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Real-Time Compensation of Instrument Transformer Dynamics Using Frequency-Domain Interpolation Techniques

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Abstract— **The dynamics of current and voltage transformers often limit the accuracy of measurements in high-voltage power systems. These dynamics may not be well known, but frequency-response data obtained on the instrument transformer may be available for a finite set of frequencies. A compensating digital filter that approximately inverts the instrument transformer's frequency response is calculated using frequency-domain interpolation techniques. The Power Quality Analyser makes use of the compensating filter technique for improved accuracy of its real-time power quality calculations and analyses.**

I. Introduction

THE effect of field instrument transformers on harmonics, power, power factor, unbalance and flicker measurements accuracy has been reviewed as part of the development of a new real-time Power Quality Analyser (PQA) for arc furnace operations and utilities. The overall measuring system performance is determined in part by the accuracy of all the transducers and transformers in the signal path.

The main components of a typical measuring system as shown in Fig. 1 are:

- field instrument transformers
- step-down voltage transformers (VTs) and current transformers (CTs) used to interface the sampling device with the field transformers without interrupting production
- sampling device and real-time analysis system

Accuracy requirements and specifications of field transducers and instrument transformers were discussed in [1]. In particular, it was pointed out that at frequencies higher than 60 Hz, the CTs and VTs may introduce gain and phase errors. Therefore, the measurement of higher system harmonics may be inaccurate. This paper describes a technique to compensate for the effects of instrument transformers dynamics in a computer-based power quality measurement system, the PQA.

The real-time compensation technique treated here was first introduced in [1]. A detailed procedure for compensating digital filter design is given. The dynamics of CTs and VTs may not be well-known, but frequency-response (FR) data obtained on them may be available for a finite set of frequencies. Compensating filters that approximately invert the instrument transformers frequency responses are calculated using frequency-domain interpolation techniques. The main idea is to interpolate the inverse of the complex FR data with a proper, stable transfer function on the imaginary axis. Then this transfer function can be implemented as a digital filter so that approximate cancellation takes place and the resulting combined FR is close to unity magnitude and zero phase in the frequency band of interest. The interpolating transfer function is mapped into the z-domain via the bilinear transformation with the appropriate sampling period. The resulting discrete-time filter can be implemented as a difference equation or as a first-order, discrete-time state-space matrix equation in the measuring system's software performing real-time computations.

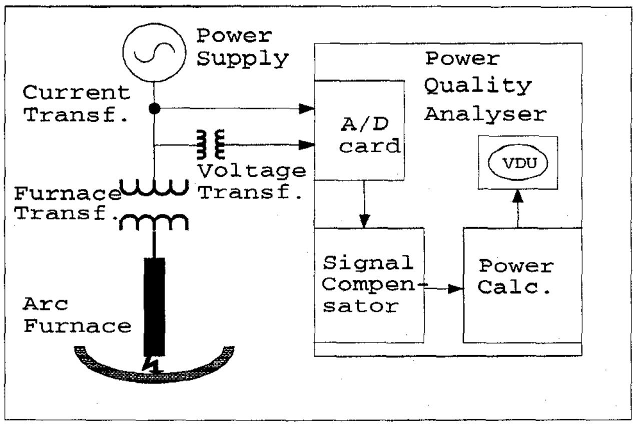

Fig. 1. Diagram of a Typical Power Quality Measurement System

Nevanlinna-Pick interpolation theory is described and used to invert the frequency response of the instrument transformer at frequencies of interest in the measured periodic signal, such as significant harmonic frequencies.

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The calculated interpolating filter is guaranteed to be stable. In case its order is too large for practical implementation, model reduction techniques such as Moore's balanced truncation [2] or frequency-weighted optimal Hankel-norm approximation [3], [4] can be used to reduce the order of the filter. The exact inversion property at the sample frequencies would then be lost, but lower-order filters with reasonably accurate frequency responses may nevertheless be obtained.

The Power Quality Analyser developed by Hatch Associates Ltd. makes use of the compensating filter technique for improved accuracy of its calculations, including real-time harmonics, power, unbalance, and flicker data calculations. The PQA VTs' frequency responses needed for filter design were measured. Experimental sinusoidal voltage data measured with the PQA are used to compare results with and without the VT compensating filters. It is shown that the digital filters can significantly reduce the measurement errors.

II. Dynamics of Instrument Transformers and Antialiasing Filters

The dynamics of instrument transformers can be characterized by frequency-response data obtained from equivalent circuit models or direct measurements. To develop an accurate compensator, the complex FR of the instrument transformer must be obtained. One way to get the FR is to measure it directly at N frequencies. For instance, define the complex FR data points obtained on an instrument transformer as:

$$\beta_i := \frac{\nu_{out}(j\omega_i)}{\nu_{in}(j\omega_i)}, \ \ i = 1, \ldots, N, \tag{1}$$

where the input and output signals (currents or voltages) $\nu_{in}(j\omega_i)$ and $\nu_{out}(j\omega_i)$ are complex-valued.

Analog antialiasing filters are used at the inputs of the PQA to attenuate high-frequency noise corrupting the signals before sampling. The magnitudes of these 4^{th}-order elliptic antialiasing filters are quite flat in the passband, but their phases are significant. Elliptic filters have standard FRs; it is easy to compute the data points at the frequencies ω_i's:

$$\psi_i := F(j\omega_i), \ \ i = 1, \ldots, N, \tag{2}$$

where $F(s)$ is the filter's transfer function.

Finally, the A/D cards used in the PQA exhibit a short delay of typically 10 μs between each channel sampled in burst mode. Ideally, all channels would be sampled at the same time. Since we want to bring all channel samples to the same sampling instant, these delays in sampling can be thought of as *time advances*. For example, if sampling starts at channel 0, then channel 1, and assuming both channels sample the same signal, then if both points are to represent the signal at the exact same sampling instant, the sampled signal of channel 1 will appear to *lead* the sampled signal of channel 0. Hence, digital compensation can be achieved by introducing a delay of 10 μs on channel 1. In light of the above, the FR data at frequencies $\{\omega_i\}_{i=1}^N$, corresponding to a 10 μs advance, can be defined as:

$$\delta_i := e^{j10^{-5}\omega_i}, \ \ i = 1, \ldots, N. \tag{3}$$

All of these data points can be combined to form a global FR of the signal path from its source to the input of the A/D card:

$$\phi_i := \beta_i \psi_i \delta_i, \ \ i = 1, \ldots, N. \tag{4}$$

Our goal is to build a stable compensating filter whose FR interpolates the points $\{\phi_i^{-1}\}_{i=1}^N$ and is "nice" and "smooth". Since we are dealing with an AC power supply, frequencies of interest will be its fundamental frequency (e.g., 60 Hz in North America, 50 Hz in Europe) and its harmonic frequencies.

III. Compensating Filter Design using Nevanlinna-Pick Interpolation

A. Problem Formulation

One way to design filters that interpolate given FR data points is to set up a *model-matching* problem and to use Nevanlinna-Pick (NP) interpolation theory to solve it. The model-matching problem is standard in $\mathcal{H}_\infty$ control theory as it arises in the design of $\mathcal{H}_\infty$-optimal controllers [5]. The model-matching problem is also ubiquitous in broadband equalization [6].

We want to design a stable discrete-time filter that interpolates the inverse of the FR data $\{\phi_i^{-1}\}_{i=1}^N$ at the complex points on the unit circle (assuming the bilinear transformation is used): $\left\{e^{j\theta_i} = \frac{1+\frac{T_s}{2}j\omega_i}{1-\frac{T_s}{2}j\omega_i} : i = 1, \ldots, N\right\}$, where T_s is the sampling period. We will first design a continuous-time filter interpolating ϕ_i^{-1} at $\alpha + j\omega_i$ in the open right-half plane for a given $\alpha > 0$, shift this filter to the left in the complex plane by α so that interpolation is now on the imaginary axis, and finally map it to a discrete-time filter via the bilinear transformation. Define $\mathcal{H}_\infty$ to be the space of stable transfer functions bounded on the $j\omega$-axis. Let $\mathcal{RH}_\infty$ and $\mathcal{CH}_\infty$ be the subspaces of $\mathcal{H}_\infty$ composed of real-rational and complex-rational functions respectively. The ∞-norm of a function g in any of these spaces is defined as its maximum gain on the $j\omega$-axis:

$$\|g(s)\|_\infty := \sup_{w \in \mathbb{R}} |g(j\omega)|. \tag{5}$$

Let $g(s) \in \mathcal{RH}_\infty$ be the filter to be designed, $m(s) \in \mathcal{RH}_\infty$ be a "model" transfer function with desired smoothness properties on the imaginary axis and that roughly interpolates the data pairs $(j\omega_i, \phi_i^{-1})$, and $w(s)$, $w^{-1}(s) \in \mathcal{RH}_\infty$ be a weighting function. Then the problem to be solved can be formulated as follows:

Problem I: Design a filter $g(s) \in \mathcal{RH}_\infty$ such that
(i) $g(j\omega_i) = \phi_i^{-1}, \ i = 1, \ldots, N$ and
(ii) $\|w(g-m)\|_\infty$ is minimized.

The minimization of $\|w(g-m)\|_\infty$ implies that the resulting $g(s)$ will be close to $m(s)$ on the $j\omega$-axis, especially at those frequencies where $|w(j\omega)|$ is large. Boundary NP interpolation theory provides a solution to this problem in $\mathcal{CH}_\infty$, and we will see later how to use this solution to compute a function in $\mathcal{RH}_\infty$ satisfying the conditions of Problem I.

Problem I can be recast into the following simplified problem which is very close to the classical NP interpolation problem:

Problem II: Find a function $q(s) \in \mathcal{CH}_\infty$ such that
(i) $q(j\omega_i) = w(j\omega_i)[\phi_i^{-1} - m(j\omega_i)], \ i = 1, \ldots, N$ and
(ii) $\|q\|_\infty$ is minimized.

Once a $q(s) \in \mathcal{CH}_\infty$ solving Problem II has been computed, it is a simple matter to obtain a real-rational function $q_R(s) \in \mathcal{RH}_\infty$ that also satisfies conditions (i) and (ii). This is done by computing the "real part" of $q(s)$ as follows:

$$q_R(s) = \frac{1}{2}[q(s) + q(s^*)^*], \tag{6}$$

where for a complex number s, s^* denotes its conjugate (conjugate transpose for matrices). If we let (A, B, C, D) be a state-space realization of $q(s)$, then $q_R(s)$ is given by:

$$q_R(s) = \mathrm{Re}\{D\} + [C \ B^*]\left(sI - \begin{bmatrix} A & 0 \\ 0 & A^* \end{bmatrix}\right)^{-1} \begin{bmatrix} B \\ C^* \end{bmatrix}. \tag{7}$$

The state-space realization in (7) is still complex, but the resulting transfer function has real coefficients. A real state-space realization can be computed if desired. Once $q_R(s)$ has been calculated, it is a simple matter to get $g(s) = w^{-1}(s)q_R(s) + m(s)$. The resulting $g(s)$ is in $\mathcal{RH}_\infty$ as required. In general, the order of $g(s)$ will be less than or equal to $4N + order[w(s)] + order[m(s)]$.

B. Solution Using NP Interpolation Theory

The classical NP interpolation problem in the right-half plane scaled with a constant $\gamma > 0$ can be formulated as follows (classical result is for $\gamma = 1$):

Problem III: Given a set of M complex numbers $\{\gamma^{-1}b_i \in \mathbb{C} : |\gamma^{-1}b_i| \leq 1, i = 1, \ldots, M\}$ and M distinct complex numbers in the open right-half plane $\{a_i \in \mathbb{C} : Re\{a_i\} > 0, i = 1, \ldots, M\}$, Find an interpolating function $q(s) \in \mathcal{CH}_\infty$ such that
(i) $q(a_i) = \gamma^{-1}b_i, \ i = 1, \ldots, M$ and
(ii) $\|q\|_\infty \leq 1$.

Pick's famous theorem provides a simple way to check if Problem III has a solution. Define the Hermitian matrices

$$A := \left[\frac{1}{a_i + a_j^*}\right]_{i,j=1,\ldots,M}, \quad B := \left[\frac{b_i b_j^*}{a_i + a_j^*}\right]_{i,j=1,\ldots,M}$$

Theorem 1: There exists an interpolating function satisfying Problem III if and only if the Pick matrix $Q := A - \gamma^{-2}B$ is positive semidefinite.

A well-known related result [5] states that the minimum achievable γ, call it γ_{opt}, such that Problem III has a solution is equal to the square root of the largest eigenvalue of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$, where $A^{-\frac{1}{2}}$ is the positive definite inverse of $A^{\frac{1}{2}}$ which satisfies $A = A^{\frac{1}{2}}A^{\frac{1}{2}}$. The corresponding function $\gamma_{opt}q(s)$ is the minimum-norm function interpolating the b_i's.

When Q is positive semidefinite, one can construct an interpolating function q using Nevanlinna's algorithm [5], [7]. But before this algorithm can be described, a few definitions are in order. Let $\mathcal{D}$ be the open unit disk and $\partial\mathcal{D}$ be the unit circle. A *Möbius function* has the form

$$M_b(z) := \frac{z - b}{1 - zb^*}, \quad \text{where } |b| < 1.$$

Some properties of M_b are: $M_b \in \mathcal{CH}_\infty$; $|M_b(z)| = 1$ on $\partial\mathcal{D}$; M_b maps $\mathcal{D}$ onto $\mathcal{D}$ and $\partial\mathcal{D}$ onto $\partial\mathcal{D}$; and $M_b^{-1} = M_{-b}$. An *all-pass function* is defined as follows:

$$A_a(s) := \frac{s - a}{s + a^*}, \quad \mathrm{Re}\{a\} > 0.$$

Note that for every ω, $|A_a(j\omega)| = 1$.

Nevanlinna's Algorithm

The algorithm presented here is adapted from [5]. It is based on the fact that for $M = 1$ with the data a and b, all solutions are given by

$$\{q(s) \ : \ q(s) = M_{-b}[q_1(s)A_a(s)], \ q_1 \in \mathcal{CH}_\infty, \|q_1\|_\infty \leq 1\}, \tag{8}$$

and the case of M points reduces to the case of $M - 1$ points. Mathematical induction can then be used to give a recursive solution to the general case of M data points. The algorithm consists of two stages. The first stage can be looked at as data formatting for successive transformed problems with 1 point, then 2 points, etc., up to M points to be solved, and the second stage uses these data to solve the problems until the solution emerges with the M data points. Here is a succinct, but complete description of the algorithm.

Stage 1
Let $b_j^0 := b_j$, the original data points. Compute the array

of complex numbers

$$\begin{array}{ccccc} b_1 & b_2 & b_3 & \cdots & b_M \\ & b_2^1 & b_3^1 & \cdots & b_M^1 \\ & & b_3^2 & \cdots & b_M^2 \\ & & & \ddots & \vdots \\ & & & & b_M^{M-1} \end{array}$$

as follows:

$$b_j^i = \frac{M_{b_i^{i-1}}(b_j^{i-1})}{A_{a_i}(a_j)}, \quad i = 1, \ldots, M-1, \; j = i+1, \ldots, M \,. \tag{9}$$

Stage 2
Pick $q_M(s) \in \mathcal{CH}_\infty$ to be a "seed" function with $\|q_M\|_\infty \leq 1$. Then compute successively

$$\begin{aligned} q_{M-1}(s) &= M_{-b_M^{M-1}}[q_M(s)A_{a_M}(s)]\,, \\ q_{M-2}(s) &= M_{-b_{M-1}^{M-2}}[q_{M-1}(s)A_{a_{M-1}}(s)]\,, \\ &\vdots \\ q_0(s) &= M_{-b_1^0}[q_1(s)A_{a_1}(s)]. \end{aligned}$$

The solution to Problem III is $q(s) = q_0(s)$.

We can now detail the design procedure to obtain digital compensating filters.

Filter Design Procedure

Step 0: Select a "model" function $m(s) \in \mathcal{RH}_\infty$ that approximately interpolates the points $\{\phi_i^{-1}\}_{i=1}^N$ on the $j\omega$-axis and that has a smooth FR with desired properties. Then select a weighting function $w(s) \in \mathcal{RH}_\infty$ such that its inverse is stable and proper, and whose magnitude on the $j\omega$-axis appropriately weights regions where deviation between the FRs of the filter to be designed and $m(s)$ should be small.

Step 1: Measure the FRs, compute the ϕ_i's and define the complex numbers

$$\rho_i := w(j\omega_i)[\phi_i^{-1} - m(j\omega_i)]\,.$$

Select a positive number α, let $\omega_i := -\omega_{i-N}$ for $i = N+1, \ldots, 2N$ and then form the sets

$$\begin{aligned} \mathcal{A} &:= \{a_i := \alpha + j\omega_i \,:\, i = 1, \ldots, 2N\}\,, \\ \mathcal{B}_0 &:= \{b_i' \,:\, b_i' = \rho_i,\, i = 1, \ldots, N, \\ &\qquad b_i' = \rho_{i-N},\, i = N+1, \ldots, 2N\}\,. \end{aligned}$$

Let $M := 2N$.

Step 2: Compute the minimum norm γ_{opt} achievable in Problem III with the data in $\mathcal{A}$ and $\mathcal{B}_0$. Then, scale the data in $\mathcal{B}_0$ and define a new set $\mathcal{B}$ as follows:

$$\mathcal{B} := \left\{b_i \,:\, b_i = \gamma_{opt}^{-1} b_i',\, b_i' \in \mathcal{B}_0\right\}\,.$$

Step 3: Use Nevanlinna's algorithm with the data sets $\mathcal{A}$ and $\mathcal{B}$ to compute a proper, stable complex-rational interpolating function $q(s)$. Note that $q(s)$ interpolates the elements of $\mathcal{B}$ at the a_i's on the line $\mathrm{Re}\{s\} = \alpha$ which lies in the right-half plane.

Step 4: Compute the real-rational function $q_R(s)$ corresponding to $q(s)$ using (7). Shift the function to the left in the complex plane: $\hat{q}_R(s) := q_R(s+\alpha)$. Note that by the maximum modulus theorem, $\|\hat{q}_R(s)\|_\infty \leq \|q_R(s)\|_\infty$. Moreover, the proper, stable, real-rational function $\hat{q}_R(s)$ interpolates ρ_i at $\pm j\omega_i$ and all of its poles lie to the left of the line $\mathrm{Re}\{s\} = -\alpha$.

Step 5: Compute $g(s) = w^{-1}(s)q_R(s) + m(s)$. This is the full-order continuous-time filter's transfer function. If the order is not a concern, discretize the filter using, e.g., the bilinear transformation:

$$f(z) := g(s)|_{s=\frac{2}{T_s}\frac{z-1}{z+1}}\,.$$

Step 6: If the order of $g(s)$ is too high, use a model reduction technique until a satisfactory tradeoff between model order and fit of the FR on the data is achieved. Then discretize the filter as in Step 5.

IV. Experimental Results with the Power Quality Analyser

A. Filter Design

A frequency-response experiment from 60 Hz to 2100 Hz (35^{th} harmonic) was performed on a 120 V/3.5 V VT used with the PQA. Then, eight points of that FR were selected: at the 60 Hz fundamental frequency, and at its harmonics: 3, 5, 9, 15, 21, 27, 33, to design an NP filter. The FR data points obtained contained the effect of the 10 μs delay between channels 0 and 1 of the 8-channel A/D card used for the experiment. It was found that the VT has a flat magnitude over the frequency band of interest, but it has a positive phase of up to four degrees around 2000 Hz. Antialiasing filters were not yet available when this research was conducted.

The digital filter was implemented for channel 7 of the PQA which measured the output voltage of the VT, while the input signal to the VT was directly measured on channel 0. Hence, an additional 60 μs advance between channels 1 and 7 on the A/D card accounted for the burst mode rate.

The combined PT FR data and phase lead effect of the burst mode rate were then used to design an NP compensating filter using the procedure outlined above. The selected model and weighting functions were:

$$m(s) = \frac{34.72}{(\frac{1}{40000}s+1)(\frac{1}{30000^2}s^2 + \frac{2\cdot 0.7}{30000}s+1)}\,, \tag{10}$$

$$w(s) = \frac{.00001s+10}{.0001s+1}\,. \tag{11}$$

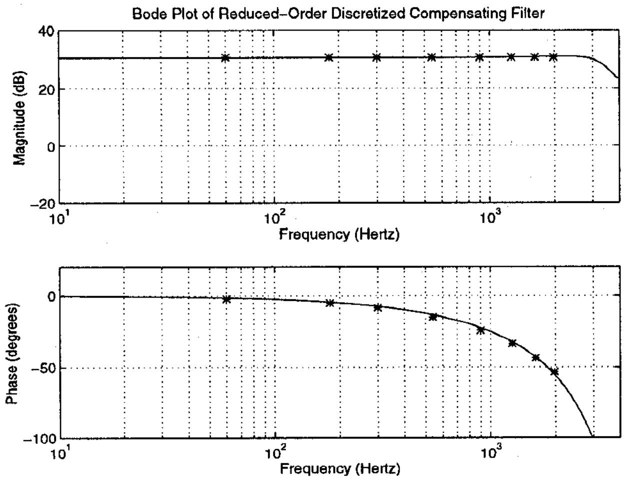

Fig. 2. Bode Plot of Reduced 4^{th}-order Discrete-Time Compensating Filter

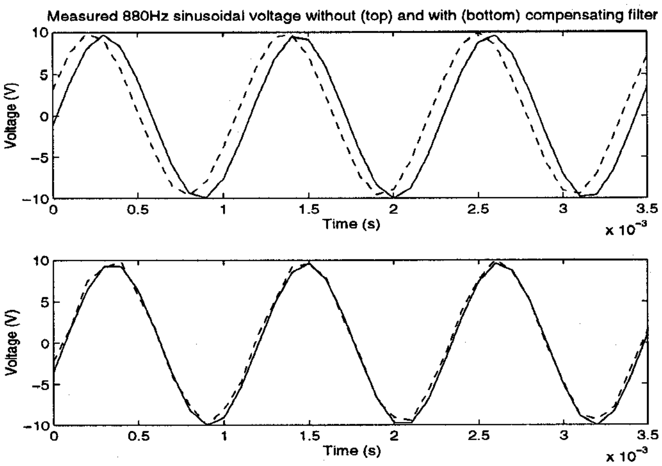

Fig. 3. Measured 880 Hz sinusoidal input and output of VT without and with filtering (dashed line: output of VT)

Then, Steps 1 to 4 of the design procedure were carried out in Matlab^{TM} using the μ-Analysis and Synthesis Control Toolbox [8]. The resulting continuous-time real-rational filter $g(s)$ was of the 36^{th} order, as expected. In Step 5, a 4^{th}-order filter was obtained by using the frequency-weighted optimal Hankel-norm approximation technique [4]. Such a large reduction in the order was achievable because of the smoothness of the FR data. Finally, this filter was discretized via the bilinear transformation with a sampling frequency of 10,000 Hz and a prewarping frequency of 1800 Hz. Its Bode plot is shown in Fig. 2 along with the interpolated inverse FR data points $\{\phi_i^{-1}\}_{i=1}^{8}$.

B. Experiments

The compensating filter was implemented in the PQA as a state-space difference equation with the "A" matrix in block-diagonal form. A sinusoidal input voltage of frequency 880 Hz (between the 14^{th} and the 15^{th} harmonics) was applied to channel 0 of the PQA and to the VT whose output was connected to channel 7. The signals were sampled and filtered at a sampling rate of 10 kHz in the PQA. Fig. 3 shows a comparison of the unfiltered and filtered digital signals, where the dashed curve is the measured output voltage of the VT, and the solid curve is the input voltage. The time reference is taken to be when channel 0 is sampled. It is seen that the phase lag of the filter shifts the signal to the right and brings it on top of the input signal, as desired.

Another test was conducted using an 1800 Hz sinusoidal voltage (the 30^{th} harmonic). The results in Fig. 4 clearly show the benefits of the compensating filter even for frequencies relatively close to the Nyquist frequency. The phase lead observed in the top plots of Fig. 3 and 4 is detrimental to calculations that are sensitive to phase errors between current and/or voltage signals, such as active and reactive power, and 3-phase system unbalance calculations. An investigation of the sensitivity of different power quality calculations to the dynamics of instrument transformers and data acquisition systems will be presented in a forthcoming paper.

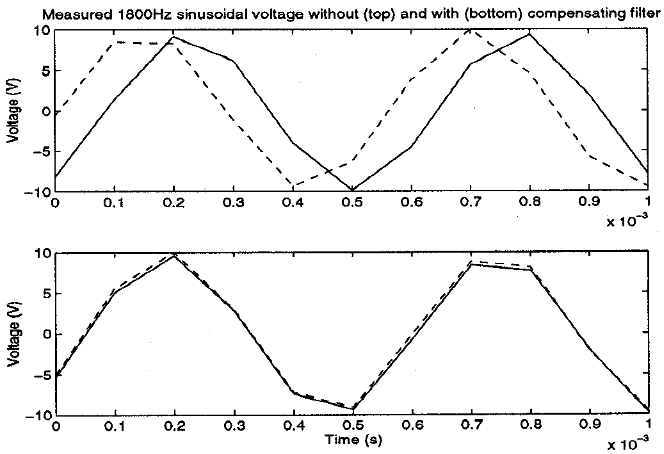

Fig. 4. Measured 1800 Hz sinusoidal input and output of VT without and with filtering (dashed line: output of VT)

V. Conclusion

A frequency-domain compensating filter design technique based on the model-matching problem and Nevanlinna-Pick interpolation theory was presented. Experimental frequency-response data of instrument transformers and sampling devices are used to design real-time digital filters that interpolate the inverted FR data. These filters can compensate for dynamics affecting the signals prior to sampling. An important application for these filters is in high-voltage power quality measurements where small phase or magnitude errors between signals can have a significant effect on computed quality measures. Compensating filters were implemented in the Power Quality Analyser to improve the accuracy of its calculations, which include real-time harmonics, power, unbalance, and flicker.

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