POWER DELAY PROFILE

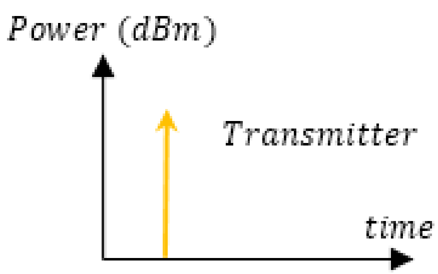
The Power Delay Profile $p(\tau)$ gives the distribution of signal power received over a multipath channel as a function of propagation delays. It is obtained as the spatial average of the complex baseband channel impulse response as

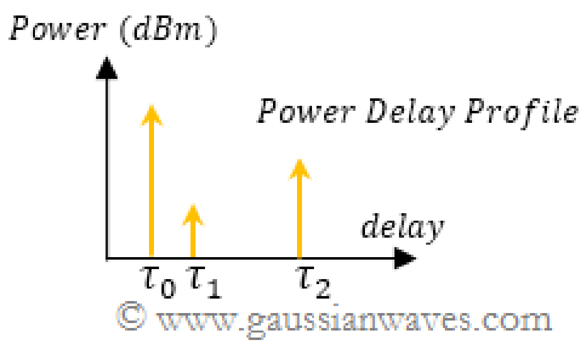
$$p(\tau)=R_{hh}(0,\tau)=E[|h(t,\tau)|^2] \quad (1)$$

As discussed in the previous post, it can also be derived from *scattering function* as given below

$$p(\tau)=\int_{-\infty}^{\infty} S(f,\tau)df \quad (2)$$

Where f denotes Doppler Frequency, τ denotes multipath propagation delay, $S(f,\tau)$ denotes the scattering function. In a Power Delay Profile plot, the signal power of each multipath is plotted against their respective propagation delays. A sample power delay profile plot, shown below, indicates how a transmitted pulse gets received at the receiver with different signal strength as it travels through a multipath channel with different propagation delays (τ_0, τ_1 and τ_2).





Power Delay Profile is usually supplied as a table of values obtained from empirical data and it serves as a guidance to system design. Nevertheless, it is not an accurate representation of the real environment in which the mobile is destined to operate at.

Maximum Excess Delay Definition and applications

With Power Delay Profile, one can classify a multipath channel into frequency selective or frequency non-selective category. The derived parameter, namely, Maximum Excess Delay together with the symbol time of each transmitted symbol, can be used to classify the channel into frequency selective or non-selective channel.

Power Delay Profile can be used to estimated the average power of a multipath channel, measured from the first signal that strikes the receiver to the last signal whose power level is above certain threshold. This threshold is chosen based on receiver design specification and is dependent on receiver sensitivity and noise floor at the receiver.

Maximum Excess Delay, also called Maximum Delay Spread, denoted as T_m, is the relative time difference between the first signal component arriving at the receiver to the last component whose power level is above some threshold. Maximum Delay Spread T_m and the symbol time peroid T_{sym} can be used to classify a channel into frequency selective or non-selective category. This classification can also be done using Coherence Bandwidth (a derived parameter from Spaced Frequency Correlation Function which in turn is the frequency domain representation of power delay profile).

Maximum Excess Delay is also an important parameter in mobile positioning algorithm. The accuracy of such algorithm depends on how well the Maximum Excess Delay parameter conforms with measurement results from actual environment.

When a mobile channel is modeled as a FIR filter (tapped delay line implementation), as in CODIT channel model**[1]**, the number of taps of the FIR filter is determined by the product of maximum excess delay and the system sampling rate.

The Cyclic Prefix in a OFDM system is typically determined by the maximum excess delay or by the RMS delay spread of that environment **[2]**.

Classification of Channel – signal spreading in Time domain:

A channel is classified as Frequency Selective, if the maximum excess delay is greater than the symbol time period, i.e, $T_m > T_{sym}$. This introduces Inter Symbol Interference (ISI) into the signal that is being transmitted, thereby distorting it. This occurs since the signal components (whose power are above the threshold or the maximum excess delay) due to multipath extend beyond the symbol time. ISI can be mitigated at the receiver by an equalizer.

In a frequency selective channel , the channel output can be expressed as the convolution of input signal and the channel impulse response plus some noise.

$$y(t)=h(t,\tau)\circledast x(t)+w(t)(3)$$

On the other hand, if the maximum excess delay is less than the symbol time period, i.e, $T_m < T_{sym}$, the channel is classified as frequency non-selective or "flat" channel. Here, all the scattered signal components (whose power are above the specified threshold or the maximum excess delay) due to the multipath, arrive at the receiver within the symbol time. This will not introduce any ISI, but the received signal is distorted due to inherent channel effects like SNR condition. Equalizers in the receiver are not needed. A time varying non-frequency selective channel is obtained by assuming that the impulse response $h(t,\tau)=h(t)\delta(\tau)$. Thus the output of the channel can be expressed as

$$y(t)=h(t,\tau)\circledast x(t)+w(t)=\int h(t,\tau)x(t-\tau)d\tau+w(t)=\int h(t)\delta(\tau)x(t-\tau)d\tau+w(t)=h(t)x(t)+w(t)(4)$$

Note, that the output of the channel can be expressed simply as product of time varying channel response and the input signal. If the channel impulse response is a deterministic constant, i.e, time in-varying, then the non-frequency selective channel is expressed as follows by assuming $h(t,\tau)=h\delta(\tau)$

$$y(t)=hx(t)+w(t)(5)$$

This is the simplest situation that can occur. In addition to that, if the noise in the above equation is white Gaussian noise, the channel is called ***Additive White Gaussian Noise (AWGN) channel***.

Characterization of Frequency Selective Channels:

Average delay and the RMS delay spread are two most important parameters that characterize a frequency selective channel. They are derived from Power Delay Profile.

Average delay:

Simply the statistical mean of the delay that a signal undergoes when transmitted over a multipath channel. For a frequency selective WSSUS channel, the average delay is equal to the first moment of the power delay profile $p(\tau)$. For a discrete channel it is calculated as

$$\bar{\tau}=\frac{\sum \tau p(\tau)}{\sum p(\tau)} \quad (6)$$

If the given PDP values are continuous in terms of time delays (as given in the sample plot below), replace the summation with integral and integrate it with respect to $d\tau$.

RMS delay spread:

RMS delay spread is equal to the second central moment of power delay profile $p(\tau)$. It is similar to the standard deviation of a statistical distribution. For a discrete channel it is given by

$$\sigma_{\tau}=\sqrt{\overline{\tau^2}-(\bar{\tau})^2} \quad (7)$$

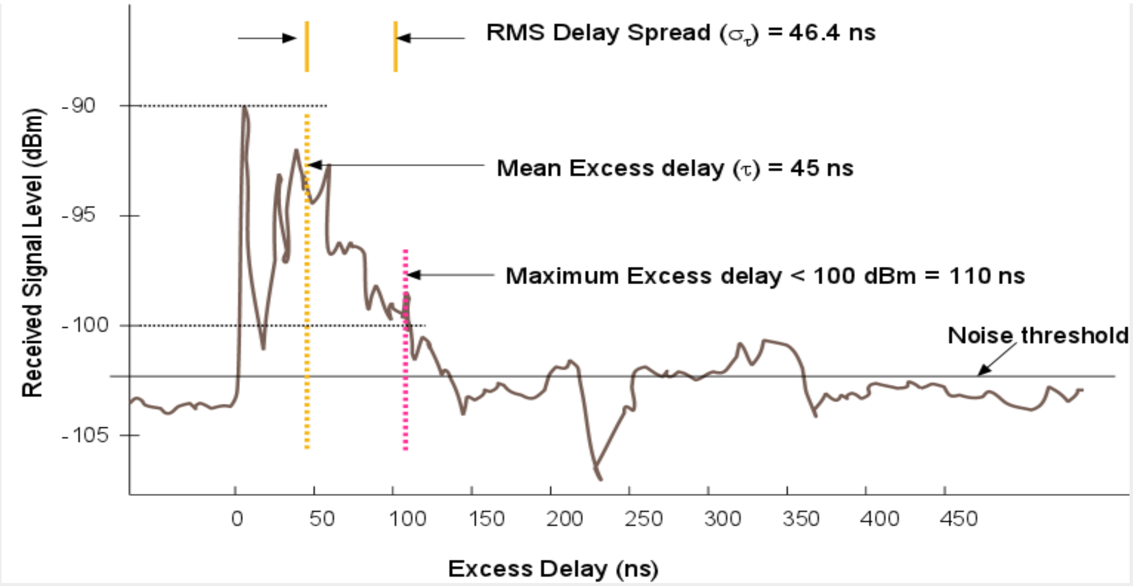
where,

$$\overline{\tau^2}=\frac{\sum \tau^2 p(\tau)}{\sum p(\tau)} \quad (8)$$

If the given PDP values are continuous in terms of time delays (as given in the sample plot below), replace the summation with integral and integrate it with respect to $d\tau$.

The ratio of RMS delay spread and symbol time duration quantifies the strength of Inter Symbol Interference. This ratio determines the complexity of the equalizer required at the receiver. Typically, when the symbol time period is greater than 10 times the RMS delay spread, no ISI equalizer is needed in the receiver.

A sample power delay profile is plotted below. Average delay, RMS delay spread, and the maximum excess delay (given a threshold of 100 dBm) are all marked.



Power Delay Profile with Mean delay, RMS delay spread, Maximum Excess Delay

Source: http://www.gaussianwaves.com/2014/07/power-delay-profile/