

# MULTI OBJECTIVE ECONOMIC DISPATCH USING PARETO FRONTIER DIFFERENTIAL EVOLUTION

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## Abstract:

**Multi Objective Economic dispatch (MOED) problem has gained recent attention due to the deregulation of power industry and environmental regulations. So generating utilities should optimize their emission in addition to the operating cost. In this paper a Pareto frontier Differential Evolution (PDE) technique is developed to solve MOED problem, which provides a set of feasible solutions to the problem. To evaluate the performance and applicability of the proposed method, it is implemented on the standard IEEE-30 bus system having six generating units including valve point effects. The results obtained demonstrate the effectiveness of the proposed method for solving the Multi Objective economic dispatch problem considering security constraints.**

*Keywords: Multi objective economic dispatch, emission dispatch, valve point effects, Pareto frontier differential evolution.*

## 1. Introduction

Economic dispatch (ED) is one of the most important problems to be solved in the operation and planning of a power system. The main goal of the ED is to meet the load demand at minimum operating cost by maintaining proper schedule of the committed generating unit outputs while satisfying all unit and system equality and inequality constraints [1]. The generation of electricity is mainly obtained from the fossil fuel and it releases several contaminants, such as sulphur oxides ( $\text{SO}_2$ ), and oxides of nitrogen ( $\text{NO}_x$ ) into the atmosphere. These gaseous pollutants cause harmful effects on human beings as well as on plants and animals [1]. The Clean Air Act Amendments of 1990 (CAAA) mandates that the electric utility industry should reduce its  $\text{SO}_2$  emission by 10 million ton/year and the  $\text{NO}_x$  by 2 million ton/year from the 1980 level [2].

Nowadays pollutants become an inescapable problem to control and govern emissions of power plants because, day by day energy conservation and pollution emissions reduction attains increasingly social attentions [3]. To decrease the atmospheric emissions, several policies can be adopted [4]. Therefore, economic dispatch problem is no more a single objective problem. When the emission constraint is also taken into account, the problem becomes a multi-objective problem, with conflicting objectives. Many works are in literature as to solve the combined economic and emission dispatch problem.

Various methodologies have been proposed to solve economic power dispatch problem considering emission dispatch in the current literature [5]-[14]. A novel fuzzy optimal search technique [5] has developed by D. Srinivasan et.al for solving multi-objective generation scheduling problem. A pattern recognition technique is used for assessing the stability of the interconnected system at each load level, and for evaluating the security

transfer limits between neighboring systems. A summary of economic/emission dispatch algorithms has presented in [6]. In [7], an elitist multi-objective evolutionary algorithm based on the Non-dominated sorting genetic Algorithm had been used for solving the environmental/economic dispatch problem. Elitism ensures the best solution does not deteriorate in the next generations.

A fuzzy multi-objective genetic algorithm (FMOGA) approach for the multi-objective economic power dispatch problem is presented in [8]. FMOGA employs a fuzzy evaluation factor to the fitness function of MOGA, which can prevent the premature convergence of the genetic algorithm. As well FMOGA can deal with “gene drift” caused by large bound of objectives. The evolutionary programming technique was developed to solve CEED problem in [9] where two objective functions (the economy and emission objectives) were converted into single objective function.

A niched Pareto genetic algorithm (NPGA) based approach to solve the multi-objective environmental/economic dispatch (EED) problem was presented in [10]. The EED problem was formulated as a non-linear constrained multi-objective optimization problem. A multi-objective optimization technique using Bacteria Foraging Optimization (BFO) [11] has been proposed to solve environmentally constrained economic dispatch problem. Multi objective load dispatch problem has solved by Nondominated sorting differential evolution Algorithm [12], which is organic integration of Pareto nondominated sorting and differential evolution algorithm. NSDE improves the crowding distance mechanism and mutation strategic in evolution effectively. A novel binary successive approximation-based evolutionary search strategy [13] had been proposed to solve the economic-emission load dispatch problem by searching the generation pattern of committed units. Inequality constraints are taken care of during the search of a generation pattern. To solve emission constrained economic power dispatch (ECEPD) problem, a differential evolution optimization algorithm has been proposed in [14].

In this paper the authors developed a Pareto Frontier Differential Evolutionary Algorithm (PDEA) to solve multi objective Economic dispatch problem considering security constraints. The proposed algorithm is implemented for different cases such as consideration of valve point effects, losses. The various results of different cases are analyzed and also compared.

## 2. Problem Formulation

The main aim of Multi objective ED problem is to find best possible combination of real power of different units within their feasible operating region for the minimization of two conflicting objectives (fuel cost and emission) while satisfying all system equality and inequality constraints including valve point effects.

### 2.1. Problem Objective Functions

#### 2.1.1 Economic dispatch objective function

The economic dispatch problem minimizes the total operating cost of a power system while meeting the system equality and inequality constraints. The total fuel cost  $F_T(P_i)$  for the entire power system may be expressed as the sum of the quadratic cost model of each generating unit as:

$$\begin{aligned} \text{Minimize } F_T &= \sum_{i=1}^{N_G} F_i(P_i) \\ \text{i.e. } F_T(P_i) &= \sum_{i=1}^{N_G} [a_i P_i^2 + b_i P_i + c_i] \text{ \$/h} \end{aligned} \quad (1)$$

However, to model the cost function of generators in a more practical manner, valve point effect is considered where the input-output curve is not linear but consists of ripples as a result of the sharp increase in losses due to the wire drawing effects which occur as each steam admission valve starts to open. The cost function is obtained based on the ripple curve for more accurate modelling. Thus the equation (1) can be modified using a sine function to model the ripples to valve point effect as

$$\text{Minimize } F_T(P_i) = \sum_{i=1}^{N_G} [a_i P_i^2 + b_i P_i + c_i] + [g_i \sin\{m_i (P_i^{\min} - P_i)\}] \text{ \$/h} \quad (2)$$

Where,

$N_G$  is the number of generating units

$a_i, b_i, c_i$  are fuel cost coefficients of unit  $i$

$g_i, m_i$  are the valve point effect co-efficient of unit  $i$

$P_i$  is the power output of generating unit 'i'

$F_i(P_i)$  is the operation cost of unit  $i$ , \$/h

$F_T(P_i)$  is the total operation cost, \$/h

### 2.1.2 Emission dispatch objective function

The atmospheric pollutants such as sulphur oxides (SO<sub>x</sub>) and nitrogen oxides (NO<sub>x</sub>) caused by fossil-fuelled thermal units can be modelled separately. However for comparison purposes, the total ton/h emission  $E_T(P_i)$  of these pollutants can be expressed as:

$$\text{Minimize } E_T(P_i) = \sum_{i=1}^{N_G} [d_i P_i^2 + e_i P_i + f_i] + [\xi_i \exp(\lambda_i P_i)] \quad (3)$$

Where  $d_i, e_i, f_i, \xi_i, \lambda_i$  are emission coefficients of unit  $i$

## 2.2. Problem constraints

### 2.2.1 Real power balance constraint

As there are few facilities to store electrical energy, the total power generation must clearly track its total demand  $P_D$ , plus real power loss in transmission lines  $P_{loss}$ , Hence

$$\sum_{i=1}^{N_G} P_i = P_D + P_{loss} \quad (4)$$

The most popular approach for finding an approximate value of the losses is by Kron's formula as given in (9), which represents the losses as a function of the output level of the system generators.

$$P_{loss} = \sum_{i=1}^{N_G} \sum_{j=1}^{N_G} P_i B_{ij} P_j + \sum_{i=1}^{N_G} B_{oi} P_i + B_{oo} \quad (5)$$

$B_{ij}, B_{oi}, B_{oo}$  are the transmission loss coefficients.

### 2.2.2 Generation capacity constraint

For stable operation, real power output should be restricted by lower and upper limits as follows

$$P_i^{min} < P_i < P_i^{max} \quad (6)$$

Where,

$P_i^{min}$  is lower permissible limit of real power generation of unit  $i$

$P_i^{max}$  is higher permissible limit of real power generation of unit  $i$

## 3. Differential Evolution

DEA is an evolutionary computational algorithm that was originally introduced by Storn and Price in 1995 [15]. The DEA optimisation process is carried out by applying the following three basic genetic operations; mutation, recombination (also known as crossover) and selection. After the population is initialised, the operators of mutation, crossover and selection create the population of the next generation  $pop^{(G+1)}$  by using the current population  $pop^{(G)}$ .

### 3.1 Differential Evolution Algorithm Optimisation Process

#### a) Initialisation

In the first step of the DEA optimisation process, the population of candidate solutions must be initialised.

The initial population of candidate is generated within its corresponding feasible limits as follows

$$X_{i,d}^{(G=0)} = \lambda_{min} + (\lambda_{max} - \lambda_{min}) * rand \quad (7)$$

where  $i = 1, 2, \dots, N_{pop}$  and  $d = 1, 2, \dots, D$

$$X = [X_1 \ X_2 \ \dots \ X_i \ \dots \ X_{N_{pop}}] \quad (8)$$

Where,  $N_{pop}$  = Number of population,  $D$  = Number of decision variables and  $G$  = Number of current generation

#### b) Mutation

The mutation operator generates mutant vectors ( $v_i^G$ ) by perturbing a randomly selected vector ( $X_{r1}$ ) with the difference of two other randomly selected vectors ( $X_{r2}$  and  $X_{r3}$ ). DEA has several strategies to generate mutant vectors but in this paper, the simplest and most popular Differential Evolution method is used.

$$V_i^G = X_{r_1}^G + F(X_{r_2}^G - X_{r_3}^G), i = 1, 2, \dots, N_{pop} \quad (9)$$

Vector indices  $r_1, r_2$  and  $r_3$  are randomly chosen, where  $r_1, r_2$  and  $r_3$  belong to  $\{1 \dots N_{pop}\}$  and  $r_1 \neq r_2 \neq r_3 \neq i$ .  $F$  is a user-defined constant known as the “scaling mutation factor”, which lies between 0 and 1.

*c) Crossover*

Crossover operation helps to increase the diversity among the mutant parameter vectors and aids the algorithm to escape from local optima. At the generation  $G$ , the crossover operation creates trial vectors ( $U_i$ ) by mixing the parameters of the mutant vectors ( $V_i$ ) with the target vectors ( $X_i$ ) according to a selected probability distribution.

$$U_i^{(G)} = u_{j,i}^{(G)} = \begin{cases} v_{j,i}^{(G)}, & \text{if } \text{rand}_j \leq CR \\ x_{j,i}^{(G)}, & \text{otherwise} \end{cases} \quad (10)$$

The crossover constant  $CR$  is a user-defined value lies in between 0 and 1.

*d) Selection*

The selection operator selects the vectors that are going to compose the population in the next generation. This operator compares the fitness of the trial vector and the corresponding target vector and selects the one that provides the best solution and advance it into the next generation according to equation below.

$$X_i^{(G+1)} = \begin{cases} U_i^{(G)}, & \text{if } f(U_i^{(G)}) \leq f(X_i^{(G)}) \\ X_i^{(G)}, & \text{otherwise} \end{cases} \quad (11)$$

The overall optimisation process is stopped whenever maximum number of generations is reached or other predetermined convergence criterion is satisfied.

**4. Pareto Frontier Differential Evolution**

A general version of the Pareto Frontier Differential Evolutionary Algorithm [16] (PDE) similar to the normal Differential evolutionary algorithm with the following modifications:-

- Generate the initial population within the given limits
- Reproduction is undertaken only among nondominated solutions in each generation.
- Offspring are placed into the population if they dominate the main parent.

The algorithm works as follows. An initial population is generated at random from a Gaussian distribution. All dominated solutions are removed from the population. The remaining non-dominated solutions are retained for reproduction. If the number of non-dominated solutions exceeds certain threshold, a distance metric relation is used to remove those parents who are very close to each others. Three parents are selected at random. A child is generated from the three parents and placed into the population if it dominates the first selected parent; otherwise a new selection process takes place. This process continues until the population is completed.

If the maximum number of non-dominated solutions exceeds a preset value( $\alpha$ ), the following nearest neighbour distance function is adopted:

$$D(x) = \frac{(\min\|x-x_i\| + \min\|x-x_j\|)}{2} \quad (12)$$

Where  $x \neq x_i \neq x_j$ , It is indicated that, the nearest neighbour distance is the average Euclidean distance between the closest two points. The non-dominated solution with the smallest neighbour distance is removed from the population until the total number of non-dominated solutions is retained to pre set value.

**4.1 Multi-objective Economic Dispatch using PDE**

The step wise procedure to implement PDE to solve Multi-Objective ED problem is outlined as follows:

- (1) Read all the parameters such as generator fuel cost coefficients ( $a_i, b_i, c_i$ ), Emission coefficients ( $d_i, e_i, f_i$ ), B-loss coefficients, valve point coefficients, etc.
- (2) Set control parameters of PDEA optimization process those are  
 Population size,  $N_{pop} = 40$   
 Number of decision variables,  $ND = 1$  (*i.e. only incremental cost  $\lambda$* )  
 Scaling mutation factor  $F = 0.8$   
 Greediness factor  $k = 0.6$   
 Crossover probability  $CR = 0.7$

Convergence tolerance  $\varepsilon = 10^{-4}$   
 Maximum number of generations  $G^{max} = 500$

(3) Generate the initial population for each individual within their feasible limits, i.e. power generations of different units within their minimum and maximum power limits

(4) **START LOOP**

(5) Calculate power generations of different units; fuel cost and emission using (2), (3)

(6) Calculate power loss using (5) and power mismatch ( $\Delta P$ ) using

$$\Delta P = \left| P_D + P_{loss} - \sum P_i \right|$$

(7) If  $\Delta P < \varepsilon$  print the result else **GOTO** step 8

(8) Increment iteration count by  $G = G + 1$

(9) Remove all dominated solutions from current population; copy the nondominated solutions into the vector  $P_{nd}$  (set of non dominated solutions)

(10) If number of nondominated solutions in  $P_{nd} > \alpha$  remove excess solutions using (12)

(11) Apply mutation operation as

Select three parents  $r_1, r_2, r_3 \in (1, 2, \dots, \infty)$ , i.e. from nondominated solutions.

Where  $r_1 \neq r_2 \neq r_3$

Calculate mutation vector  $V_i^G = X_{r_1}^G + F(X_{r_2}^G - X_{r_3}^G), i = 1, 2, \dots, N_{pop}$

(12) Apply crossover operation as

Select target vector  $i \in (1, 2, \dots, N_{pop})$  i.e. from main population

$$\text{Trial vector } U_i^{(G)} = \begin{cases} V_i^G, & \text{if } rand_j \leq CR \\ X_i^G, & \text{otherwise} \end{cases}$$

(13) Apply selection operation as

$$X_i^{(G+1)} = \begin{cases} U_i^{(G)}, & \text{if } f(U_i^{(G)}) \leq f(X_i^{(G)}) \\ X_i^{(G)}, & \text{otherwise} \end{cases}$$

(14) Update the population

(15) **END LOOP**

## 5. Results and Discussions

The proposed Pareto Frontier DE algorithm is tested on the standard IEEE-30 bus system having six generating units. The system data has taken from [17]. The algorithm is written in MATLAB 7.7 on Intel(R) CORE2Duo processor, 2.20GHz with 3GB RAM. The B loss coefficients of the system are calculated in p.u. on 100 MVA base and given below. To demonstrate the effectiveness of the proposed algorithm, three different cases are considered as follows:

*Case1:* Losses are not included in equality constraint and Valve point effects are neglected

*Case2:* Losses are included in equality constraint but Valve point effects are neglected

*Case3:* Losses as well as Valve point effects are considered

Table.1. Results for CASE1

	Minimum fuel	Minimum emission	Minimum loss	Compromised Results
P1	0.05	0.30900747	0.153527282	0.05
P2	0.384172287	0.479758623	0.370842481	0.490822081
P3	0.738597179	0.678560601	0.843753094	0.665293479
P4	0.734440325	0.408464508	0.330030749	0.644450969
P5	0.584193349	0.484091185	0.815369962	0.610544407
P6	0.342305781	0.474126588	0.320501939	0.372871869
TG	2.833708922	2.834008975	2.834025508	2.833982805
$P_L$	0.027992885	0.027697000	0.023493873	0.028661424
FC	608.032426	632.4585581	636.7895699	614.2463286
EC	0.212302999	0.196085055	0.209996490	0.207880531
TIME	2.460687	2.113552	1.793391	1.871933

Where, TG=Total generation (per unit);  $P_L$  =Transmission power losses (p.u.)

FC=Fuel cost (\$/h); EC= Emission (ton/h); ITR= Number of iterations;

TIME= CPU Computation Time (sec)

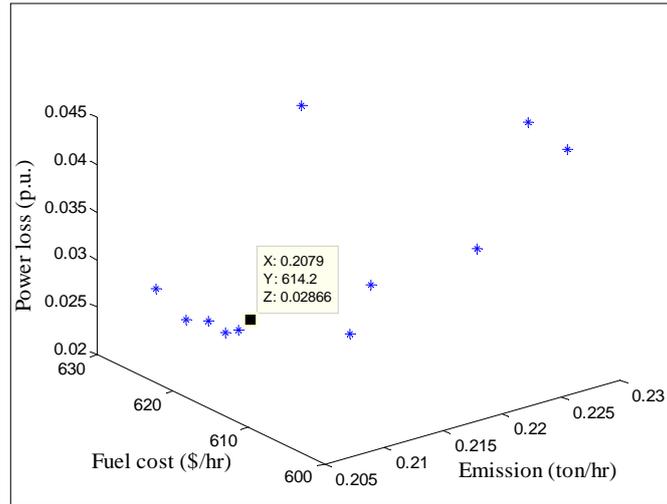


Fig.1. Nondominated solutions corresponding to CASE1

Table.2. Results for CASE2

	Minimum fuel	Minimum emission	Minimum loss	Compromised Results
P1	0.105328324	0.360390890	0.397676136	0.103167922
P2	0.291729787	0.447005032	0.30101718	0.283363935
P3	0.734655487	0.596788756	0.874850369	0.745016383
P4	0.812249991	0.311377634	0.406541792	0.771797834
P5	0.590283992	0.719413890	0.708720951	0.703759317
P6	0.329928025	0.429039943	0.169341803	0.256890729
TG	2.864175606	2.864016109	2.858148231	2.863996121
$P_L$	0.030190434	0.02997993	0.024153049	0.029968056
FC	611.3470079	647.6783538	645.9642641	614.6836317
EC	0.21438345	0.196726533	0.209756953	0.216292916
TIME	1.947973	2.152292	1.663565	1.932421

Where, TG=Total generation (per unit);  $P_L$  =Transmission power losses (p.u.)

FC=Fuel cost (\$/h); EC= Emission (ton/h); TC=Total operating cost (\$/h)

TIME= CPU Computation Time (sec)

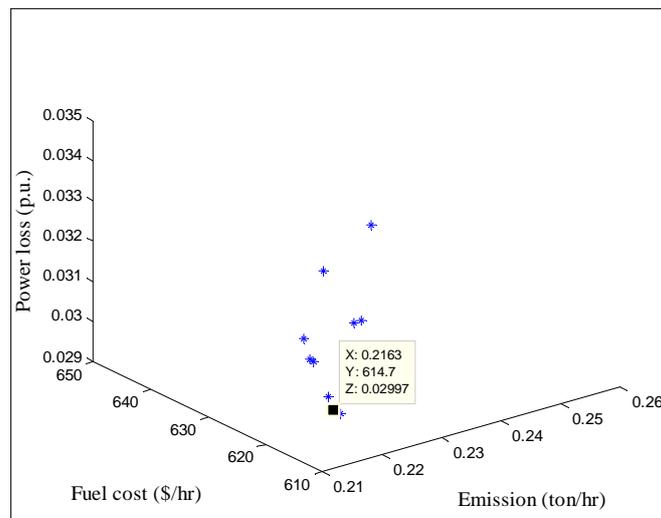


Fig.2. Nondominated solutions corresponding to CASEE2

Table.3. Results for CASE3

	Minimum fuel	Minimum emission	Minimum loss	Compromised Results
P1	0.14067754	0.37754009	0.44635507	0.14067754
P2	0.41934981	0.44389366	0.26013339	0.41934981
P3	0.69474355	0.54164328	0.77578188	0.69474355
P4	0.80798973	0.46300017	0.49430183	0.80798973
P5	0.40891673	0.54595415	0.28768752	0.40891673
P6	0.39277248	0.494472504	0.6	0.39277248
TG	2.86444986	2.866503863	2.86425971	2.86444986
$P_L$	0.03041904	0.032479199	0.03025971	0.03041904
FC	627.029793	671.312348	678.968947	627.029793
EC	0.21000169	0.194533499	0.20377436	0.21000169
TIME	1.810112	1.804893	1.739174	1.801288

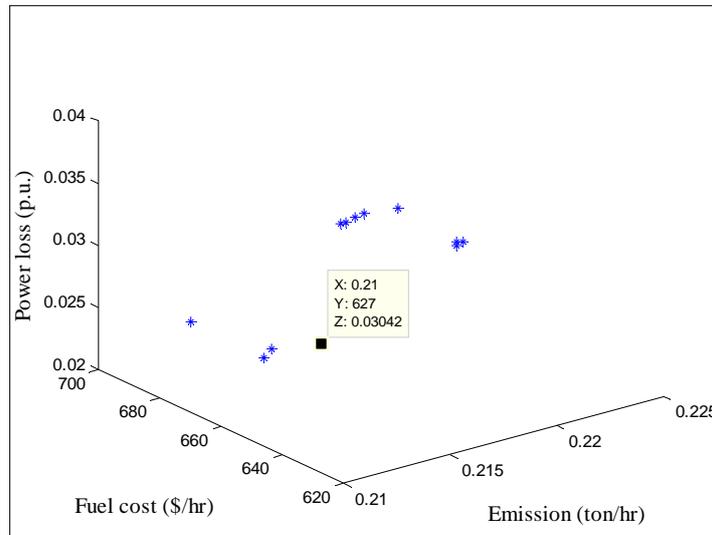


Fig.3. Nondominated solutions corresponding to CASE3

In each case, results are arranged corresponding to minimum fuel cost, minimum emission and minimum power loss. The compromised results are also shown. From the results it can be observed that, PDEA is providing better fuel cost emission and power loss. The computation time is also very less in case of PDEA. From Table.1 to Table.3, it can be found that with the introduction of nonlinearities from case 1 to case 3 fuel cost and emission are increasing continuously.

The set of non dominated compromised solutions are plotted for each case with the help of three dimensional figures shown in Fig.1 to Fig.3 respectively. The three axes fuel cost, emission and power loss of three dimensional figure indicates that, we are trying to choose an optimum point which satisfying all three objectives. From the Fig.1 to Fig.3 it can be seen that, Fuel cost and Emission both are conflicting each other i.e. if one will increase the other is decreasing. The compromised solution is shown in box within the each figure.

Under different practical situations for various cases priority or requirement of fuel cost, emission or losses can be different. Hence the designer should have a set of solutions so that as per requirement and specifications, Engineer can select the correct solution set easily. From the Figures it can be noted that, PDEA provides flexibility to the designer so that he/she can choose the optimized results according to the situation i.e. whether they need minimum fuel cost, minimum emission, minimum loss or all three..

**6. Conclusions**

In this paper, the Pareto Frontier Differential Evolutionary Algorithm is presented as an effective tool for solving constrained multi objective optimization problem. The proposed algorithm has been tested on IEEE 30-bus test system and the results for different cased are compared. The transmission losses are included. Emission as well as operational cost incurred due to valve-point effect is higher and this result fits in for a real time situation. The computational time is also very less. The proposed algorithm shows its effectiveness providing results in finding optimal or near optimal solution for a non-linear function. For better readability and analysis different cases are considered and also compared which is may be helpful to the designer.

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