Optimal Particle Swarm Based Placement and Sizing of Static Synchronous Series Compensator to Maximize Social Welfare

Somayeh Hajforoosh*, Seyed M. H Nabavi† and Mohammad A. S. Masoum**

Abstract – Social welfare maximization in a double-sided auction market is performed by implementing an aggregation-based particle swarm optimization (CAPSO) algorithm for optimal placement and sizing of one Static Synchronous Series Compensator (SSSC) device. Dallied simulation results (without/with line flow constraints and without/with SSSC) are generated to demonstrate the impact of SSSC on the congestion levels of the modified IEEE 14-bus test system. The proposed CAPSO algorithm employs conventional quadratic smooth and augmented quadratic nonsmooth generator cost curves with sine components to improve the accurate of the model by incorporating the valve loading effects. CAPSO also employs quadratic smooth consumer benefit functions. The proposed approach relies on particle swarm optimization to capture the near-optimal GenCos and DisCos, as well as the location and rating of SSSC while the Newton based load flow solution minimizes the mismatch equations. Simulation results of the proposed CAPSO algorithm are compared to solutions obtained by sequential quadratic programming (SQP) and a recently implemented Fuzzy based genetic algorithm (Fuzzy-GA). The main contributions are inclusion of customer benefit in the congestion management objective function, consideration of nonsmooth generator characteristics and the utilization of a coordinated aggregation-based PSO for locating/sizing of SSSC.

Keywords: Congestion management, Social welfare, SSSC, CAPSO, GA and fuzzy

1. Introduction

Competition in a deregulated power system is expected to set a fair market structure and motivate all participants to maximize their own individual profits. This will allow the market to behave in a manner that will eventually maximize the profit of most participants. In addition to the deregulation challenges, electrical loads are rapidly growing and some transmission lines are congested.

Many artificial intelligent (AI) congestion management approaches have been proposed in the literature that are mainly based on market models [1], particle swarm optimizations (PSOs) (for generation rescheduling and/or load shedding) [2], genetic algorithms (GAs) [3] and sensitivity analysis using transmission line susceptances [4]. Several studies have also been performed on congestion management to maximize social and individual welfare [5, 6], as well as, social welfare maximization considering reactive power and congestion management in the deregulated environment [7].

Recent solutions for managing power flow in transmission lines are based on flexible AC transmission systems (FACTS) through the use of large power converters [8]. Different approaches, based on sensitivity methods have also been proposed for optimal locating of FACTS devices in both vertically integrated and unbundled power systems [9-12].

A trial-and-error (T&E) based congestion management method for the placement of FACTS controllers is presented in [13]. Congestion management by interline power flow controller (IPFC) and unified power flow controller (UPFC) are performed in [14, 15].

These references simplify the optimization problem by assuming given sizes of FACTS devices and/or using second order objective benefit functions without considering the sine components due to the valve point loading effects. In addition, the impacts of SSSC and its economical effects in deregulated power systems have not been truly explored and will be investigated in this paper.

This paper proposes a new PSO-based algorithm for alleviating congestion and maximizing social benefit in a double-sided auction market by optimal placement and sizing of one Static Synchronous Series Compensator (SSSC) unit. Simulations are performed to investigate the impact of SSSC on congestion levels of the modified 14-bus test system with quadratic smooth and quadratic nonsmooth (with sine components due to valve point

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loading effect) generator cost curves and quadratic smooth benefit functions for loads. The proposed method shows the benefits of SSSC in a deregulated power market and demonstrates how it may be utilized by ISO to prevent congestion and improve the overall social welfare.

2. Mathematical Model of SSSC

While several types of FACTS devices can be used to solve the optimal power flow (OPF) problem, this paper focuses on the utilization of one SSSC to maximize the social welfare. SSSC injects a series voltage in quadratic with the line current. With the application of SSSC, the effective line reactance can be modified to become inductive or capacitive.

2.1 Power injection model of SSSC

In this paper, the Newton-Raphson (N-R) power flow formulation is used and SSSC is represented using the power injection model (PJM) [16]. This is a convenient approach for the integration of SSSC devices in the existing power system software tools as it retains the symmetrical structure of the admittance matrix. As shown in Fig. 1, the change in the line flow due to the introduction of SSSC is represented as a line with active and reactive powers injected at the receiving and sending ends.

\[
P_{ij} = V_i^2 g_{ij} - V_i V_j \left[ g_{ij} \sin \delta_i - b_{ij} \cos \delta_i \right] - V_i V_{se} \left[ g_{ij} \cos \left( \delta_i - \theta_{se} \right) - b_{ij} \sin \left( \delta_i - \theta_{se} \right) \right] \]

\[
Q_{ij} = V_i^2 b_{ij} - V_i V_j \left[ g_{ij} \sin \left( \delta_i - \theta_{se} \right) - b_{ij} \cos \left( \delta_i - \theta_{se} \right) \right]
\]

\[
P_{ij} = V_j^2 g_{ij} - V_i V_j \left[ g_{ij} \cos \delta_j + b_{ij} \sin \delta_j \right] + V_j V_{se} \left[ g_{ij} \cos \left( \delta_j - \theta_{se} \right) + b_{ij} \sin \left( \delta_j - \theta_{se} \right) \right]
\]

\[
Q_{ij} = V_j^2 b_{ij} - V_i V_j \left[ g_{ij} \sin \delta_j - b_{ij} \cos \delta_j \right] + V_j V_{se} \left[ g_{ij} \sin \left( \delta_j - \theta_{se} \right) - b_{ij} \cos \left( \delta_j - \theta_{se} \right) \right]
\]

where \( g_{ij} + j b_{ij} = 1/Z_{se} \). As the SSSC device neither absorbs nor injects real power with respect to the AC system, the real power exchange via the DC link is zero:

\[
\text{Re} \left\{ V_{se} I_{ji}^{*} \right\} = 0
\]

2.2 Cost of SSSC

In this paper, the cost of SSSC is included in the form of the following linear equation:

\[
\text{Cost (SSSC)} = \text{SSSC Capacity} \times \eta \times \text{CRF}
\]

\[
\text{Capital Recovery Factor (CRF)} = \frac{i(1+i)^n}{(1+i)^n-1}
\]

where \( \eta \) is the investment cost coefficient (per-MVA) of SSSC, \( i \) is a discount rate or present worth rate, \( n \) is the life time of SSSC (in years). The annual capital payment (ACP) represents the uniform annual amount payments for the useful life of the project. In this paper, it is assumed that \( \eta = 50,000$/MVA, \( n = 15 \) years and \( i = 6\% \) Per-year. Therefore, the CRF and ACP will be 0.10296 and 5148$/MVA-year, respectively.

3. Problem Formulation

3.1 The objective function

In the double-sided auction market model, both DisCos and GenCos participate in the market and offer their bid-quantity packages to the market operator. In the literature, the GenCos cost and DisCos benefit functions are usually assumed to be quadratic. In this paper, GenCos cost functions are improved by adding the valve point loading effects that introduces ripples in the actual input–output curve. This is done by considering an additional sine term to account for the valve effects [17]. Therefore, the objective of market operator is to maximize the social welfare, including load flow equality and operational inequality constraints:
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\[
\begin{align*}
\text{Max} & \quad \left\{ \sum_{i=1}^{N_G} \left( a_i + b_i P_i + c_i P_i^2 \right) \right. \\
& \quad - \sum_{i=1}^{N_G} \left( a_i + b_i P_i + c_i P_i^2 + e_i \sin(f_i \times (P_i - P_{\text{max}}))) \right) \\
& \quad \left. - \sum_{i=1}^{N_L} C_i d_i \right\}
\end{align*}
\]

Subject to:

\[
f(V, \delta, P, Q) = 0
\]

\[
P_{\text{max}}^{i} \leq P_i \leq P_{\text{min}}^{i} \quad i = 1, 2, \ldots, N_G
\]

\[
Q_{\text{min}}^{i} \leq Q_i \leq Q_{\text{max}}^{i} \quad i = 1, 2, \ldots, N_G
\]

\[
P_{\text{max}}^{i} \leq P_i \leq P_{\text{min}}^{i} \quad i = 1, 2, \ldots, N_D
\]

\[
Q_{\text{min}}^{i} \leq Q_i \leq Q_{\text{max}}^{i} \quad i = 1, 2, \ldots, N_D
\]

\[
S_{i} (\theta, V_i) \leq \left( S_{\text{max}}^{i} \right)^2 \quad i = 1, 2, \ldots, N_L
\]

\[
V_{\text{min}}^{i} \leq V_i \leq V_{\text{max}}^{i} \quad i = 1, 2, \ldots, N_B
\]

\[
\theta_{\text{min}}^{i} \leq \theta_i \leq \theta_{\text{max}}^{i}
\]

where \( f(V, \delta, P, Q) \) is the power flow equation; \( V \) and \( \delta \) vectors of magnitude and angle of bus voltages; \( P \) and \( Q \) are vectors of bus active and reactive power injection; \( P_{\text{gi}} \) and \( Q_{\text{gi}} \) are dispatched generations at bus \( i \), with maximum and minimum limits of active and reactive power equal to \( P_{\text{max}}^{i} \) and \( P_{\text{min}}^{i} \), \( Q_{\text{max}}^{i} \), \( Q_{\text{min}}^{i} \), respectively. \( N_G, N_D, \) and \( N_B \) are the number of generators, loads, and buses, respectively. \( P_{Dj} \) is the dispatched load at bus \( j \), \( a_{gi}, b_{gi}, \) and \( c_{gi} \) are the coefficients of benefit functions, while \( a_{gi}, b_{gi}, c_{gi}, e_{gi}, \) and \( f_{gi} \) are cost coefficients in GenCos.

Fig. 2 shows the effect of considering the valve point effects on the GenCos fuel cost functions. Note that even when considering only two units, there are multiple peaks and differentiable valleys [17].

It is obvious that considering the valve point loading changes the operating points of the generators and their generation costs. Therefore, the optimization problem with the addition of the sine components has become too complex to be solved by conventional approaches. Furthermore, accurate modeling adds more challenges to most derivative-based optimization algorithms in finding the global solution since the objective is no longer convex nor differentiable everywhere.

### 4. Proposed PSO algorithm

PSO is a global search technique, based on the mechanisms of natural selection and genetics capable of simultaneously searching several possible solutions [17]. PSO starts with a random generation of initial particles and uses velocity vectors for updating until the termination criterion has been satisfied. It requires the evaluation of fitness function (FF) to assign a quality value to every solution produced. PSO has been applied to many problems including optimal location and parameters setting of UPFC for increasing loadability [18], optimum placement and sizing of DG using a binary PSO algorithm to achieve the minimum electricity cost for consumers [19] and economic dispatch with non-smooth cost functions [20].

The social welfare optimization problem (Eq. (8)) is a complex, large-scale nonlinear programming dilemma that cannot easily be solved by most conventional approaches. This paper proposes a coordinated aggregation-based PSO (CAPSO) algorithm for the optimal placement and sizing of SSSC to capture the (near) global solution.

### 5. Development of the Coordinated aggregation-based PSO (CAPSO) algorithm

The proposed coordinated aggregation-based PSO (CAPSO) approach is similar to the algorithm developed in [20] for economic load dispatching. The main difference is on a proposed new and novel updating approach for the velocity vector. The steps for the proposed CAPSO as a population based algorithm are presented in the next section.

#### 5.1 Proposed initial population and structure of particles

To begin the CAPSO, two random number generators are used to select the initial particle position vector and velocity vector within the range of the designated control variables. In this paper, the selected particle position vector and velocity vector structures contain generation and demand levels, as well as SSSC location and compensation level (Fig. 3).
5.2 Proposed fitness function

CAPSO algorithm involves the evaluation of the objective (fitness) function to measure the quality of the solutions. A solution with a better quality (e.g., higher fitness value) will be included in the new population, while low quality solutions are discarded. In this paper, exponential penalty functions, as shown in Table 1, for each generated particle are calculated for lines that have power overflows and/or reach voltage, generation and load limits, based on respective penalty functions as follows:

\[ F_{\text{Fitness}} = F_{\text{Line flow}} \times F_{\text{Bus voltage}} \times F_{\text{Generation}} \times F_{\text{Load}} \]  

(9)

where \( F_{\text{Line flow}} = \prod_{j=1}^{N_f} F_{f_j} \), \( F_{\text{Bus voltage}} = \prod_{j=1}^{N_b} F_{v_j} \), \( F_{\text{Generation}} = \prod_{j=1}^{N_g} F_{g_j} \), and \( F_{\text{Load}} = \prod_{j=1}^{N_d} F_{d_j} \). where \( F_{\text{fitness}} \) is the fitness function value for each particle.

Table 1. The proposed penalty functions

<table>
<thead>
<tr>
<th>Penalty function</th>
<th>( x &lt; x_{\text{min}} )</th>
<th>( x_{\text{min}} \leq x &lt; x_{\text{max}} )</th>
<th>( x = x_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_1 )</td>
<td>( e^{\alpha_1 x} - 1 )</td>
<td>1</td>
<td>( e^{\alpha_1 x_{\text{max}} - x_{\text{max}}} )</td>
</tr>
<tr>
<td>( F_2 )</td>
<td>( e^{\alpha_2 x} - 1 )</td>
<td>1</td>
<td>( e^{\alpha_2 x_{\text{max}} - x_{\text{max}}} )</td>
</tr>
<tr>
<td>( F_3 )</td>
<td>( e^{\alpha_2 x} - 1 )</td>
<td>1</td>
<td>( e^{\alpha_2 x_{\text{max}} - x_{\text{max}}} )</td>
</tr>
<tr>
<td>( F_4 )</td>
<td>( e^{\alpha_3 x} - 1 )</td>
<td>1</td>
<td>( e^{\alpha_3 x_{\text{max}} - x_{\text{max}}} )</td>
</tr>
</tbody>
</table>

*) where \( \alpha_1, \alpha_2, \alpha_3, a_1, a_2, a_3, D_1, D_2, a_1, a_2 \) are the coefficients used to adjust the slope of the penalty functions.

6. The Proposed CAPSO Algorithm

The problem defined by Eq. (8) is solved using the CAPSO algorithm of Fig. 4. The main steps are as follows:

**Step 1:** Prepare input power system parameters (e.g., system topology, line and load specifications, generation limits, line flow limits and cost coefficient parameters).

**Step 2:** Assume a suitably population size \( (N_{\text{particle max}}) \) and maximum number of iterations \( (N_{\text{it max}}) \). Set initial counters and parameter values (e.g., \( N_{\text{particle}} = N_{\text{it}} = 1 \)). At the first iteration, a random generated population is used as the initial position vector \( S(0) \) of each particle (including initial vector of generators’ real power outputs, loads demands, locations and sizes of FACTS devices), and is set as the best position \( P_i(0) \) of particle \( i \). Then, a random generator is used to prepare the initial velocity vector. Note that at each of the following iterations, the particles’ positions are updated based on their speeds.

**Step 3 (Fitness Process):**

**Step 3A:** Run power flow for each particle and determine voltage magnitudes and phase angles at all buses. Calculate power flow in each transmission line of the system. Compute the objective function (Eq. (8)).

**Step 3B:** Compute the proposed penalty functions (Table 1) using outputs of the applied power flow. Compute fitness functions (Eq. (9)) for particle \( N_{\text{particle}} \) and determine its best position.

**Step 3C:** If the obtained position has a better fitness function compared to the current fitness value \( (P_{\text{best}}) \), then replace it in \( P_{\text{best}} \). Otherwise, do not update the current \( P_{\text{best}} \).

**Step 3E:** If \( N_{\text{particle}} < N_{\text{particle max}} \), go to Step 3A, otherwise, if the obtained position has a better fitness function than the current \( G_{\text{best}} \), replace the new position in \( G_{\text{best}} \) \( f(S_i(0)) = \text{Max} f(S_i(0)) \).

**Step 3F:** Update the position vector (generators’ real power outputs, load demands, locations and sizes of FACTS devices) for each particle using the following steps.

**Step 4 (Reproduction Process):**

**Step 4A:** At each generation, the position of all individuals (except the best ones identified by \( G_{\text{best}} \)) will be improved by regulating their velocities based on the following equation:

\[ V_i(t+1) = \left( V_i(t) + c(P_i - S_i(t)) \right) + \sum_j \left( \text{rand}_j \times \omega_j (t) \times (S_j(t) - S_i(t)) \right), \quad j \in T_i \]

(10)

where \( i = 1, 2, ..., N_{\text{particle}} \) is the iteration number, \( T_i \) represents the set of particles-\( j \) with better achievements than the set of particles-\( i \), \( c \) is the cognitive parameter, and \( P_i \) is the best position of the particle-\( i \), while \( S_i(t) \) and \( S_j(t) \) are the positions of particles-\( i \) and particle-\( j \), respectively. The random parameters \( \text{rand}_j \) and \( \text{rand}_j \) are used to maintain the diversity of the population and are uniformly distributed within the ranges \([0, 0.999, 1]\) and \([0, 1]\), respectively. \( \omega_j (t) \) are the weighting factors of the coordinates as follow:

\[ \omega_j = \frac{f(S_j) - f(S_i)}{\sum f(S_j) - f(S_i)}, \quad j, l \in T_i \]

(11)

where \( f(S_j) \) is the achievement of particle-\( j \) and \( f(S_i) \) is the better achievement by particle-\( j \).

**Step 4B:** The velocity of the best particle in the swarm is updated using a random coordinator which is calculated between its position and the position of a randomly selected particle in the swarm:
Input power system parameters. Assume initial population size ($N_{\text{particle, max}}$) and maximum generation number ($N_{\text{it, max}}$).

Set initial population and generation counter ($N_{\text{particle}} = N_{\text{it}} = 1$).

Use a random generator to set the initial position and velocity vectors. For each particle, set its position as the best position.

Run N-R power flow for each particle. Compute objective function (Eq. 8).

If any limits are violated then apply the proposed penalty functions (using Table 1). Compute fitness function (Eq. 9).

Determine its best position. Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

Set initial population and generation counter ($N_{\text{particle}} = N_{\text{it}} = 1$).

If any of the limits are violated then apply proposed penalty functions (using Table 1). Compute fitness function (Eq. 9).

Determine its best position. Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

Velocity Updating Initialization

Fitness Process

If fitness ($P_{\text{particle}}$) > fitness ($P_{\text{best}}$),

Set $P_{\text{best, particle}} = P_{\text{particle}}$.

Set $N_{\text{particle}} = 1$

Yes

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

No

Yes

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

Yes

Update the position of the particle using Eq. (16).

Are position limits violated?

Yes

Apply penalty functions to the fitness function according to Table 1.

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

No

Run N-R power flow for each particle. Compute objective function (Eq. 8).

If any of the limits are violated then apply proposed penalty functions (using Table 1). Compute fitness function (Eq. 9).

Determine its best position. Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

Best Particle selection

Global Best Particle selection

Fitness (P_{\text{best, particle}}) > Fitness (P_{\text{best}})

Run N-R power flow for each particle. Compute objective function (Eq. 8).

If any of the limits are violated then apply proposed penalty functions (using Table 1). Compute fitness function (Eq. 9).

Determine its best position. Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

No

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

Yes

Fitness (P_{\text{particle}}) > Fitness (P_{\text{best}})

Set $P_{\text{best, particle}} = P_{\text{particle}}$.

Set $N_{\text{particle}} = 1$

Yes

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

No

Update the position of the particle using Eq. (16).

Are position limits violated?

Yes

Apply penalty functions to the fitness function according to Table 1.

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

No

Run N-R power flow for each particle. Compute objective function (Eq. 8).

If any of the limits are violated then apply proposed penalty functions (using Table 1). Compute fitness function (Eq. 9).

Determine its best position. Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

Best Particle selection

Global Best Particle selection

Fitness (P_{\text{best, particle}}) > Fitness (P_{\text{best}})

Set $N_{\text{particle}} = 1$

Yes

Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$.

No

Update the velocity of the best particle by a random coordinator, using Eq. (12).

Improve the position vectors by regulating their velocities using Eq. (10).

Print optimal generation and demand levels, optimal SSSC size and location.

Fig. 5. The proposed CAPSO algorithm for maximizing social welfare in double-sided auction market by placement and sizing of SSSC. Step 3D: Find a particle which has the maximum fitness function, the global best position ($G_{\text{best}}$). Set $N_{\text{particle}} = N_{\text{particle, max}} + 1$. 
where $S_i(t)$ is a randomly selected particle and $r$ is a random digit in the range of [0, 1].

To explore local, as well as, global feasible spaces, the inertia weight (at each iteration) is varied according to the following rule:

$$W = \frac{W_{\text{max}} - W_{\text{min}}}{N_{\text{it}_{\text{max}}}} \times N_{\text{it}}$$

where $N_{\text{it}}$ is the current iteration, while $W_{\text{max}}$ and $W_{\text{min}}$ are maximum and minimum values of inertia weights, respectively. The updating of best particle of the group behaves in a crazy way and helps the (best particle) swarm to escape from local optimal points.

**Step 5 (Velocity Bounds’ Oscillations):** Check if the bounds of velocities (Eqs. (14)-(15)) are enforced. If the bounds are violated then they are replaced with the respective limits to prevent excessively large steps during the initial phases of the search. The velocities of the $i$th particle in the $n$-dimensional decision space are limited by:

$$[-V_i^{\text{max}_l}, +V_i^{\text{max}_l}]$$

where the maximum velocity in the $l$th dimension of the search space is proposed as:

$$V_i^{\text{max}_l} = \frac{S_{i_{\text{max}}^{\text{max}_l}} - S_{i_{\text{min}}^{\text{min}_l}}}{N_{\text{r}}}$$

where $S_{i_{\text{max}}^{\text{max}_l}}$ and $S_{i_{\text{min}}^{\text{min}_l}}$ are the upper and lower limits in the $l$-dimension of the search space, while $N_{\text{r}}$ is the selected number of the search intervals which is an important parameter in the CAPSO algorithms. A small value of $N_{\text{r}}$ facilitates global exploration (searching new areas), while a large value tends to facilitate local exploration (fine-tuning of the current search area). A suitable value for $N_{\text{r}}$ will usually provide a balance between global and local exploration abilities and consequently results in a reduction in the number of required iterations to capture a near-global optimum solution. In this paper, $N_{\text{r}}$ is considered to be 15.

**Step 6 (Position Update):** Update the current positions (generators’ real power outputs, load demands and FACTS parameters) according to the following equation:

$$S_i(t+1) = S_i(t) + V_i(t+1)$$

**Step 7 (Position Limits):** Check if the limits of particles’ positions are enforced. If any of the limits are violated then a high penalty function is applied to the fitness function.

**Step 7A:** Run power flow for each particle and determine voltage magnitudes and phase angles at all buses. Calculate power flow in each transmission line of the system.

**Step 7B:** Compute the proposed penalty functions (Fig. 3) using outputs of the power flow. Compute fitness functions (Eq. (10)) for each particle, and find the best position for each particle. If the obtained position has better fitness function than the current $P_{\text{best}}$, then replace the new position in $P_{\text{best}}$. Otherwise, do not update the current $P_{\text{best}}$. Set $N_{\text{particle}} = N_{\text{particle}} + 1$.

**Step 7C:** If $N_{\text{particle}} < N_{\text{particle-max}}$ go to Step 7A; otherwise, if the obtained position has a better fitness function than the current $G_{\text{best}}$, replace the new position in $G_{\text{best}}$.

**Step 8 (Stopping Decisive Factor):** If the maximum number of iterations is achieved then print solution and stop; otherwise, go to Step 4.

### 7. Simulation Results

This section presents the basic operation of the modified IEEE 14-bus test system (Fig. 6) [13], as well as, locating and sizing of SSSC unit with smooth/nonsmooth generators cost curves (Eq. (8)), without/with transmission line flow constraints using the proposed CAPSO algorithm of Fig. 5.

In this test system, the objective function consists of 16 variables for the 5 generation and 8 demand nodes. In addition, three SSSC parameters are added to the variables of the objective function in each test system. There are 20 possible locations to place a SSSC unit in the modified IEEE 14-bus test system.

In addition, it is assumed that the SSSC reactance is 0.05 pu, maximum SSSC voltage is 0.1 pu and its phase angle
can change from 0 to 180 degrees.

In this paper, six cases are studied to demonstrate the ability of the proposed CAPSO algorithm (Fig. 5):

**Cases A1 and A2:** Base operation without line flow constraints & without SSSC using smooth (A1) and nonsmooth (A2) generation cost functions. Simulation results are presented in Tables 2-6.

**Cases B1 and B2:** Base operation with line flow constraints & without SSSC using smooth (B1) and nonsmooth (B2) generation cost functions. Simulation results are provided in Tables 2-6.

**Cases C1 and C2:** Base operation with line flow constraints and SSSC using smooth (C1) and nonsmooth (C2) generation cost functions (Tables 5, and 7-10).

Selected parameters for the proposed CAPSO algorithm are: number of generations= 1000, number of populations= 95 and the cognitive parameter = 0.7.

### 7.1 Validation of the proposed CAPSO algorithm

Two case studies (A1, B1) are performed to validate the proposed method. Tables 2 and 3 present comparisons of the achieved locational marginal prices (LMPs) and bus voltages by the proposed CAPSO algorithm. Note that the conventional SQP approach is not applicable (N/A) for this operating condition, while the proposed approach can effectively account for the nonsmooth cost curve characteristics and solve the optimization problem (Eq. (8)). Compared to other optimization techniques such GA and Fuzzy-GA [23, 24], the proposed CAPSO achieves better solutions without/with one SSSC unit (Tables 2-3).

According to columns 5 and 8 of Table 2, there is negligible difference (<1%) between the results of the two algorithms for social welfare. This difference is calculated using:

$$\Delta SW(\%) = \left( \frac{SW_{CAPSO} - SW_{FGA}}{SW_{FGA}} \right) \times 100$$

### 7.2 Operation with enforcement of transmission line constraints and without SSSC

Enforcement of transmission line constraints alleviates the congestion on the transmission lines (Table 4, columns 3 and 4). However, as expected, line flow constraints cause a significant decrease in social welfare (Tables 2 and 3, row 4). Therefore, line flow constraints are the main causes of low social benefit and low loading levels.

Moreover, some power consumers will have to bid higher prices in power markets since they cannot access cheaper power due to transmission limits. Table 5 shows the achieved locational marginal prices (LMPs) and bus voltages by the proposed CAPSO algorithm. Note that the line flow constrains show significant impacts on the LMPs.

### Table 2. Cost-benefit analysis by CAPSO, Fuzzy GA, GA and SQP algorithms with smooth cost curves.

<table>
<thead>
<tr>
<th>Case</th>
<th>A1</th>
<th>B1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAPSO (Fig. 5)</td>
<td>Fuzzy-GA [23, 24]</td>
</tr>
<tr>
<td>Gen Cost ($/h)</td>
<td>1650.51</td>
<td>1665.13</td>
</tr>
<tr>
<td>Cust Benefit ($)</td>
<td>3640.65</td>
<td>3637.50</td>
</tr>
<tr>
<td>Social Benefit ($)</td>
<td>1990.145</td>
<td>1972.36</td>
</tr>
</tbody>
</table>

*) Calculated using Eq. (16).

### Table 3. Cost-benefit analysis by CAPSO, Fuzzy GA, GA and SQP algorithms with nonsmooth cost curves.

<table>
<thead>
<tr>
<th>Case</th>
<th>A2</th>
<th>B2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAPSO (Fig. 5)</td>
<td>Fuzzy-GA [23, 24]</td>
</tr>
<tr>
<td>Gen Cost ($/h)</td>
<td>1631.88</td>
<td>1666.15</td>
</tr>
<tr>
<td>Cust Benefit ($)</td>
<td>3601.81</td>
<td>3602.81</td>
</tr>
<tr>
<td>Social Benefit ($)</td>
<td>1969.13</td>
<td>1956.66</td>
</tr>
</tbody>
</table>

*) Congested transmission lines are printed in bold.
and will increase them at most buses. Some analyses of the simulation results for the modified IEEE 14-bus test system are as follow:

- Due to the weak structure of the transmission lines system, after the enforcement of transmission line constraints, the total system load reduces form the maximum value of 386.59 MW to 343.37 MW (Table 6). This indicates that the system under consideration does not have the capability of supporting the maximum load under assumed congestion conditions. As a result, the increased generation costs will reduce the total system social welfare. Therefore, improving the transmission lines is recommended.

- According to Table 4, in the modified IEEE 14-bus test system without line flow constraints, generators G1, G2, G3 and G4 are highly loaded. With line constrains, generation level of G4 is substantially decreased to about 48.7 MW. This is the maximum capacity of lines connected to this generator at node 6. As a result, the overall social benefit is decreased (Table 2, row 4).

- With unconstrained conditions, congestion occurs in lines 7-10, 13 and 14-17 (Table 4).

- Transmission line limits overcome the congestion problem (Table 4, columns 3 and 4); however, social benefit decreases from 1990.14 $/h to 1535.29 $/h and from 1969.13 $/h to 1534.74 $/h for smooth and nonsmooth cost curves, respectively (Tables 2 and 3, row 4). Therefore, line flow constrains are the main causes of low social benefit and high generation cost.

- According to Table 3 for case A1, inclusion of the sin component on the generator’s characteristics decreases the total customer benefit from 3640.65 $/h to 3601.01 $/h and decreases the social benefit from 1990.1 $/h to 1969.1 $/h, respectively. Therefore, the ISO needs to consider the actual valve setting points in the objective function by including nonsmooth characteristics to achieve more accurate results and perform realistic cost analysis.

### Table 5. Obtained LMP values and voltage levels by the proposed CAPSO for the modified IEEE 14-bus test system without SSSC

<table>
<thead>
<tr>
<th>Bus</th>
<th>Case A1</th>
<th>Case B1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LMP ($/MWh)</td>
<td>$/p.u.$*</td>
</tr>
<tr>
<td>1</td>
<td>25.31</td>
<td>1.050</td>
</tr>
<tr>
<td>2</td>
<td>26.39</td>
<td>1.041</td>
</tr>
<tr>
<td>3</td>
<td>28.03</td>
<td>1.014</td>
</tr>
<tr>
<td>4</td>
<td>27.60</td>
<td>1.016</td>
</tr>
<tr>
<td>5</td>
<td>27.21</td>
<td>1.018</td>
</tr>
<tr>
<td>6</td>
<td>27.20</td>
<td>1.060</td>
</tr>
<tr>
<td>7</td>
<td>27.63</td>
<td>1.046</td>
</tr>
<tr>
<td>8</td>
<td>27.63</td>
<td>1.060</td>
</tr>
<tr>
<td>9</td>
<td>27.64</td>
<td>1.043</td>
</tr>
<tr>
<td>10</td>
<td>27.73</td>
<td>1.039</td>
</tr>
<tr>
<td>11</td>
<td>27.59</td>
<td>1.046</td>
</tr>
<tr>
<td>12</td>
<td>27.64</td>
<td>1.045</td>
</tr>
<tr>
<td>13</td>
<td>27.80</td>
<td>1.039</td>
</tr>
<tr>
<td>14</td>
<td>28.30</td>
<td>1.023</td>
</tr>
</tbody>
</table>

*) Maximum and minimum voltage limits are set to 1.10 p.u. and 0.95 p.u.

### Table 6. Obtained optimal generation and load levels in MW by the proposed CAPSO for the modified IEEE 14-bus with smooth cost curves without optimal locating/sizing of one SSSC.

<table>
<thead>
<tr>
<th>Generator/Load</th>
<th>Smooth generation cost curve</th>
<th>Proposed CAPSO</th>
<th>Fuzzy-GA</th>
<th>SQP [22]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
<td>B1</td>
<td>A1</td>
<td>B1</td>
</tr>
<tr>
<td>G1</td>
<td>100</td>
<td>97.66</td>
<td>94.22</td>
<td>97.25</td>
</tr>
<tr>
<td>G2</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>G3</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>G4</td>
<td>86.59</td>
<td>48.71</td>
<td>92.83</td>
<td>48.94</td>
</tr>
<tr>
<td>L1</td>
<td>58.65</td>
<td>117.11</td>
<td>58.10</td>
<td>116.90</td>
</tr>
<tr>
<td>L2</td>
<td>55.20</td>
<td>124.89</td>
<td>55.63</td>
<td>125.14</td>
</tr>
<tr>
<td>L3</td>
<td>5.03</td>
<td>6.54</td>
<td>5.63</td>
<td>8.02</td>
</tr>
<tr>
<td>L4</td>
<td>20.98</td>
<td>17.08</td>
<td>21.54</td>
<td>16.86</td>
</tr>
<tr>
<td>L5</td>
<td>36.03</td>
<td>23.97</td>
<td>35.79</td>
<td>22.15</td>
</tr>
<tr>
<td>L6</td>
<td>52.11</td>
<td>30.99</td>
<td>51.88</td>
<td>31.23</td>
</tr>
<tr>
<td>L7</td>
<td>72.23</td>
<td>7.27</td>
<td>73.90</td>
<td>7.16</td>
</tr>
<tr>
<td>L8</td>
<td>62.01</td>
<td>7.81</td>
<td>62.33</td>
<td>7.664</td>
</tr>
<tr>
<td>Total</td>
<td>386.59</td>
<td>343.37</td>
<td>387.05</td>
<td>346.15</td>
</tr>
<tr>
<td>Total load</td>
<td>362.22</td>
<td>335.66</td>
<td>362.8</td>
<td>335.064</td>
</tr>
</tbody>
</table>

\[
\Delta SW = SW_{Without\, TSSC} - SW_{With\, TSSC} \quad (17)
\]

Table 8 provides the obtained LMP values by the proposed CAPSO for the modified IEEE 14-bus with SSSC. It is to be noted that the LMP may not be uniformly distributed and some participants may benefit more than others. However, SSSC will provide overall benefit to the system as a whole, while some market participants may benefit more than others. Moreover, introduction of SSSC increases social benefit and decreases the total system cost.
This demonstrates the effectiveness of optimal sizing and placement of SSSC.

Comparison of LMP values for the three case studies (A, B and C) are presented in Fig. 7. In this figure, the radius of the circles represents LMP values and bus numbers are identified on their circumferences. Based on the Fig. 7 and Tables 7-9, analysis of the results can be summarized as follows:

- If transmission line constraints are not considered, all system buses have similar LMP values represented by circular trajectories (Case A in Fig. 7 corresponding to column 2 of Table 5).
- However, after the inclusion of line constraints there is a non-uniform increase in LMP values as shown by the two noncircular trajectories (Case B in Fig. 7 corresponding to column 4 of Table 5).
- After optimal placement and sizing of SSSC by the proposed CAPSO algorithm, the LMP values decrease and their trajectories become circular again (Case C in Fig. 7 corresponding to columns 2 and 4 of Table 8).

It is to be noted that the benefits to individual consumers may be not uniformly distribute and some participants may actually face reduction in their welfare/profit. However, SSSC will provide overall benefit to the system as a whole, while some market participants may benefit more the others.

The total system cost for Cases A, B and C are presented in Tables 2-3 and 8. Note that application of SSSC has increased the social welfare from 1535.29 Table 7.

<table>
<thead>
<tr>
<th>Bus</th>
<th>Case C1: CAPSO with SSSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMP ($/MWh)</td>
<td>$1.050</td>
</tr>
<tr>
<td>$27.44</td>
<td>1.013</td>
</tr>
<tr>
<td>$30.01</td>
<td>0.970</td>
</tr>
<tr>
<td>$28.44</td>
<td>0.982</td>
</tr>
<tr>
<td>$27.37</td>
<td>0.989</td>
</tr>
<tr>
<td>$27.60</td>
<td>1.052</td>
</tr>
<tr>
<td>$28.33</td>
<td>1.010</td>
</tr>
<tr>
<td>$28.33</td>
<td>1.051</td>
</tr>
<tr>
<td>$28.28</td>
<td>1.027</td>
</tr>
<tr>
<td>$28.33</td>
<td>1.022</td>
</tr>
<tr>
<td>$28.06</td>
<td>1.036</td>
</tr>
<tr>
<td>$28.08</td>
<td>1.039</td>
</tr>
<tr>
<td>$28.25</td>
<td>1.033</td>
</tr>
<tr>
<td>$28.88</td>
<td>1.011</td>
</tr>
</tbody>
</table>

*) Calculated using Eq. (17).
S/h to 1670.49 $/h, resulting in an annual saving of $1.185 Million.

- In addition, SSSC has also improved the system voltage profile. This is evident from the comparison of bus voltage levels for the three cases as presented in Tables 5 and 8.
- After the placement and sizing of SSSC, generation of G4 is significantly increased (in order to transfer more power from node 6 to node 13), which increases the social benefit (Table 8). These results demonstrate the ability of SSSC in improving system operation with line flow constraints.
- Furthermore, after the placement of SSSC, load levels at nodes 11, 12, 13, and 14 (corresponding to loads 5-8) -previously elevated due to the line flow constraints- are now decreased (Tables 7). This is due to their lower benefit coefficients compared to the other loads.

7.4 Economical analysis of using SSSC

Considering the high cost of FACTS devices that is considered in this paper, it is important to perform cost analysis before the allocation of SSSC. This is carefully done in the optimization procedure (Eq. (8)) by including the initial investment cost, the income and the expected life of the SSSC, as well as an annual inflation cost of 6%.

For the economic analysis of SSSC, its cost is compared with the total system cost improvement in Table 7. According to Tables 7 and 8, the application of SSSC has improved the annual capital cost and the social welfare to 60.51×10^3$/year and 1185×10^3$/year, respectively; which correspond to an annual net revenue of 1124.49$/year. Therefore, installation of SSSC in the IEEE 14-bus test system offers benefit that exceeds its cost. In addition, the investment cost of SSSC is significantly less than the improvement in the total social welfare.

Fig. 8 shows the individual welfare of each consumer without/with the line flow constraints and the impact of one SSSC unit (including its best location and size) on the social benefit, considering nonsmooth cost curves. It is to be noted that the benefits to individual consumers are not uniformly distributed and some participants may actually face reduction in their welfare/profit.

Furthermore, according to Tables 6, without any line flow constraints, there are very high load demands at nodes 11-14 (corresponding to loads 5-8) due to higher benefit coefficients. However, when the line flow constraints are considered (Tables 6), there are substantial reductions in load demands and social benefit at these nodes (Fig. 8). Also, line flow constraints will substantially increase loading levels (Table 6) at nodes 4-5 (corresponding to loads 1-2) and increase their social benefits (Fig. 8). In addition, the generation level of generator G4 (located at bus 6) is decreased and therefore, the total system generation cost is increased.

In general, the simulation results of this paper indicate that optimal placement and sizing of one SSSC device using the proposed CAPSO algorithm is cost-effective as it will reduce the total annual system cost and improve the system voltage profile.

7.5 Impact of valve point-loading on social welfare

In this paper, the valve point loading effect is considered in the optimization objective function. To show its impacts on the obtained results, the following analyses are performed:

- According to Table 8 at the same load level (Cases C1 and C2), inclusion of the sin component on the generator’s characteristics increases the total generation cost and decreases the total system social welfare.
- In addition, considering valve point loading effect in objective function changes the size and investment cost of the SSSC and affects the amount of social welfare (Tables 7 and 8).

Therefore, the ISO needs to consider the actual valve setting points in the objective function by including nonsmooth characteristics to get results that are more accurate and perform realistic cost (Tables 7-9).

8. Conclusions

A coordinated aggregation based particle swarm optimization algorithm is proposed and implemented to maximize social welfare and perform congestion management in a double-sided auction market by appropriate locating and sizing of SSSC device and optimal rescheduling of generation and demand levels. The valve point loading effects is included to the quadratic smooth generator cost curves to establish more accurate model. Adding the sine part in the objective function significantly increases the degree of complexity and the difficulty of detecting the global solution. To guarantee that locational marginal prices charged at the demand buses is less than or equal to DisCos benefit, quadratic consumer benefit functions are incorporated in the objective function. In addition, SSSC investment cost versus its economical
benefits on the power systems is presented. Numerous cases are studied to show the ability of detecting best solutions using the proposed method. The sequential quadratic programming outcomes are compared with the proposed method. Main conclusions based on simulation results for the modified IEEE 14-bus test system are:

• SSSC has the ability to redistribute power flow, influence load and generation levels at different buses, and significantly increase the social benefit (Tables 8). Installation of SSSC offers benefit that exceeds its cost for the system conditions studied.
• SSSC device may have different impacts on the welfare of individual participants and may affect the double-sided auction price of each bus differently. Therefore, some participants may benefit more than others.
• The proposed method shows the benefits of SSSC in a deregulated power market and demonstrates how they may be utilized by ISO to improve the total system social welfare and prevent congestion.
• To investigate the impact of SSSC on the social welfare improvement, its costs is compared with the social welfare improvement in Table 5. According to this table, the achieved social welfare improvement using SSSC is 135.20$/h while the cost of SSSC is only 7.431$/h. Consequently, SSSC will provide overall benefit to the system as a whole.
• The smoothness of the generator cost curves shows no significant impact on line overloading; however, it will increase generation cost. This needs to be considered by ISO to get more accurate results and realistic cost analysis.
• Compared to other optimization techniques such SQP and Fuzzy-GA, the proposed CAPSO algorithm achieves better solutions without/with SSSC (Tables 2-3).

References


Appendix

The nonsmooth cost coefficients (Eq. (8)) are presented in Table A1. The test systems data is presented in [23, 24].

Table A1. Characteristics of the GenCos in the modified IEEE 14-Bus

<table>
<thead>
<tr>
<th>GenCos</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>50</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>f</td>
<td>0.063</td>
<td>0.098</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>