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Performance Analysis of the Robust Least Squares Target Localization Scheme using RDOA Measurements

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Abstract – A practical recursive linear robust estimation scheme is proposed for target localization in the sensor network which provides range difference of arrival (RDOA) measurements. In order to radically solve the known practical difficulties such as sensitivity for initial guess and heavy computational burden caused by intrinsic nonlinearity of the RDOA based target localization problem, an uncertain linear measurement model is newly derived. In the suggested problem setting, the target localization performance of the conventional linear estimation schemes might be severely degraded under the low SNR condition and be affected by the target position in the sensor network. This motivates us to devise a new sensor network localization algorithm within the framework of the recently developed robust least squares estimation theory. Provided that the statistical information regarding RDOA measurements are available, the estimate of the proposition method shows the convergence in probability to the true target position. Through the computer simulations, the omni-directional target localization performance and consistency of the proposed algorithm are compared to those of the existing ones. It is shown that the proposed method is more reliable than the total least squares method and the linear correction least squares method.

Keywords: RDOA, Target localization, Robust least squares estimation, Pseudo linear estimator, Sensor network

1. Introduction

Nowadays, it has been widely recognized that the TDOA (time difference of arrival)-based target localization is crucial for the development of a surveillance system [1-3]. This is because there are practical limitations in the existing target localization schemes using other information; the performance of the AOA (angle of arrival)-based system might be ensured when the sensor array is precisely calibrated and that of the TOA (time of arrival)-based system totally depends on the accuracy of the emission time [4].

There has been much attempt to deal with the target localization using TDOA measurements [5-7]. The TDOA-based target localization problem could be characterized by the nonlinear state estimation problem for determining a unique crossing point between two parabolic functions. To effectively handle this inherent nonlinearity of the problem, based on the Gaussian measurement noise assumption, the target localization problem was formulated within the framework of MLE (maximum likelihood estimation). Although the MLE method is one of possible choices to solve the problem, unfortunately, it has a few well-known flaws not to be overlooked. It inevitably relies on the numerical method to obtain the solution. Moreover, if one cannot choose proper initial guess for solving the problem, the ML localization algorithm could not guarantee the optimal estimation performance [8].

To overcome the above mentioned flaws, a closed-form PL (pseudo-linear) estimation technique has been studied by many researchers. Under the standing assumptions that three or more TDOA measurements are available and the measurement error is small enough, it converts the nonlinear estimation problem to the linear one by introducing an auxiliary state variable [9, 10]. For its recursive linear structure, the sensitivity problem against the initial guess could be successfully relaxed. As well, it is possible to implement the resultant localization algorithm on the cheap microprocessor. In spite of its usefulness, the localization performance of the existing PL localization schemes is severely deteriorated under low SNR conditions. This is because the noisy data is reused for constituting the measurement matrix [11-14].

In order to remove the estimation error due to the noise corrupted measurement matrix, the QCLS (quadratic correction LS) algorithm which consists of the nominal LS estimator and the corrector was proposed in [11]. As a more advanced form, the LCLS (linear correction LS) method has been also proposed in [12]. The LCLS algorithm minimizes the cost function defined by augmenting the original LS cost and the nonlinear state

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equality constraint by using the Lagrange multiplier. Both algorithms of the QCLS and the LCLS provide relatively good localization performance over the conventional PL estimator. However, to obtain the error reduced estimates, since the QCLS requires two-stage LS estimator and the LCLS needs a numerical method, they are not free from the computational burden.

Meanwhile, in view of the EIV (error-in-variable) model, the TLS (total least squares) based localization algorithm has been proposed [4, 14]. It attempts to make the set of consistent equations using correction terms for the noise component present in the measurement matrix of the PL. The TLS is a natural extension of the least squares method. However, the inevitable correlation between the noise in the measurement matrix and the measurement noise restricts the performance improvement.

To ensure the satisfactory localization performance under low SNR conditions, one may consider a robust state estimation. In 2007, the RoLS (robust least squares) estimator was proposed which can compensate the estimation error caused by the noises of the measurement matrix and the measurement [13]. By using the predefined auto-correlation of the measurement matrix noise and the cross-correlation between the measurement matrix noise and the measurement noise, the RoLS compensates the estimation error without computational burden. Additionally, the estimate of the proposed method converges to the true value with the convergence in probability.

In this paper, we build a new target localization estimator based on the RoLS algorithm. It is assumed that the localization is carried out in an open space to minimize the multipath effect of the RDOA measurement. The proposition is proven that the estimation results converge to the true target position in probability. Through the computer simulations, the omni-directional target localization performance and consistency of the proposed algorithm are compared to those of the existing ones.

2. Pseudo Linear Measurement Model using RDOA Measurement

Let's assume that the sensors in the 2-D network as Fig. 1.

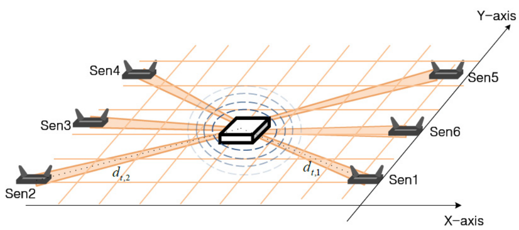


Fig. 2. Relative geometry for target localization using RDOA measurements

The positions of $n+1$ fixed sensors are known exactly as (x_j, y_j) , $j = 1, 2, \cdots, n+1$. A set of n TDOA measurements τ_j , $j = 2, \cdots, n+1$, is collected at various position. In the absence of measurement errors, the TDOA measurements at step k can be yielded to the RDOA, r_j by multiplying the propagation velocity of the source as follows:

$$r_j(k) = v_p \tau_j(k) = d_{t,j}(k) - d_{t,1}(k) \tag{1}$$

where v_p is the propagation velocity constant and $d_{t,j}$ is the distance between the target (x_t, y_t) and j^{th} sensor:

$$d_{t,j}(k) \triangleq \sqrt{\left(x_t(k) - x_j\right)^2 + \left(y_t(k) - y_j\right)^2}. \tag{2}$$

In the presence of additive measurement noises which are zero mean white noise, δr_j with variance σ_j^2 , the RDOA measurements become

$$\tilde{r}_j(k) = r_j(k) + \delta r_j(k),\ j = 2, 3, \cdots, n+1. \tag{3}$$

These n nonlinear equations can be written as a single equation with n-dimensional column vectors:

$$\tilde{\boldsymbol{r}}(k) = \boldsymbol{r}(k) + \delta \boldsymbol{r}(k) \tag{4}$$

On the other hand, by introducing an intermediate variable $d_{t,1}$ and squaring both sides of (3), we obtain the following linear measurement equations.

$$\begin{aligned}\tilde{r}_j^2(k) = -2\left[x_j - x_1 \quad y_j - y_1 \quad \tilde{r}_j(k) - \delta r_j(k)\right]\begin{bmatrix} x_t(k) \\ y_t(k) \\ d_{t,1}(k)\end{bmatrix} \\ + 2\tilde{r}_j(k)\delta r_j(k) - \left(\delta r_j(k)\right)^2 + d_{2,0}^2 - d_{1,0}^2, \\ j = 2, 3, \cdots, n+1\end{aligned}$$

where $d_{j,0}$ is the distance from the j^{th} sensor to the origin. From the above result, the linear measurement equation can also be represented by n-dimensional column vectors and matrices as follows:

$$\begin{aligned}\overline{Y}_k &= H_k X_k + \overline{v}_k \\ &= \left[\tilde{H}_k - \Delta H_k\right] X_k + \overline{v}_k\end{aligned} \tag{5}$$

where the relevant vectors and matrices are defined as

$$Y_k \triangleq \begin{bmatrix} \tilde{r}_2^2(k) - d_{2,0}^2 + d_{1,0}^2 \\ \vdots \\ \tilde{r}_n^2(k) - d_{2,0}^2 + d_{1,0}^2 \end{bmatrix},$$

$$H_k \triangleq -2\begin{bmatrix} x_2-x_1 & y_2-y_1 & r_2(k) \\ & \vdots & \\ x_n-x_1 & y_n-y_1 & r_n(k) \end{bmatrix},$$

$$H_k \triangleq -2\begin{bmatrix} x_2-x_1 & y_2-y_1 & r_2(k) \\ & \vdots & \\ x_n-x_1 & y_n-y_1 & r_n(k) \end{bmatrix},$$

$$\tilde{H}_k \triangleq -2\begin{bmatrix} x_2-x_1 & y_2-y_1 & \tilde{r}_2(k) \\ & \vdots & \\ x_n-x_1 & y_n-y_1 & \tilde{r}_n(k) \end{bmatrix}, \quad (6)$$

$$\Delta H_k \triangleq -2\begin{bmatrix} 0 & 0 & \delta r_2(k) \\ & \vdots & \\ 0 & 0 & \delta r_n(k) \end{bmatrix}, \quad (7)$$

$$X_k^T \triangleq \begin{bmatrix} x_t(k) & y_t(k) & d_{t,1}(k) \end{bmatrix}^T,$$

$$v_k \triangleq \begin{bmatrix} 2\tilde{r}_2(k)\delta r_2(k) - (\delta r_2(k))^2 \\ \vdots \\ 2\tilde{r}_n(k)\delta r_n(k) - (\delta r_n(k))^2 \end{bmatrix}. \quad (8)$$

From (7) and (8), stochastic properties for ΔH_k and v_k are given by

$$E[\Delta H_k] = O^{(n-1)\times 3},$$

$$E[v_k] = \begin{bmatrix} \sigma_2^2 \\ \vdots \\ \sigma_n^2 \end{bmatrix} \triangleq b_k, \quad (9)$$

$$E\left[\Delta H_k^T \Delta H_k\right] = diag\left(0, 0, 4\sum_{j=2}^{n}\sigma_j^2\right) \triangleq W_k, \quad (10)$$

$$E\left[\Delta H_k^T v_k\right] = \begin{bmatrix} 0 & 0 & -4\sum_{j=2}^{n}\tilde{r}_j(k)\sigma_j^2 \end{bmatrix}^T \triangleq V_k. \quad (11)$$

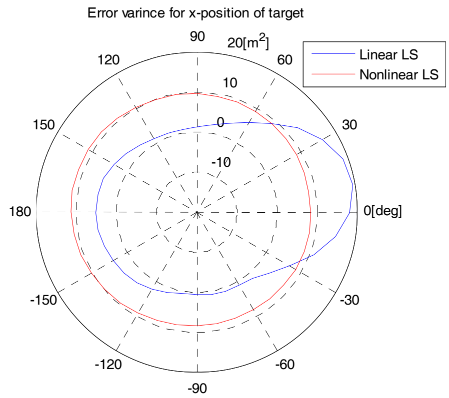
To make the problem simple, based on the assumption that the variance of the RDOA measurement noise are given, one defined the bias removed linear measurement equation as

$$Y_k = \left(\tilde{H}_k - \Delta H_k\right)X_k + v_k \quad (12)$$

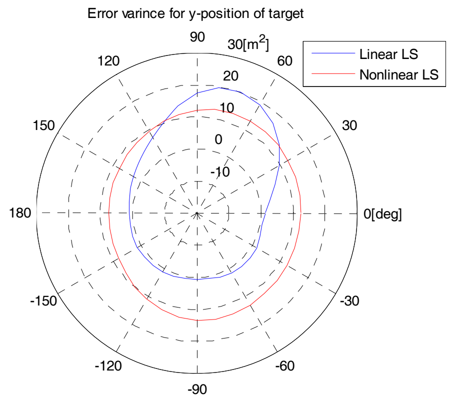
where

$$Y_k \triangleq \bar{Y}_k - b_k, \ \ v_k \triangleq \bar{v}_k - b_k$$

Remark 1. For the RDOA measurements equation, one can use the nonlinear equation from (4) or the linear equation from (12). However, there may be a discrepancy in the localization performance if the comparison is performed over whole azimuth of the sensor network. The reason is that the measurement noise of the nonlinear equation $\delta \boldsymbol{r}(k)$ is just additive RDOA measurement noise but the linear equation one v_k is not only the RDOA measurement noise but also the RDOA measurement itself as shown in (8): $v_k = f\left(\tilde{r}_j(k), \delta r_j(k)\right), j = 2, \cdots, n$.



(a) Error variance for x-position of target



(b) Error variance for y-position of target

Fig. 2. Error variance over the whole azimuth

To check the discrepancy of the performance over the whole azimuth of the sensor network, the least squares (LS) estimators have been used for each measurement model: the nonlinear LS is obtained from (4) and the linear LS is from (12). (For the theoretical comparison with the nonlinear, the linear LS has been used the true measurement matrix H_k in the simulation.) The sensor network is assumed that the sensors are at (0, 0), (0, 3000), (0, 6000), (6000, 6000), (6000, 3000), (6000, 0) [m]. Their sensor geometry is similar to [1]. The RDOA noise variance is 20000[m^2]. The target positions are selected 1500[m] far from the center of the sensor network, (3000, 3000), and we check the estimation error variance every 10 degree around the center. The iteration number of simulations is 100 and the simulation is performed to 500 steps at every iteration.

As shown at Fig. 2, the estimation error variance over the whole azimuth of the linear LS is not identical for every target position. This is because the non-uniform measurement noise of the linear equation. However, in this geometry of the network, the linear LS estimator is generally smaller than the nonlinear one.

Remark 2. The linear estimator is more advantageous than the nonlinear one at the practical aspects such as observability and real-time implementation. However, the given linear measurement Eq. (12) is an uncertain linear measurement equation with stochastic parametric uncertainties, ΔH_k. The available measurement matrix is not H_k but $\tilde{H}_k$. Therefore, it is noted that the linear LS of this simulation is not implementable in practice.

Since the true measurement matrix is not available, the NLS (nominal least squares) solution has been proposed based on the following PL measurement equation which neglects the uncertainty matrix ΔH_k from the Eq. (12) under the assumption that the noise component of the uncertainty matrix is small enough.

$$Y_k = \left(\tilde{H}_k - \Delta H_k\right) X_k + v_k \approx \tilde{H}_k X_k + v_k$$

However, this standing assumption could decline the estimation performance if the SNR of the RDOA measurement is low.

Lemma 1: **(Estimation errors with NLS)** [13] From the following vector represented measurement equation for (12)

$$\underbrace{\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_k \end{bmatrix}}_{\triangleq Y^k} = \left(\underbrace{\begin{bmatrix} \tilde{H}_1 \\ \tilde{H}_2 \\ \vdots \\ \tilde{H}_k \end{bmatrix}}_{\triangleq \tilde{H}^k} - \underbrace{\begin{bmatrix} \Delta H_1 \\ \Delta H_2 \\ \vdots \\ \Delta H_k \end{bmatrix}}_{\triangleq \Delta H^k} \right) X_k + \underbrace{\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix}}_{\triangleq v^k} \tag{13}$$

, one can define a possible cost function using the available information $\tilde{H}^k$ as

$$J_k = \left(Y^k - \tilde{H}^k X_k\right)^T \left(Y^k - \tilde{H}^k X_k\right)$$

Then, the NLS, the minimum solution of the cost function, is

$$\hat{X}_k^{NLS} = \left\{(\tilde{H}^k)^T (\tilde{H}^k)\right\}^{-1} \left\{(\tilde{H}^k)^T Y^k\right\}$$

and it has estimation errors by the uncertainties as follows:

$$\begin{aligned} \hat{X}_k^{NLS} &= \left\{(\tilde{H}^k)^T (\tilde{H}^k)\right\}^{-1} \left\{(\tilde{H}^k)^T \left(H^k X_k - v_k\right)\right\} \\ &= X_k \underbrace{-\alpha_k X_k + \beta_k}_{errors} \end{aligned}$$

where the scale-factor error

$$\alpha_k \triangleq \left\{(\tilde{H}^k)^T (\tilde{H}^k)\right\}^{-1} \left\{(\tilde{H}^k)^T \Delta H^k\right\}, \tag{14}$$

and the bias error

$$\beta_k \triangleq \left\{(\tilde{H}^k)^T (\tilde{H}^k)\right\}^{-1} \left\{(\tilde{H}^k)^T v^k\right\}. \tag{15}$$

To compensate the estimation error of the NLS, there have been several proposed linear estimators. For the vectorized uncertain linear measurement Eq. (12), one of the linear estimators is the TLS in [14] which is defined as

$$\hat{X}_k^{TLS} = \left\{(\tilde{H}^k)^T (\tilde{H}^k) - \bar{\sigma}^2 I\right\}^{-1} (\tilde{H}^k)^T Y^k \tag{16}$$

where $\bar{\sigma}$ is a minimum singular value for the matrix

$$\begin{bmatrix} \tilde{H}^k & Y^k \end{bmatrix} = \bar{U}\bar{\Sigma}\bar{V}^T. \tag{17}$$

The other one is the LCLS in [12] which is defined as

$$\hat{X}_k^{LCLS} = \left\{(\tilde{H}^k)^T (\tilde{H}^k) - \lambda M\right\}^{-1} (\tilde{H}^k)^T Y^k \tag{18}$$

where

$$M = diag\left(\begin{bmatrix} 1 & 1 & -1 \end{bmatrix}\right)$$

and λ denotes the Lagrange multiplier satisfying the constraint as follows:

$$f(\lambda) = \left(\hat{X}_k^{LCLS}\right)^T M \left(\hat{X}_k^{LCLS}\right) = 0. \tag{19}$$

The TLS and the LCLS provide compensated estimation results for the uncertain linear measurement equation, however, the compensation factors, $\bar{\sigma}$ and λ need additional computation burden which is a singular value decomposition procedure for (17) and a numerical method such as root finding for (19), respectively. These additional burdens may be an obstacle for real-time implementation. Furthermore, as shown in (16) and (18), the compensation seems to be focused on the auto-correlation $(\tilde{H}^k)^T (\tilde{H}^k)$ rather than the cross-correlation $(\tilde{H}^k)^T (v^k)$. In order to successfully eliminate the estimation errors, (14) and (15), and to flee from the burden, the RoLS approach should be considered for the localization problem [13].

3. Target Localization Algorithm Based on the Robust Least Squares

The reasons of the estimation error for the NLS are the scale-factor error and the bias error. If the errors can be represented with the available matrix and the predefined stochastic information, it is possible to remove from the NLS. With this strategy, the RoLS was developed [13]. Based on the RoLS, the target localization algorithm can be shown the convergence in probability.

Lemma 2: **(RoLS based target localization algorithm)** Given the measurement Eq. (12), the RoLS based target localization algorithm is

$$\begin{aligned}\hat{X}_k^{RoLS} &= P_k\left\{(\tilde{H}^k)^T Y^k - V^k\right\} \\ &= \left(I - \hat{\alpha}_k\right)^{-1}\left(\hat{X}_k^{NLS} - \hat{\beta}_k\right)\end{aligned} \tag{20}$$

where $P_k = \left\{(\tilde{H}^k)^T(\tilde{H}^k) - W^k\right\}^{-1}$ and the compensation factors are defined as

$$\hat{\alpha}_k \triangleq \left\{(\tilde{H}^k)^T(\tilde{H}^k)\right\}^{-1} W^k,\ \hat{\beta}_k \triangleq \left\{(\tilde{H}^k)^T(\tilde{H}^k)\right\}^{-1} V^k \tag{21}$$

where $W^k \triangleq E\left[(\tilde{H}^k)^T \Delta H^k\right]$ and $V^k \triangleq E\left[(\tilde{H}^k)^T v^k\right]$ are obtained as following, from (10) and (11), respectively.

$$W^k = W_1 + W_2 + \cdots W_k,\ V^k = V_1 + V_2 + \cdots V_k$$

For a real-time application, its recursive formula is

$$\begin{aligned}\hat{X}_k^{RoLS} &= \left(I + P_k W_k\right)\hat{X}_{k-1}^{RoLS} \\ &\quad + P_k \tilde{H}_k^T\left(Y_k - \tilde{H}_k \hat{X}_{k-1}^{RoLS} - b_k\right) - P_k V_k \\ P_k^{-1} &= P_{k-1}^{-1} + \tilde{H}_k^T \tilde{H}_k - W_k\end{aligned} \tag{22}$$

Theorem 1: **(Convergence in probability of the RoLS based target localization algorithm)** If the sequence of the RDOA measurement noises $\delta r_j(k)$ of all sensors $(j = 2, \cdots, n)$ are i.i.d. (independent and identically distributed) and the estimate of the RoLS based target localization algorithm exists due to $P_k^{-1} > 0$ [13], the estimate of the RoLS based target localization algorithm converges to the true position in probability.

$$\hat{X}_k^{RoLS} \xrightarrow{p} X$$

Proof: When the target position is fixed $X_k = X$ and the sequence of the RDOA measurement noises are i.i.d, the true measurement matrix becomes constant matrix: $H_k = H$ and the definitions of (9)-(11) are independent for time as follows:

$$E\left[v_k\right] = b,\ E\left[\Delta H_k^T \Delta H_k\right] = W,\ E\left[\Delta H_k^T v_k\right] = V,\ \forall k.$$

Because

$$H^k = \tilde{H}^k - \Delta H^k,$$

$$\begin{aligned}E\left[(\tilde{H}^k)^T(\tilde{H}^k) - W^k\right] &= E\left[\sum_{j=1}^{k} \tilde{H}_j^T \tilde{H}_j - W_j\right] \\ &= E\left[\sum_{j=1}^{k} H^T H + \Delta H_j^T H + H^T \Delta H_j + \Delta H_j^T \Delta H_j - W_j\right] \\ &= k\left(H^T H\right),\end{aligned}$$

$$\begin{aligned}E\left[(\tilde{H}^k)^T(H^k)\right] &= E\left[\sum_{j=1}^{k} \tilde{H}_j^T H\right] \\ &= E\left[\sum_{j=1}^{k} H^T H + \Delta H_j^T H\right] = kH^T H,\end{aligned}$$

$$\begin{aligned}E\left[(\tilde{H}^k)^T\left(v^k - b^k\right)\right] &= E\left[\sum_{j=1}^{k} \tilde{H}^T v - \tilde{H}^T b\right] \\ &= E\left[\sum_{j=1}^{k} H^T(v - b) + \Delta H^T(v - b)\right] = kV.\end{aligned}$$

Therefore, it is obvious that

$$\operatorname*{p\,lim}_{k\to\infty}\left\{\frac{(\tilde{H}^k)^T(\tilde{H}^k) - W^k}{k}\right\} = H^T H, \tag{23}$$

$$\operatorname*{p\,lim}_{k\to\infty}\left\{\frac{(\tilde{H}^k)^T(H^k)}{k}\right\} = H^T H, \tag{24}$$

$$\operatorname*{p\,lim}_{k\to\infty}\left\{\frac{(\tilde{H}^k)^T(v^k - b^k) - V^k}{k}\right\} = 0. \tag{25}$$

Since P_k^{-1} is invertible, the inverse function is a continuous function [16]. Then, by the continuous mapping theorem [17],

$$\operatorname*{p\,lim}_{k\to\infty}\left(kP_k\right)^{-1} = \operatorname*{p\,lim}_{k\to\infty}\left(\frac{(\tilde{H}^k)^T(\tilde{H}^k) - W^k}{k}\right)^{-1} = \left(H^T H\right)^{-1}. \tag{26}$$

Now, the RoLS based target localization can be written as

$$\hat{X}_k^{RoLS} = P_k\left\{(\tilde{H}^k)^T Y^k - V^k\right\} = P_k k \times \frac{1}{k}\left\{(\tilde{H}^k)^T Y^k - V^k\right\}$$

by the Slutsky's theorem [18, 19] and (23)-(26), it converges to the true position in probability.

$$\begin{aligned}&\operatorname*{p\,lim}_{k\to\infty} P_k k \times \frac{1}{k}\left\{(\tilde{H}^k)^T(Y^k - b^k) - V^k\right\} \\ &= \left(\operatorname*{p\,lim}_{k\to\infty} kP_k\right) \times \left(\operatorname*{p\,lim}_{k\to\infty} \frac{1}{k}\left\{(\tilde{H}^k)^T(Y^k - b^k) - V^k\right\}\right) \\ &= \left(\operatorname*{p\,lim}_{k\to 0} kP_k\right) \times \left(\operatorname*{p\,lim}_{k\to 0} \frac{\tilde{H}^k H^k}{k} X + \operatorname*{p\,lim}_{k\to 0} \frac{\tilde{H}^k(v^k - b^k) - V^k}{k}\right) \\ &= X\end{aligned} \tag{27}$$

■

Remark 3. It is easily seen that the singular value in (16) and the Lagrange multiplier in (18) are similar to the scale-factor error correction term, W^k of the RoLS based target localization algorithm (20). W^k corrects $(\Delta H^k)^T(\Delta H^k)$ from the auto-correlation term $(\tilde{H}^k)^T(\tilde{H}^k)$. However, differently with the TLS and the LCLS, the RoLS corrects not only the auto-correlation term but also the cross-correlation one by V^k. For this reason, the RoLS based target localization algorithm can be regarded more general solution in view of the filter structure for error compensation.

4. Simulation Results

The simulations, we prove the performance of the RoLS based target localization algorithm. It is compared with the TLS, LCLS, and NLS algorithm. We assume the sensor network as shown in Fig. 1 which contains 6 sensor nodes and provides the RDOA measurements. The positions of nodes are same with the assumption in Fig. 2. The iteration number of simulations is 100 and the simulation is performed to 500 steps at every iteration.

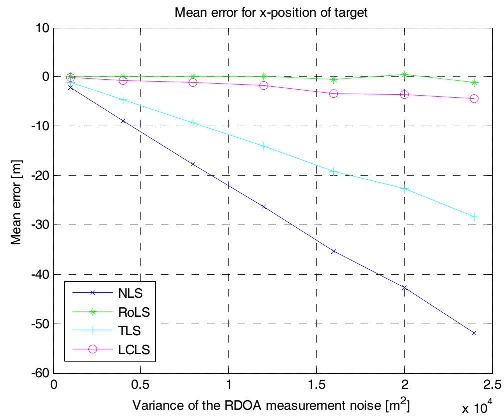
4.1 Performance comparison for the RDOA noise variance at a fixed position

To check the performance degradation by low SNR of the RDOA measurement, variances of the measurement noise are set from 1000[m^2] to 24000[m^2]. The target position is placed at 60 [deg] and 1500[m] far from the network's center. The variances are not enough small differently from [14] and [18] whose the bias b_k were neglected. In low SNR condition, the bias may affect the estimation performance. In this reason, under the assumption that the variance is given information, all of algorithms are derived with the bias removed Eq. (12).

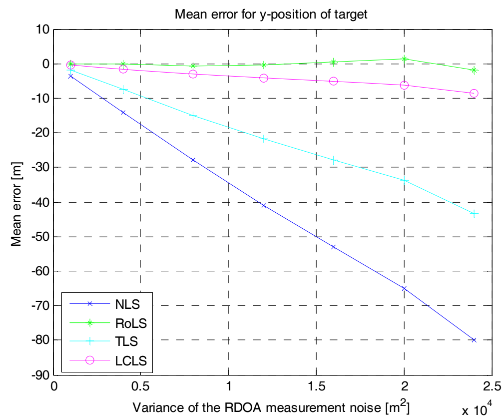
The mean error results of each estimation method are represented in Fig. 3. The errors are increased rapidly along the degradation of the SNR condition. The TLS and LCLS show more compensated results than the NLS but they still have mean error even though the bias of the measurement noise is removed. On the other hand, the results of the RoLS seems to guarantee Theorem 1. Therefore, as mentioned in Remark 3, it is thought that the compensation method of the RoLS for the scale-factor error and the bias error is more general approach for the target localization problem using the RDOA measurements.

4.2 Performance comparison over the whole azimuth

As shown in Fig. 2, estimation performance can be affected by the target position in the sensor network. To check the performance over the whole azimuth, the target positions are selected 1500[m] far from the center of the sensor network, (3000, 3000), and the performance is checked for every 10 degrees around its center. In this simulation, the variance of the RDOA measurement is set to 20000[m^2]. Estimation results of the target position over the whole azimuth as shown in Fig. 4 and Fig. 5 which represent mean error and root mean square error (RMSE), respectively.



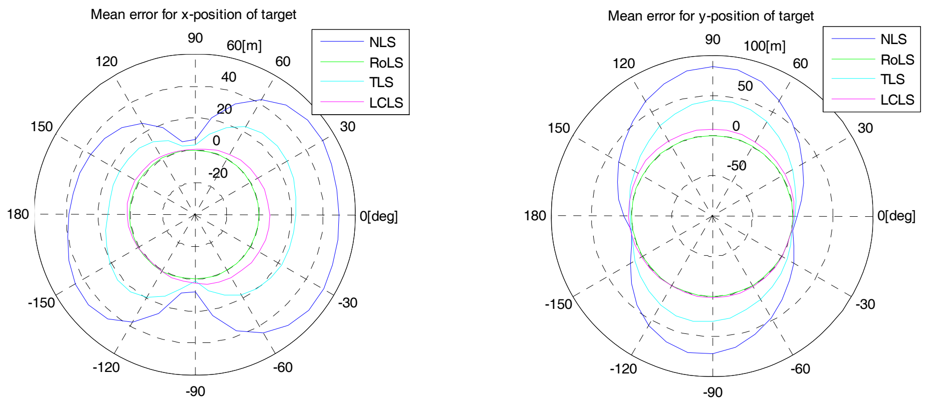
(a) Mean error for x-position of target



(b) Mean error for y-position of target

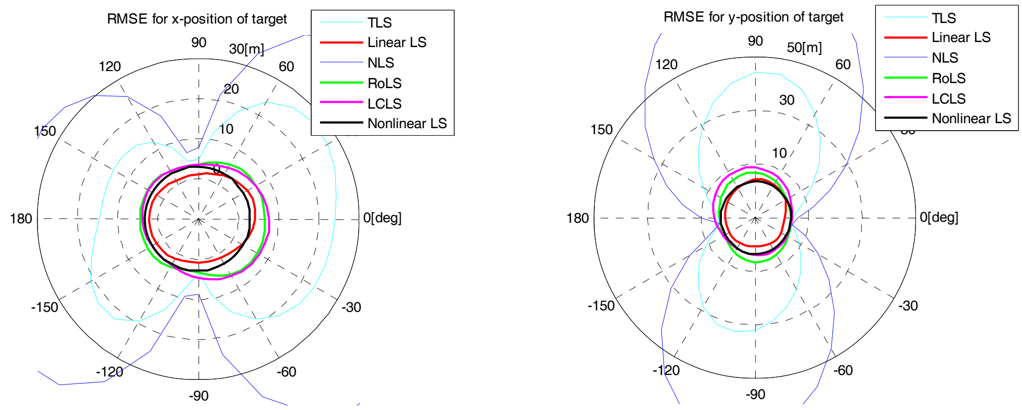
Fig. 3. Mean error of estimated target position for the noise variance of the RDOA measurements

The superiority of the mean error performance of the RoLS is also remarkable in whole azimuth simulation. The mean error result of the NLS shows the effect of the performance variation along the target location. It seems that the estimation error of the NoLS is dependent on the geometry of the sensor network. (It is worthy that the relation between the geometry and the estimation error is studied as future work.) The mean errors of the TLS and the LCLS also have affected by the target location but the RoLS shows relatively unified performance due to the small mean error. However, the RoLS does not guarantee the minimum RMSE as shown Fig. 5. It is because that, in this simulation, the cost function of the RoLS is applied the identical weight which is differently from the RWLS in [20]. This difference is described in Table 1. Therefore, by applying the optimal weight matrix, the RMSE performance of the RoLS method can be improved.



(a) Mean error for x-position of target (b) Mean error for y-position of target

Fig. 4. Mean error of estimated target position over the whole azimuth



(a) RMSE for x-position of target (b) RMSE for y-position of target

Fig. 5. RMSE of estimated target position over the whole azimuth

Table 1. Comparison of the RoLS and the RWLS

RoLS	Cost function	$J_k^{RoLS} = \{(Y^k - b^k) - \tilde{H}^k X_k\}^T \{(Y^k - b^k) - \tilde{H}^k X_k\} - X_k^T W^k X_k + (V^k)^T X_k + (X_k)^T V^k$
	Compen-sation factors	$W_k = E[\Delta H_k^T \Delta H_k]$, $V_k = E[\Delta H_k^T v_k]$
RWLS in [20]	Cost function	$J_k^{WRLS} = \{(Y^k - b^k) - \tilde{H}^k X_k\}^T \Lambda^k \{(Y^k - b^k) - \tilde{H}^k X_k\} - X_k^T \bar{W}^k X_k + (\bar{V}^k)^T X_k + (X_k)^T \bar{V}^k$
	Compen-sation factors	$\bar{W}_k = E[\Delta H_k^T \Lambda_k \Delta H_k]$, $\bar{V}_k = E[\Delta H_k^T \Lambda_k v_k]$

5. Conclusion

The RoLS based target localization algorithm has been proposed using the uncertain linear measurement equation which is given the stochastic information of the RDOA measurements. Since the linear estimator suffers from the estimation error due to the uncertainty caused by the RDOA measurement noise, the newly developed RoLS algorithm has been applied to compensate the estimation error by using the stochastic information. For aspects of the estimation error compensation, the proposition has more generous form than the TLS and the LCLS estimators and can be shown the convergence in probability with mathematical proof. In the simulation, the performance of the target localization is affected by the SNR of the measurement and the target's position in the sensor network but the proposition shows superior performance than that of the TLS, LCLS, and NLS. Owing to the significantly small mean error characteristic and its recursive form, the proposition will be utilizable for real-time and low cost localization system.

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