Energy of motion is called *kinetic energy*. (The root of the word is the same as the word “cinema” -- in French, kinetic energy is “énergie cinétique.”) How does an object's kinetic energy depend on its mass and velocity? Joule attempted a conceptually simple experiment on his honeymoon in the French-Swiss Alps near Mt. Chamonix, in which he measured the difference in temperature between the top and bottom of a waterfall. The water at the top of the falls has some gravitational energy, which isn't our subject right now, but as it drops, that gravitational energy is converted into kinetic energy, and then into heat energy due to internal friction in the churning pool at the bottom:

\[
\text{gravitational energy} \rightarrow \text{kinetic energy} \rightarrow \text{heat energy}
\]

In the logical framework of this book's presentation of energy, the significance of the experiment is that it provides a way to find out how an object's kinetic energy depends on its mass and velocity. The increase in heat energy should equal the kinetic energy of the water just before impact, so in principle we could measure the water's mass, velocity, and kinetic energy, and see how they relate to one another.\textsuperscript{4}
Although the story is picturesque and memorable, most books that mention the experiment fail to note that it was a failure! The problem was that heat wasn't the only form of energy being released. In reality, the situation was more like this:

\[
\text{gravitational energy} \rightarrow \text{kinetic energy} \rightarrow \text{heat energy} + \text{sound energy} + \text{energy of partial evaporation.}
\]

The successful version of the experiment, shown in figures d and f, used a paddlewheel spun by a dropping weight. As with the waterfall experiment, this one involves several types of energy, but the difference is that in this case, they can all be determined and taken into account. (Joule even took the precaution of putting a screen between himself and the can of water, so that the infrared light emitted by his warm body wouldn't warm it up at all!)
The result is

\[ K = \frac{1}{2}mv^2 \text{[kinetic energy]} \].

Figure f: A realistic drawing of Joule's apparatus, based on the illustration in his original paper. The paddlewheel is sealed inside the can in the middle. Joule wound up the two 13-kg lead weights and dropped them 1.6 meters, repeating this 20 times to produce a temperature change of only about half a degree Fahrenheit in the water inside the sealed can. He claimed in his paper to be able to measure temperatures to an accuracy of 1/200 of a degree.

Whenever you encounter an equation like this for the first time, you should get in the habit of interpreting it. First off, we can tell that by making the mass or velocity greater, we'd get more kinetic energy. That makes sense. Notice, however, that we have mass to the first power, but velocity to the second. Having the whole thing proportional to mass to the first power is necessary on theoretical grounds, since energy is supposed to be additive. The dependence on \( v^2 \) couldn't have been predicted, but it is sensible.
For instance, suppose we reverse the direction of motion. This would reverse the sign of \( v \), because in one dimension we use positive and negative signs to indicate the direction of motion. But since \( v^2 \) is what appears in the equation, the resulting kinetic energy is unchanged.

What about the factor of \( 1/2 \) in front? It comes out to be exactly \( 1/2 \) by the design of the metric system. If we'd been using the old-fashioned British engineering system of units (which is no longer used in the U.K.), the equation would have been \( K = (7.44 \times 10^{-2} \text{ Btu} \cdot \text{s}^2/\text{slug} \cdot \text{ft}^2)mv^2 \). The version of the metric system called the SI,\(^6\) in which everything is based on units of kilograms, meters, and seconds, not only has the numerical constant equal to \( 1/2 \), but makes it unitless as well. In other words, we can think of the joule as simply an abbreviation, \( 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2 \). More familiar examples of this type of abbreviation are \( 1 \text{ minute} = 60 \text{ s} \), and the metric unit of land area, \( 1 \text{ hectare} = 10000 \text{ m}^2 \).

Source:
http://physwiki.ucdavis.edu/Fundamentals/02._Conservation_of_Energy/2.1_Energy