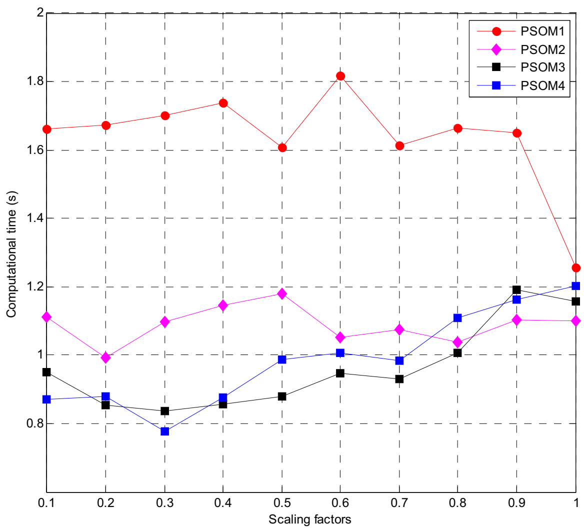

Fig.8 variation of scaling factors versus computational time.

VII. Conclusions

The developments of the original PSO for solving the ED problem with the generator constraints by using mutation operators are presented. Four scenarios of mutation operators are introduced for performances enhancing of the PSO in term of increasing diversity exploration. Ones scenario will have been activated if particle's velocity slides out of boundary or nearly to zero. The effectiveness of the proposed approaches is compared with other approaches such as PSO, TSA, GA, BA and methods reported in literatures. The results show that PSOMs' had the best solutions quality in term of minimum generation cost and mean generation cost. The proposed approaches can converge to the minimum generation cost faster than the rest approaches.

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