PHASE SPACE ENTROPY

There is a problem with making this description of entropy into a mathematical definition. The problem is that it refers to the number of possible states, but that number is theoretically infinite. To get around the problem, we coarsen our description of the system. For the atoms in figure a, we don't really care exactly where each atom is. We only care whether it is in the right side or the left side. If a particular atom's left-right position is described by a coordinate x, then the set of all possible values of x is a line segment along the x axis, containing an infinite number of points. We break this line segment down into two halves, each of width Δx, and we consider two different values of x to be variations on the same state if they both lie in the same half. For our present purposes, we can also ignore completely the y and zcoordinates, and all three momentum components, px, py, and pz.

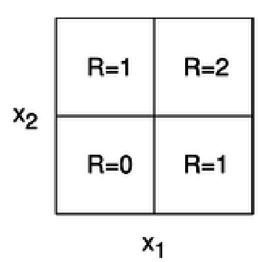


Figure d: The phase space for two atoms in a box.

Now let's do a real calculation. Suppose there are only two atoms in the box, with coordinates x1 and x2. We can give all the relevant information about the state of the system by specifying one of the cells in the grid shown in figure d. This grid is known as the *phase space* of the system. The lower right cell, for instance, describes a state in which atom number 1 is in the right side of the box and atom number 2 in the left. Since there are two possible states with R=1 and only one state with R=2, we are twice as likely to observe R=1, and R=1 has higher entropy than R=2.

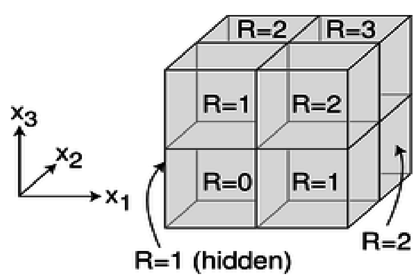
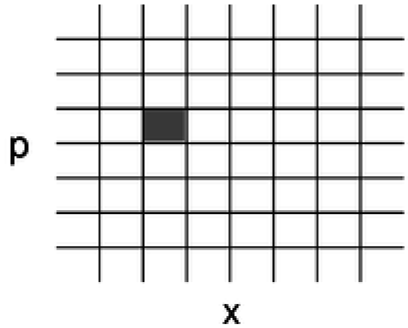


Figure e: The phase space for three atoms in a box.

Figure e shows a corresponding calculation for three atoms, which makes the phase space three-dimensional. Here, the R=1 and 2 states are three times more likely than R=0 and 3. Four atoms would require a four-dimensional phase space, which exceeds our ability to visualize. Although our present example doesn't require it, a phase space can describe momentum as well as position, as shown in figure f. In general, a phase space for a monoatomic gas has six dimensions per atom (one for each coordinate and one for each momentum component).



f / A phase space for a single atom in one dimension, taking momentum into account.

Source:
http://physwiki.ucdavis.edu/Fundamentals/05._Thermodynamics/5.4_Entropy_As_a_Microscopic_Quantity