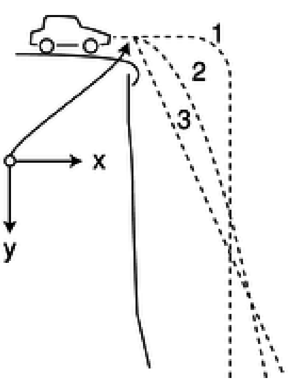
PREDICTING THE DIRECTION OF MOTION

Kinetic energy doesn't depend on the direction of motion. Sometimes this is helpful, as in the high road-low road example (p. 84, example 9), where we were able to predict that the balls would have the same final speeds, even though they followed different paths and were moving in different directions at the end. In general, however, the two conservation laws we've encountered so far aren't enough to predict an object's path through space, for which we need conservation of momentum (chapter 3), and the mathematical technique of vectors. Before we develop those ideas in their full generality, however, it will be helpful to do a couple of simple examples, including one that we'll get a lot of mileage out of in section 2.3.

Suppose we observe an air hockey puck gliding frictionlessly to the right at a velocity v, and we want to predict its future motion. Since there is no friction, no kinetic energy is converted to heat. The only form of energy involved is kinetic energy, so conservation of energy, $\Delta E=0$, becomes simply $\Delta K=0$.

There's no particular reason for the puck to do anything but continue moving to the right at constant speed, but it would be equally consistent with conservation of energy if it spontaneously decided to reverse its direction of motion, changing its velocity to −v. Either way, we'd have ΔK=0. There is, however, a way to tell which motion is physical and which is unphysical. Suppose we consider the whole thing again in the frame of reference that is initially moving right along with the puck. In this frame, the puck starts out withK=0. What we originally described as a reversal of its velocity from v to −v is, in this new frame of reference, a change from zero velocity to −2v, which would violate conservation of energy. In other words, the physically possible motion conserves energy in all frames of reference, but the unphysical motion only conserves energy in one special frame of reference.



r / A car drives over a cliff.

For our second example, we consider a car driving off the edge of a cliff (r). For simplicity, we assume that air friction is negligible, so only kinetic and gravitational energy are involved. Does the car follow trajectory 1, familiar from Road Runner cartoons, trajectory 2, a parabola, or 3, a diagonal line? All three could be consistent with conservation of energy, in the ground's frame of reference.

For instance, the car would have constant gravitational energy along the initial horizontal segment of trajectory 1, so during that time it would have to maintain constant kinetic energy as well. Only a parabola, however, is consistent with conservation of energy combined with Galilean relativity. Consider the frame of reference that is moving horizontally at the same speed as that with which the car went over the edge. In this frame of reference, the cliff slides out from under the initially motionless car. The car can't just hover for a while, so trajectory 1 is out. Repeating the same math as in example 8 on p. 83, we have

$$x{\ast}=0\ ,y{\ast}=12gt2$$

in this frame of reference, where the stars indicate coordinates measured in the moving frame of reference. These coordinates are related to the ground-fixed coordinates (x,y) by the equations

$$x=x{\ast}+vtandy=y{\ast},$$

where v is the velocity of one frame with respect to the other. We therefore have

$$x=vt\ ,y=12gt2,$$

in our original frame of reference. Eliminating t, we can see that this has the form of a parabola:

$$y=g2v2x2.$$

Source: http://physwiki.ucdavis.edu/Fundamentals/02._Conservation_of_Energy/2.1_Energy