Abstract - We report on further development of a new method for frequency control of the external cavity diode laser with the use of FM sideband heterodyne technique where AOM operating in the Raman-Nath diffraction mode is employed as an external phase modulator. It is shown that the AOM design can be considerably simplified and its size can be minimized if the Raman-Nath diffraction mode is the dominant diffraction mode of AOM. In our experiment, the acousto-optic interaction length of the AOM designed primarily for operation in such a mode was 2 mm. Such compact devices can be used as phase modulators in a wide range of applications in laser spectroscopy, where the frequency noise reduction of a variety of the lasers is essential.

INTRODUCTION

A new technique for frequency control of an external cavity diode laser without the use of direct injection current modulation has been presented in [1,2]. FM sideband heterodyne spectroscopy with AOM operating in the Raman-Nath diffraction mode as an external phase modulator and the saturated absorption resonances within D_2 absorption line of Cs atom as the frequency references were used to produce the error signals for high speed servo-loop.

It is known that the free running line width of the external cavity diode laser (ECDL) is of the order of a few megahertz and is limited by the mechanical and acoustical vibrations of the external cavity. Such frequency fluctuations can be suppressed by the electronic feedback to the laser diode (LD) injection current. At present, a conventional FM sideband technique [3] with the electro-optical modulator (EOM) as an external phase modulator is used to provide the error signals for servo-loops. The saturated absorption resonances lie in the area intermediate between the limit cases of the operation in the Raman-Nath diffraction or in the Bragg reflection mode. We have replaced the 1205-C2 AOM by the using "ISOMET" model 1205-C2 AOM as an external phase modulator.

It has been shown in [1,2] that the acousto-optic modulator (AOM) operating in a Raman-Nath diffraction mode as an external phase modulator has some features advantageous in comparison with EOMs which are conventionally used for the same purposes. By varying only the local oscillator frequency, the error signals with the appropriate shape and slope sign have been obtained in a wide frequency range. The power of the driving local oscillator signal, sufficient to get few percent of the ratio of powers of each ±1st diffraction order sidebands to the carrier power, did not exceeded 100 mW. Due to the fact that the available optical phase modulators do not produce the pure FM spectrum, a careful control of the input and output polarizations of the light passing through AOM is needed to avoid residual amplitude noise. In case of using AOM as an external phase modulator, there is no need to take care about the polarization of the light passing through AOM.

THEORY

Fig. 1 shows the typical experimental configuration of the conventional FM sideband spectroscopy [3]. A single-frequency laser provides the radiation at frequency \( \omega \) and with the electric field \( E = (1/2) E_0 \exp(\iota \omega t) + c.c. \). The optical beam is passed through a phase modulator driven the sinusoidal RF signal at frequency \( \Omega \). It is known that the output radiation has a pure FM spectrum:

\[
E_1(t) = \frac{E_0}{2} \sum_{N=\pm \infty} J_0(M) \exp \left[ \iota \left( \omega + N \Omega \right) t \right] + c.c.,
\]

where \( M \) is the modulation index and \( J_0 \) – are the Bessel functions of order \( N \). If \( M \ll 1 \), \( J_0(M) \approx 1, J_1(M) = \pm 1 \).
After that the beam is passed through the dispersive medium of length \( l \), whose intensity-absorption coefficient \( \alpha \) and index of refraction \( n \), are the functions of optical frequency. To take into consideration the amplitude attenuation \( \delta_n \) and optical phase shift \( \phi_n \) experienced by each frequency component, the following quantities are introduced in [3]: \( T_2 = \exp(-\delta_n - i\phi_n) \), \( \delta_n = \alpha n l / 2 \) and \( \phi_n = n \eta (\omega + \Omega) / c \), where \( N = -1, 0, 1 \) denotes the values at frequencies \( \omega - \Omega, \omega \) and \( \omega + \Omega \), respectively. The output FM radiation will be described by (1/2)\( E_A(t) + c. c. \), where

\[
E_2 = E_0 \left[ T_2 e^{i \omega t} + T_{-2} e^{i (\omega + \Omega) t} - T_{-2} e^{i (\omega - \Omega) t} \right]
\]  

The photodetector electrical signal \( S(t) \) is proportional to the intensity of the light coming onto photodetector \( I_2(t) = c |E_2(t)|^2 / 8 \pi \) and will contain a beat signal at modulation frequency \( \Omega \) if all the \( \delta_n \) or if all the \( \phi_n \) are not equal. It is shown in [3] that

\[
I_2(t) = c |E_2|^2 / 8 \pi = \frac{c E_0^2}{8 \pi} e^{-2 \delta_0} \times
\]

\[
\times \left[ 1 + \cos(\phi_1 - \phi_2) M \sin \Omega t \right]
\]

where the in-phase component (cos \( \Omega \)) of the beat signal is proportional to the difference in loss experienced by the upper and lower sidebands, whereas the quadrature (sin \( \Omega \)) component is proportional to the difference between phase shift experienced by the carrier and the average of the phase shifts experienced by the sidebands. The main feature of the FM spectroscopy is the condition when the modulation frequency \( \Omega \) is large compared with the line width of the probed optical resonance. When a single isolated sideband (for instance, the upper one) probes the optical transition, the losses and phase shift experienced by the carrier and lower sideband remain constant values so \( \tilde{\delta} = \delta_0 = \delta_1 \) and \( \tilde{\phi} = \phi_0 = \phi_1 \), where \( \tilde{\delta} \) and \( \tilde{\phi} \) are the constant background loss and phase shift, respectively. If we denote \( \Delta \tilde{\delta} = \delta_1 - \tilde{\delta} \) and \( \Delta \tilde{\phi} = \phi_1 - \tilde{\phi} \) as the values describing the deviations from the background values caused by the optical transition, then Eq. (2) simplifies to:

\[
I_2(t) = c |E_2|^2 / 8 \pi = \frac{c E_0^2}{8 \pi} e^{-2 \tilde{\delta}} \times
\]

\[
x \left[ 1 + \Delta \tilde{\delta} M \cos \Omega t + \Delta \tilde{\phi} M \sin \Omega t \right]
\]

Thus the cos\( \Omega \) and sin\( \Omega \) components are directly proportional to the absorption and dispersion, respectively.

The rf beat signal results from heterodyning of the FM sidebands and thus the photodetector electrical signal \( S(t) \) is proportional to the geometrical mean of the intensities of one sideband and of the carrier. If we assume for simplicity that a purely absorptive feature is probed with a single isolated sideband and no background absorption is present, that means that \( \Delta \phi = \tilde{\phi} = 0 \), so the optical power incident on a photodetector of area \( A \) is given by:

\[
P_2(t) = P_0 \left( 1 - \Delta \tilde{\delta} M \cos \Omega t \right)
\]

where \( P_2(t) = A |E_2(t)|^2 / 8 \pi \) is the total laser power. The photodetector current \( i(t) \) of quantum efficiency \( \eta \) and gain \( g \) is

\[
i(t) = i(t) + i_s(t)
\]

where the dc photocurrent is given by:

\[
i_s(t) = -g e \eta \frac{P_0}{h \omega} \Delta \tilde{\delta} M \cos \Omega t
\]

It has been reported in [1] that, for the first time, phase-sensitive detection of the sub-Doppler resonances within \( \text{D}_2 \) absorption line of Cs atom has been carried out by means of conventional FM sideband heterodyne spectroscopy with acousto-optic modulator operating in the Raman-Nath diffraction mode as an external phase modulator. This technique has been used to obtain the error signals for wide bandwidth servo-loop for frequency control of the ECDL as well.

In this paper we present the theoretical background of the AOM in the Raman-Nath diffraction regime in order to confirm its validity for use as phase modulator in the FM sideband heterodyne spectroscopy.

Let the sinusoidal elastic wave penetrates through the homogeneous medium and a single-frequency laser beam is directed perpendicular to the wave vector of the elastic wave (Fig. 2). Acoustic wave running in the \( y \) direction induces periodical varying of the index of refraction compared with its initial mean value \( n_0 [4] \):

\[
n = n_0 + \Delta n \sin \Omega \left( t - \frac{y}{v} \right)
\]

where \( \Omega \) is acoustic wave frequency and \( v \) is the acoustic wave velocity.

For light wave directed on \( x \), the folloing expression is valid:
The starting counting point of the phase angle is chosen at front edge of the acoustic beam \((x = 0)\); \(\lambda_0\) is a laser radiation wavelength in vacuum. Electric field of the light passing through acoustic beam of width \(L\) is given by:

\[
E_i = E_0 \cos \left[ \omega t + \Phi_0 + \Delta \Phi \sin \Omega \left( t - \frac{y}{v} \right) \right],
\]

where \(\Phi_0 = - \frac{2\pi n}{\lambda_0} L\) is a phase shift determined by mean value \(n_0\) of the index of refraction and \(\Delta \Phi = - \frac{2\pi n}{\lambda_0} \Delta n\) is a phase shift induced by varying part \(\Delta n\) of the index of refraction. Thus, at any point on the acoustic wave edge \((x = L, \ y = \text{const})\), the light beam is phase modulated, and this modulation depends only on the time \(t\). The electrical field \(E_1\) of the optical radiation passed through AOM will be described as:

\[
E_1 = E_0 J_0(\Delta \Phi) \cos(\omega t + \Phi_0) + E_0 \sum_{n=1}^{N} J_n(\Delta \Phi) \times
\]

\[
\cos(\omega t + N \Omega t + \Phi_0) + (-1)^n \cos(\omega t - N \Omega t + \Phi_0)
\]

Here \(\tau = t - y/v\).

Thus light wave spectrum has a maximum at carrier frequency \(\omega\) and sidebands at \(\omega \pm N \Omega\), since \(\tau = t - y/v\), where \(N\) is integer number. The amplitude dependence of the carrier and of the sidebands on the phase shift \(\Delta \Phi\) is given by Bessel functions \(J_n(\Delta \Phi)\).

At the moment the carrier phase angle \(\Phi_0 = - \frac{2\pi n}{\lambda_0} L\) does not depend on \(y\). This is not valid for the waves corresponding to the sidebands. For example, for the wave

\[
J_N(\Delta \Phi) \cos \left( \Omega t + \Phi_0 - \frac{N \Omega y}{v} \right)
\]

the phase \(\Phi_N\) depends on \(y\), e.g. on the point where light beam crosses acoustic beam edge. Indeed,

\[
\Phi_N = \Phi_0 - \frac{N \Omega y}{v} = \Phi_0 - 2\pi \frac{N y}{\Lambda},
\]

where \(\Lambda\) is sound wavelength. The interference of the waves of the same order irradiated from the acoustic beam rear edge \((y = L)\), leads to their addition only if they are directed at angle \(\theta_N\) defined by expression:

\[
\sin \theta_N = N \lambda / \Lambda.
\]

Sidebands of the order \(N\) at frequency \(\omega + N \Omega\) are reflected at angle \(\theta_N\), and the sidebands at frequency \(\omega - N \Omega\) experience symmetrical reflections. Thus the input light beam, directed perpendicular to the acoustic wave direction, is split in a row of the beams diverging at the angles \(\theta_N\) symmetrically relative to the incidence direction of the light. These angles are defined by : \(\sin \theta_N = \pm N \lambda / \Lambda\).

The previous considerations are valid only if acoustic wave beam width \(L\) does not exceed some critical value \(L_{cr}\). It is shown in [4] that it should not exceed critical value for the first order sideband which equals to \(L_{cr} = L_1 = \Lambda^2 / \lambda\).

For example, in case of crystal TeO\(_2\) with index of refraction \(n = 2,367\) and with acoustic wave velocity \(v = 4,2\ \text{km/s}\), at \(\lambda_0 = 850\ \text{nm}\), at modulation frequency \(\Omega = 40\ \text{MHz}\), the acoustic wave critical width is \(L_{cr} = 31\ \text{mm}\).

Dropping out in (7) the terms describing constant phase shift, we get the expression similar to (1):

\[
E_i = E_0 \cos(\omega t + \Delta \Phi \sin \Omega t) =
\]

\[
i = 0 \sum_{n=1}^{\infty} J_n(\Delta \Phi) \cos(\omega t + N \Omega t) + (-1)^n \cos(\omega t - N \Omega t)
\]

The phase modulation \(\Delta \Phi = - (2\pi L / \lambda_0) \Delta n\) plays now the role of the modulation index \(M\). The amplitude of modulation of the index of refraction is equalled:

\[
[\Delta n] = \left[ \frac{n_0 + p / 2}{\sqrt{1 / \rho v^2}} \right] \gamma / L
\]

where \(I = P / L d\) is sound wave intensity, \(\rho\) is the volume density of the crystal material, \(p\) is a photoelastic constant, \(d\) is a width of the acousto-optic interaction area or light beam diameter. The index of refraction varying is proportional to the square root of acoustic power and, note that important fact, is inversely proportional to the square root of the acousto-optic interaction length. Thus light wave phase modulation will be proportional to the square root of the acoustic-optic interaction length:

\[
\Delta \Phi = - (2\pi L / \lambda_0) \Delta n \sim (L)^{1/2}.
\]

For example, in case of crystal TeO\(_2\) with the following values of:

\(n = 2,367, \ v = 4,2\ \text{km/s}, \ \lambda_0 = 850\ \text{nm}, \ P = 40\ \text{mW}, \ \rho = 6,0,10^3\ \text{kg/m}^3, \ p = 0,24, \ d = 2\ \text{mm}, \ \Delta \Phi\) value equals to: \(\Delta \Phi = - (2\pi L / \lambda_0) \Delta n = -0,47\) for \(L = 17\ \text{mm}\) and \(\Delta \Phi = -0,16\) for \(L = 2\ \text{mm}\).

If we assume that \(J_0(\Delta \Phi) \approx 1\), then \(J_n(\Delta \Phi) = \pm \Delta \Phi / 2\), and the terms of higher order are vanished and Eq.(8) simplifies to:

\[
E_i = E_0 \cos(\omega t) + E_0 \Delta \Phi / 2 \times
\]

\[
\cos(\omega t) - \cos(\omega t - \Delta \Phi)
\]
In our experiments, the dispersive medium of length for the intensity of the light ±1st diffraction order at frequencies ω ± Ω with calculated diffraction angle δ_1 ≈ 0.2° at modulation frequency Ω = 40 MHz.

Thus the spectrum of the optical radiation passed through AOM operating in the Raman-Nath diffraction mode will be consisted of the strong carrier at frequency ω and of two weak sidebands at ω ± Ω and with amplitudes \( |E_0(ΔΦ/2)| \) diverging at the angle δ_1, symmetrically relative to the incidence direction of the input light. The ratio of powers of each sidband of ±1st diffraction order to the carrier power is \( P_{±1}/P_0 = (ΔΦ/2)^2 \). In the experiments where we used “ISOMET” AOM [1,2] with single crystal made of PbMoO_4, having properties similar to those of crystal TeO_2, and with parameters : \( L = 20 \) mm, \( d = 2 \) mm, at power of the signal driving AOM \( P = 40 \) mW, the ratio \( P_{±1}/P_0 \) did not exceed 4%, that gives for ΔΦ the values smaller than 0.4.

In our experiments, the dispersive medium of length \( l \) is a cesium saturated vapor cell. The saturated absorption resonances of \( D_2 \) line of Cs atom were probed in the field of two counter-running radiations, one of them, saturating the absorption, had the carrier frequency \( ω_c \) and the second one, probing the transition, consisted of three beams corresponding to the carrier and to the sidebands of ±1st diffraction order at frequencies ω ± Ω with calculated diffraction angle δ_1 ≈ 0.2° at modulation frequency Ω = 40 MHz.

Introducing, as we did before when we described the principles of the FM spectroscopy, amplitude attenuation \( δ_1 \) and phase shift \( φ_0 \), experienced by each frequency component \( (2) \), we will find the similar to (3) expression for the intensity of the light \( I_c(t) \) incident on a photodetector:

\[
I_c(t) = c |E^2_0| / 8\pi = c E^2_0 e^{-2\delta} \times
\]

\[
\left[ 1 + Δδ \ ΔΦ \ \cos Ω t + Δϕ \ ΔΦ \ \sin Ω t \right]
\]

and the beat signal photocurrent \( i_b(t) \) is now proportional to ΔΦ and, as followed from (11), to \((L)^{1/2}\).

**EXPERIMENT**

The experimental set-up is shown in Fig.3. Grating stabilized diode laser in the Littrow configuration provided tunable single-frequency input radiation at 851nm.

The modulator was ISOMET model 1205-C2 AOM, with 80 MHz center frequency and with sweep bandwidth of 40 MHz, driven by 40 mW of rf power at frequencies large compared to the 5 MHz of natural line width of the optical transitions. The AOM output consisted only of three beams corresponding to the carrier light and to the nearest sidebands of the ±1st diffraction order.

The AOM single pass output presenting pure frequency modulated and spatially separated optical spectrum was used as a probe beam for saturated absorption spectroscopy in a magnetically shielded cesium vapor cell at room temperature and was focused on a p-i-n photo-detector PD1, which had a bandwidth of 40 MHz.

The ratio of powers of each ±1st diffraction order sidebands to the carrier power in the front input window of the Cs cell was equal to \( P_{±1}/P_0 = 65 \) µW / 1.75 mW ≈ 0.04.

The rf beat was detected by the heterodyne detection using double-balanced mixer, which produced dc signal served as an error signal for our servo system. Fig.4 and Fig.5 show dispersion like shaped correction signals obtained when the laser frequency is scanned across Doppler profile of caesium \( D_2 \) absorption line and coincides with the optical transition frequencies 6S\(_{1/2}\), F = 4 – 6P\(_{3/2}\), F' = 3, 4, 5, where F and F' are the total angular momenta of the atom in the ground and excited states.

As it was reported in [1], by varying only the local oscillator frequency, we found the error signals with the appropriate shape and slope sign in a wide frequency range from about 10 MHz up to 40 MHz. Acceptable level of the obtained in a wide AOM modulation frequency range mixer output signals allowed to use them as the error signals for a high-speed servo-loop whose output correction signal was added to the laser current. A second, low-frequency servo-loop was used to compensate for slow drifts caused by thermal and mechanical perturbations.

In the experiments described in [1,2] we used ISOMET model 1205-C2 AOM. This AOM is designed primarily for use as the Bragg deflector, with 80 MHz center frequency and with 30 MHz sweep bandwidth. At lower modulation frequencies, its diffraction properties lie in the area intermediate between the limit cases of operation in the Raman-Nath diffraction or in the Bragg reflection mode. According to [4], the limit conditions are defined by the dimensionless Klein-Cook factor: Q = 2πLλ/\( \Lambda^2 \), where \( L \) is the acousto-optic interaction length, \( λ \) and \( \Lambda \) are the light and the acoustic wavelengths, respectively. The area Q < 0.3 corresponds to the Raman-Nath case, and the Bragg diffraction mode is dominant at Q > 4π.
If AOM does not produce the pure FM spectrum, as it should happen in case of operation in the Raman-Nath diffraction mode at normal incidence of the input optical beam, small imbalance in the amplitudes of the sidebands or their relative phase shift can prevent the beat frequency from vanishing exactly and can introduce nonzero baseline of the error signals, as one can see in Fig. 4, and additional AM noise.

To avoid this problem, we have replaced the 1205-C2 AOM by the developed at VNIIFTRI, designed for operation in the Raman-Nath diffraction mode, AOM with 40 MHz center frequency and with 40 MHz sweep bandwidth. Its acousto-optic crystal material is TeO$_2$ (the 1205-C2 crystal material is PbMoO$_4$).

By using (11), without taking into account the acoustic power losses, we have calculated $\Delta \Phi = -(2 \pi L/\lambda_0) \Delta n$ for acousto-optic interaction lengths $L = 17$ mm and $L = 2$ mm: $\Delta \Phi = -0.47$ and $\Delta \Phi = -0.16$, respectively.

The 2 mm acousto-optic interaction length is about one order smaller than that of 1205-C2 AOM. New AOM has $Q=0.41$ at 40 MHz modulation frequency. At such Q value, Raman-Nath diffraction mode is dominant, and Bragg reflection of the light mainly into one beam of the first diffraction order is not visually observed yet. At the same 40 mW of the driving AOM rf power, the zero background level of the error signals was achievable for all of the sub-Doppler resonances without noticeable losses in the amplitude of the signals (Fig. 5) compared with those obtained when 1205-C2 AOM was used.

Finally, we have removed the case, which the TeO$_2$ crystal was placed in, and have removed the impedance matching circuit. The same 40 MHz of rf power of the local oscillator fed the piezo-electric transducer directly through rf cable. We have not lost much in the correction signals amplitude (Fig. 5). The Fig 6 represents schematic drawing of the 1205-C2 AOM and of the final version of “VNIIFTRI” AOM design.

Thus, the shortening of the acousto-optic crystal allowed to minimize considerably the size of AOM designed for operation in the Raman-Nath diffraction mode.

CONCLUSIONS

We reported on the AOM designed primarily for operation in the Raman-Nath diffraction mode and used as an external phase modulator in FM sideband heterodyne spectroscopy for frequency control of the ECDL. It is shown that the AOM design can be considerably simplified and its size can be minimized if the Raman-Nath diffraction mode is the dominant
diffraction mode of AOM. In our experiment, the acousto-optic interaction length of the AOM designed primarily for operation in such a mode was 2 mm. Such compact devices can be used as phase modulators in a wide range of applications in laser spectroscopy, where the frequency noise reduction of a variety of the lasers is essential.

REFERENCES