420 Journal of Electrical Engineering & Technology Vol. 7, No. 3, pp. 420~428, 2012
http://dx.doi.org/10.5370/JEET.2012.7.3.420

Intelligent Digital Redesign for Nonlinear Interconnected Systems using Decentralized Fuzzy Control

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Abstract – In this paper, a novel intelligent digital redesign (IDR) technique is proposed for the nonlinear interconnected systems which can be represented by a Takagi-Sugeno (T-S) fuzzy model. The IDR technique is to convert a pre-designed analog controller into an equivalent digital one. To develop this method, the discretized models of the analog and digital closed-loop system with the decentralized controller are presented, respectively. Using these discretized models, the digital decentralized control gain is obtained to minimize the norm between the state variables of the analog and digital closed-loop systems and stabilize the digital closed-loop system. Its sufficient conditions are derived in terms of linear matrix inequalities (LMIs). Finally, a numerical example is provided to verify the effectiveness of the proposed technique.

Keywords: Nonlinear interconnected systems, Takagi-Sugeno (T-S) fuzzy model, Decentralized control, Intelligent digital redesign (IDR), Linear matrix inequality (LMI)

1. Introduction

Recently, many practical systems, such as power systems, communication networks, and economic models, have the high dimensionality and nonlinearity. In order to control and analyze these systems, many researchers have con-cerned about stability and stabilization of nonlinear inter-connected systems [1-9, 18, 19]. Because of the high dimensionality and the structural constraint, the traditionally centralized control method is not suitable for the interconnected systems. To solve this problem, various control approaches have been developed for the nonlinear interconnected systems. Among them, the decentralized fuzzy control technique using the Takagi-Sugeno (T-S) fuzzy model is regarded as a powerful technique and has gathered many research interests [4-6].

Apart from the nonlinearity issue, because of the ubiquitousness and its application, there are many cases to control the interconnected system using the digital device such as a computer. Thus, the study for the digital control is essential for the interconnected system. Among many digital control methods, a digital redesign (DR) is widely used as the predominant digital controller design technique [10-12]. It executes as a tool which converts an original analog controller into a new digital one maintaining the state-matching condition.

In order to apply DR method in the nonlinear system, an intelligent digital redesign (IDR), which is merged with the fuzzy control scheme and the DR method, was proposed in [13]. Also, the robust IDR problem was studied for nonlinear systems with parametric uncertainties [14]. However, these papers dealt with the local state-matching without concerning the global one. The global approaches of the IDR problem were presented in [15-17], but they did not consider the interconnection of the nonlinear systems. The decentralized digital controllers for the interconnected systems were proposed in [18-20]. But there were no studies about the IDR problem for the nonlinear interconnected systems.

In this paper, we propose a novel IDR method for the nonlinear interconnected systems which can be modelled by T-S fuzzy systems. The discretized models of the analog and digital closed-loop system with the decentralized controller are presented, respectively. Based on these discretized models, sufficient conditions are achieved for both the stability of the digital closed-loop system and the state-matching condition between analog and digital closed-loop systems. Its constructive conditions are presented in terms of linear matrix inequalities (LMIs). Finally, it shows the validity of the proposed ideas, techniques and procedures, through simple example.

This paper is organized as follows: Section 2 describes the discretized models of the T-S fuzzy interconnected systems. The stability and state-matching conditions are proposed with the LMI form in Section 3. In Section 4, simulation example is provided to demonstrate the design procedures. Finally, the conclusions are given in Section 5.

2. T-S Fuzzy Modeling and Discretization for Nonlinear Interconnected System

Consider an interconnected T-S fuzzy system consisting of q systems, in which the i th IF-THEN rule of the k th

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Received: November 19, 2010; Accepted: March 2, 2012

subsystem is represented by the following form:

$$R_i^k : \text{IF } z_1^k \text{ is } \Gamma_{i1}^k \text{ and} \cdots \text{and } z_p^k \text{ is } \Gamma_{ip}^k,$$
$$\text{THEN } \dot{x}_k(t) = A_i^k x_k(t) + B_i^k u_k(t) + \sum_{l=1,l\neq k}^{q} A_i^{kl} x_l(t) \tag{1}$$

where z_h, $h \in \mathcal{I}_p$ is the premise variable; $x_k \in \mathbb{R}^{n_k}$ and $u_k \in \mathbb{R}^{m_k}$ are the state and control input for the k th subsystem, respectively; Γ_{ih}^k, $(i,h,k) \in \mathcal{I}_r \times \mathcal{I}_p \times \mathcal{I}_q$, is a fuzzy set for z_h^k; A_i^k and B_i^k denote nominal system matrices with appropriate dimensions for the i th rule in the k th subsystem and A_i^{kl} is the interconnection matrix with the k th and the l th subsystem.

Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics is inferred as

$$\dot{x}_k(t) = \sum_{i=1}^{r} \mu_i^k(t)\Big(A_i^k x_k(t) + B_i^k u_k(t) + \sum_{l=1,l\neq k}^{q} A_i^{kl} x_l(t)\Big) \tag{2}$$

where

$$\mu_i^k(t) = \omega_i^k(z^k(t)) \Big/ \sum_{i=1}^{r} \omega_i^k(z^k(t)),$$
$$\omega_i^k(z^k(t)) = \prod_{h=1}^{p} \Gamma_{ih}^k(z_h^k(t))$$

in which $\Gamma_{ih}^k(z_h^k(t))$ is the fuzzy membership grade of z_h^k in Γ_{ih}^k.

Suppose a pre-designed analog fuzzy controller for k th subsystem (2)

$$u_{c_k}(t) = \sum_{i=1}^{r} \mu_i^k(t) K_{c_i}^k x_{c_k}(t) \tag{3}$$

where $K_{c_i}^k$ denotes the analog control gain and the subscript 'c' means the analog control. Substituting (3) into (2), k th sub-closed-loop system is written as

$$\dot{x}_{c_k}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i^k(t)\mu_j^k(t) \times\Big(\big(A_i^k + B_i^k K_{c_j}^k\big)x_{c_k}(t) + \sum_{l=1,l\neq k}^{q} A_i^{kl} x_{c_l}(t)\Big) \tag{4}$$

Now, consider a digital fuzzy controller for (2)

$$u_{d_k}(t) = u_{d_k}(nT) = \sum_{i=1}^{r} \mu_i^k(nT) K_{d_i}^k x_{d_k}(nT) \tag{5}$$

where $u_{d_i}(t)$ is the digital control input to be determined in time interval $t \in [nT, nT+T$, $n \in \mathbb{Z}_{\geq 0}$ and $T \in \mathbb{R}_{>0}$ is a samp-ling period and the subscript 'd' denotes the digital control. Substituting (5) into (2), the k th sub-closed-loop system is obtained by

$$\dot{x}_{d_k}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i^k(t)\mu_j^k(nT) \times\Big(A_i^k x_{d_k}(t) + B_i^k K_{d_j}^k x_{d_k}(nT) + \sum_{l=1,l\neq k}^{q} A_i^{kl} x_{d_l}(t)\Big) \tag{6}$$

Supposing the sampling period T, closed-loop fuzzy system (4) and (6) are approximately discretized as

$$\begin{aligned} x_{c_k}(nT+T) &\approx \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i^k(nT)\mu_j^k(nT) \times\Big(\Phi_{ij}^k x_{c_k}(nT) + \sum_{l=1,l\neq k}^{q} \hat{G}_{ij}^{kl} x_{c_l}(nT)\Big) \\ &= \sum_{l=1,l\neq k}^{q}\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i^k(nT)\mu_j^k(nT) \times\begin{bmatrix} \frac{1}{q-1}\Phi_{ij}^k & \hat{G}_{ij}^{kl} \end{bmatrix}\begin{bmatrix} x_{c_k}(nT) \\ x_{c_l}(nT) \end{bmatrix} \end{aligned} \tag{7}$$

where

$$\Phi_{ij}^k = \exp\big((A_i^k + B_i^k K_{c_j}^k)T\big),$$
$$\hat{G}_{ij}^{kl} = (\Phi_{ij}^k - I)(A_i^k + B_i^k K_{c_j}^k)^{-1} A_i^{kl}$$

and

$$\begin{aligned} x_{d_k}(nT+T) &\approx \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i^k(nT)\mu_j^k(nT) \times\Big(\big(G_i^k + H_i^k K_{d_i}^k\big)x_{d_k}(nT) + \sum_{l=1,l\neq k}^{q} G_i^{kl} x_{d_l}(nT)\Big) \\ &= \sum_{l=1,l\neq k}^{q}\sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i^k(nT)\mu_j^k(nT) \times\begin{bmatrix} \frac{1}{q-1}(G_i^k + H_i^k K_{d_i}^k) & G_i^{kl} \end{bmatrix}\begin{bmatrix} x_{d_k}(nT) \\ x_{d_l}(nT) \end{bmatrix} \end{aligned} \tag{8}$$

where

$$G_i^k = \exp(A_i^k T),$$
$$H_i^k = (G_i^k - I)(A_i^k)^{-1} B_i^k,$$
$$G_i^{kl} = (G_i^k - I)(A_i^k)^{-1} A_i^{kl}.$$

Remark 1: There are fruitful research works [3-6] focusing on the design of the decentralized fuzzy controller in the continuous-time or the discrete-time domains. However, the study of IDR for the interconnected fuzzy system has not been sufficiently achieved.

3. New IDR Based on Discretized T-S Fuzzy System

In this paper, our main problem is addressed as follows:

Problem 1: Suppose that the equilibrium point $x_c(t)=0$ of (4) is asymptotically stable by a well-constructed analog controller (3). Then digital controller (5) has to satisfy the following conditions:

- The state-matching error between $x_{c_k}(nT)$ of (7) and $x_{d_k}(nT)$ of (8) is minimized for any $k \in \mathbb{Z}_{\geq 0}$.
- The digital closed-loop system (6) and its discretized system (8) are globally asymptotically stable.

Before proceeding to main results, the following matrix inequality and proposition will be needed throughout the proof:

Lemma 1([21]): Given any matrices Y and $P=P^T \succ 0$, we have

$$-Y^T P^{-1} Y \leq P - Y^T - Y .$$

Proposition 1: In (6), there exists some constant $\eta > 0$ such that

$$\sum_{k=1}^{q}\left\|x_{d_k}(t)\right\| \leq \eta \sum_{k=1}^{q}\left\|x_{d_k}(nT)\right\|$$

where

$$\eta=\sum_{k=1}^{q}(1+T\beta_k)\mathrm{e}^{(\alpha_k+(q-1)\gamma_k)T},$$

$$\alpha_k=\sup_{i\in\mathcal{I}_r}\left\|A_i^k\right\|,$$

$$\beta_k=\sup_{(i,j)\in\mathcal{I}_r\times\mathcal{I}_r}\left\|B_i^k K_{d_j}^k\right\|,$$

$$\gamma_k=\sup_{(i,l)\in\mathcal{I}_r\times\mathcal{I}_q}\left\|A_i^{kl}\right\|$$

for $t\in[nT,nT+T)$.

Proof : Integrating (6) from nT to t, the solution is given by

$$x_{d_k}(t)=x_{d_k}(nT)+\int_{nT}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i^k(\tau)\mu_j^k(nT)\times\Big(A_i^k x_{d_k}(\tau)+B_i^k K_{d_j}^k x_{d_k}(nT)+\sum_{l=1,l\neq k}^{q}A_i^{kl}x_{d_l}(\tau)\Big)d\tau$$

for $t\in[nT,nT+T$. Taking the norms and the sum of all subsystems on both sides yields

$$\begin{aligned}\sum_{k=1}^{q}\left\|x_{d_k}(t)\right\| &\leq \sum_{k=1}^{q}\left\|x_{d_k}(nT)\right\|+\sum_{k=1}^{q}\left\|\int_{nT}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i^k(\tau)\mu_j^k(nT)\times\Big(A_i^k x_{d_k}(\tau)+B_i^k K_{d_j}^k x_{d_k}(nT)+\sum_{l=1,l\neq k}^{q}A_i^{kl}x_{d_l}(\tau)\Big)d\tau\right\|\\
&\leq \sum_{k=1}^{q}\left\|x_{d_k}(nT)\right\| +\sum_{k=1}^{q}\int_{nT}^{t}\alpha_k\left\|x_{d_k}(\tau)\right\|+\beta_k\left\|x_{d_k}(nT)\right\|+\gamma_k\sum_{l=1,l\neq k}^{q}\left\|x_{d_l}(\tau)\right\|d\tau\\
&=\sum_{k=1}^{q}\left\|x_{d_k}(nT)\right\| +\sum_{k=1}^{q}\int_{nT}^{t}\big(\alpha_k+(q-1)\gamma_k\big)\left\|x_{d_k}(\tau)\right\|+\beta_k\left\|x_{d_k}(nT)\right\|d\tau\end{aligned}$$

An application of the Gronwall-Bellman inequality to $\sum_{k=1}^{q}x_{d_k}(t)$ results in

$$\begin{aligned}\sum_{k=1}^{q}\left\|x_{d_k}(t)\right\| &\leq \sup_{(i,j,l)\in\mathcal{I}_r\times\mathcal{I}_r\times\mathcal{I}_q}\sum_{k=1}^{q}\big(1+T\beta_k\big)\mathrm{e}^{(\alpha_k+(q-1)\gamma_k)T}\left\|x_{d_k}(nT)\right\|\\
&\leq \eta\sum_{k=1}^{q}\left\|x_{d_k}(nT)\right\|\end{aligned}$$

□

The main results are summarized to solve Problem 1 in the following theorem:

Theorem 1: If there exist some symmetric and positive matrices P^k, P^l, some symmetric matrices a_{ii}^{kl}, b_{ii}^{kl}, c_{ii}^{kl}, and some matrices a_{ij}^{kl}, b_{ij}^{kl}, c_{ij}^{kl}, d_{ij}^{kl}, M_i^k, Y^k, such that the following LMIs have optimal solutions, then, $x_d(nT)$ of (8) closely matches $x_c(nT)$ of (7), and origin of (8) is globally asymptotically stable.

$$\underset{P^k,M^k,Y^k}{\text{minimize}}\ \gamma \ \text{ subject to}$$

$$\begin{bmatrix}-\gamma P^k & * & * \\ 0 & -\gamma P^l & * \\ \frac{1}{q-1}(\Phi_{ij}^k Y^k-\Psi_{ij}^k) & \hat{G}_{ij}^{kl}Y^k-G_i^{kl}Y^k & I-\gamma Y^k-\gamma(Y^k)^T\end{bmatrix}\prec 0 \tag{10}$$

$$\begin{bmatrix}-\omega P^k+a_{ii}^{kl} & * & * \\ b_{ii}^{kl} & -\frac{\alpha}{4}P^k+c_{ii}^{kl} & * \\ \frac{1}{\sqrt{q-1}}\Psi_{ii}^k & G_i^{kl}Y^k & P^k-Y^k-(Y^k)^T\end{bmatrix}\prec 0 \tag{11}$$

$$\begin{bmatrix}-4\omega P^k+2a_{ij}^{kl}+2(a_{ij}^{kl})^T & * & * \\ 2b_{ij}^{kl}+2(d_{ij}^{kl})^T & -\alpha P^k+2c_{ij}^{kl}+2(c_{ij}^{kl})^T & * \\ \frac{1}{\sqrt{q-1}}(\Psi_{ij}^k+\Psi_{ji}^k) & G_i^{kl}Y^k+G_j^{kl}Y^k & P^k-Y^k-(Y^k)^T\end{bmatrix}\prec 0 \tag{12}$$

$$\tilde{X}^{kl}=(\tilde{X}^{kl})^T=\big[X_{ij}^{kl}\big]_{r\times r}\succ 0 \tag{13}$$

where

$$\Psi_{ij}^k=G_i^kY^k+H_i^kM_j^k, \qquad \omega=\tfrac{1}{q-1}-\tfrac{\alpha}{4},$$

$$X_{ii}^{kl}=(X_{ii}^{kl})^T=\begin{bmatrix}a_{ii}^{kl} & * \\ b_{ii}^{kl} & c_{ii}^{kl}\end{bmatrix},\quad X_{ij}^{kl}=(X_{ji}^{kl})^T=\begin{bmatrix}a_{ij}^{kl} & d_{ij}^{kl} \\ b_{ij}^{kl} & c_{ij}^{kl}\end{bmatrix}$$

for $\forall\{(i,j,k)\in\mathcal{I}_r\times\mathcal{I}_r\times\mathcal{I}_q \mid 1\leq i\leq j\leq r\}$.

Proof : In order to closely match $x_{d_k}(nT)$ of (8) to $x_{c_k}(nT)$ of (7), consider the norm minimization problem in the following form:

$$\left\|\begin{bmatrix}\frac{1}{q-1}\Phi_{ij}^k & \hat{G}_{ij}^{kl}\end{bmatrix}-\begin{bmatrix}\frac{1}{q-1}(G_i^k+H_i^kK_{d_j}^k) & G_i^{kl}\end{bmatrix}\right\|\le\|\hat{\gamma}\| \quad (14)$$

for some $\hat{\gamma}>0$. Using the norm property, the above inequality holds

$$\begin{bmatrix}\frac{1}{q-1}(\Phi_{ij}^k-G_i^k-H_i^kK_{d_j}^k) & \hat{G}_{ij}^{kl}-G_i^{kl}\end{bmatrix}^T \times\begin{bmatrix}\frac{1}{q-1}(\Phi_{ij}^k-G_i^k-H_i^kK_{d_j}^k) & \hat{G}_{ij}^{kl}-G_i^{kl}\end{bmatrix}\prec\gamma^3\begin{bmatrix}P^k & *\\ 0 & P^l\end{bmatrix} \quad (15)$$

where $\gamma>0$ and $P^k=(P^k)^T\succ 0$ with $\hat{\gamma}I\prec\gamma^3\mathrm{diag}\{P^k,P^l\}$. Taking the Schur complement and the congruence trans-formation with $\mathrm{diag}\{I,I,Y^k\}$, (15) yields

$$\begin{bmatrix}I & * & *\\ 0 & I & *\\ 0 & 0 & Y^k\end{bmatrix}^T\begin{bmatrix}-\gamma P^k & * & *\\ 0 & -\gamma P^l & *\\ \frac{1}{q-1}(\Phi_{ij}^k-G_i^k-H_i^kK_{d_j}^k) & \hat{G}_{ij}^{kl}-G_i^{kl} & -\gamma^2 I\end{bmatrix} \times\begin{bmatrix}I & * & *\\ 0 & I & *\\ 0 & 0 & Y^k\end{bmatrix}\prec 0 \quad (16)$$

where Y^k is any full rank matrix with appropriate dimension. Applying Lemma Lemma 1 and denoting $K_{d_i}^kY^k=M_i^k$, inequality (16) yields LMI (10).

Apart from the state-matching problem, for solving the stability problem, suppose the Lyapunov function $V(nT)=\sum_{k=1}^q x_{d_k}^T(nT)P^kx_{d_k}(nT)$. The rate of increase of $V(nT)$ along (8) is then

$$\begin{aligned}&V(nT+T)-V(nT)\\ &\le\frac{1}{4}\sum_{k=1}^q\sum_{i=1}^r\sum_{j=1}^r\mu_i^k(nT)\mu_j^k(nT)\\ &\quad\times\Big(\big((G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k)x_{d_k}+\sum_{l=1,l\ne k}^q(G_i^{kl}+G_j^{kl})x_{d_l}\big)^TP^k\\ &\quad\times\big((G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k)x_{d_k}+\sum_{l=1,l\ne k}^q(G_i^{kl}+G_j^{kl})x_{d_l}\big)\\ &\quad-4x_{d_k}^TP^kx_{d_k}\Big)\end{aligned}$$

We have used $\alpha\sum_{k=1}^q\sum_{l=1,l\ne k}^q x_{d_l}^TP^lx_{d_l}=\alpha(q-1)\sum_{k=1}^q x_{d_k}^TP^kx_{d_k}$ with $\alpha\in\mathbb{R}_{>0}$, then

$$\begin{aligned}&V(nT+T)-V(nT)\\ &\le\frac{1}{4}\sum_{k=1}^q\sum_{l=1,l\ne k}^q\sum_{i=1}^r\sum_{j=1}^r\mu_i^k(nT)\mu_j^k(nT)\begin{bmatrix}x_{d_k}\\ x_{d_l}\end{bmatrix}^T\\ &\times\left(\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k) & G_i^{kl}+G_j^{kl}\end{bmatrix}^TP^k\right.\\ &\times\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k) & G_i^{kl}+G_j^{kl}\end{bmatrix}\\ &\left.+\begin{bmatrix}-\left(\frac{4}{q-1}-\alpha\right)P^k & 0\\ 0 & -\alpha P^l\end{bmatrix}\right)\begin{bmatrix}x_{d_k}\\ x_{d_l}\end{bmatrix}\\ &=\sum_{k=1}^q\sum_{l=1,l\ne k}^q\sum_{i=1}^r(\mu_i^k(nT))^2\begin{bmatrix}x_{d_k}\\ x_{d_l}\end{bmatrix}^T\\ &\times\left(\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+H_i^kK_{d_j}^k) & G_i^{kl}\end{bmatrix}^TP^k\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+H_i^kK_{d_j}^k) & G_i^{kl}\end{bmatrix}\right.\\ &\left.+\begin{bmatrix}-\left(\frac{4}{q-1}-\alpha\right)P^k & 0\\ 0 & -\alpha P^l\end{bmatrix}\right)\begin{bmatrix}x_{d_k}\\ x_{d_l}\end{bmatrix}\\ &+\sum_{k=1}^q\sum_{l=1,l\ne k}^q\sum_{i<j}^r\mu_i^k(nT)\mu_j^k(nT)\begin{bmatrix}x_{d_k}\\ x_{d_l}\end{bmatrix}^T\\ &\times\left(\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k) & G_i^{kl}+G_j^{kl}\end{bmatrix}^TP^k\right.\\ &\times\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k) & G_i^{kl}+G_j^{kl}\end{bmatrix}\\ &\left.+\begin{bmatrix}-\left(\frac{4}{q-1}-\alpha\right)P^k & 0\\ 0 & -\alpha P^l\end{bmatrix}\right)\begin{bmatrix}x_{d_k}\\ x_{d_l}\end{bmatrix}\end{aligned}$$

If there exists $\tilde{X}^{kl}=(\tilde{X}^{kl})^T\succ 0$ such that the following inequalities are satisfied

$$\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+H_i^kK_{d_j}^k) & G_i^{kl}\end{bmatrix}^TP^k\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+H_i^kK_{d_j}^k) & G_i^{kl}\end{bmatrix}+\begin{bmatrix}-\left(\frac{1}{q-1}-\frac{\alpha}{4}\right)P^k & 0\\ 0 & -\frac{\alpha}{4}P^l\end{bmatrix}+\tilde{P}^{kl}X_{ii}^{kl}\tilde{P}^{kl}\prec 0 \quad (17)$$

$$\begin{aligned}&\frac{1}{4}\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k) & G_i^{kl}+G_j^{kl}\end{bmatrix}^TP^k\\ &\times\begin{bmatrix}\frac{1}{\sqrt{q-1}}(G_i^k+G_j^k+H_i^kK_{d_j}^k+H_j^kK_{d_i}^k) & G_i^{kl}+G_j^{kl}\end{bmatrix}\\ &+\begin{bmatrix}-\left(\frac{1}{q-1}-\frac{\alpha}{4}\right)P^k & 0\\ 0 & -\frac{\alpha}{4}P^l\end{bmatrix}+\frac{1}{2}\tilde{P}^{kl}(X_{ij}^{kl}+X_{ji}^{kl})\tilde{P}^{kl}\prec 0\end{aligned} \quad (18)$$

then ΔV is majorized by

$$\Delta V\le-\sum_{k=1}^q\sum_{l=1,l\ne k}^q\tilde{x}_{kl}^T\Theta_k^T\hat{P}^{kl}\tilde{X}^{kl}\hat{P}^{kl}\Theta_k\tilde{x}_{kl} \quad (19)$$

where

$$\Theta_k=\begin{bmatrix}\tilde{\theta}_1^k,\tilde{\theta}_2^k,\ldots,\tilde{\theta}_r^k,\end{bmatrix}^T$$
$$\tilde{\theta}_i^k=\mathrm{diag}\{\theta_i^k,\cdots,\theta_i^k\}\in\mathbb{R}^{2n_k\times 2n_k}$$

$$\tilde{P}^{kl} = \mathrm{diag}\{P^k, P^l\}$$
$$\hat{P}^{kl} = \mathrm{diag}\{\tilde{P}^{kl}, \cdots, \tilde{P}^{kl}\} \in \mathbb{R}^{2n_k r \times 2n_k r}$$

Applying Schur complement to (17) and the congruence transformation with $\mathrm{diag}\{I, I, Y^k\}$, we obtain

$$\begin{bmatrix} -\left(\frac{1}{q-1} - \frac{\alpha}{4}\right)P^k + a_{ii}^{kl} & * & * \\ b_{ii}^{kl} & -\frac{\alpha}{4}P^l + c_{ii}^{kl} & * \\ \frac{1}{\sqrt{q-1}}\left(G_i^k Y^k + H_i^k K_{d_j}^k Y^k\right) & G_i^{kl} Y^k & -(Y^k)^T (P^k)^{-1} Y^k \end{bmatrix} \prec 0 \tag{20}$$

Applying Lemma 1 and denoting $K_i^k Y^k = M_i^k$ result in (11).

We can again establish a similar argument to (18), in order to obtain (11), as follows:

(18)
$$\Leftrightarrow \begin{bmatrix} -\left(\frac{4}{q-1} - \alpha\right)P^k + 2a_{ij}^{kl} + 2a_{ji}^{kl} & * & * \\ 2b_{ij}^{kl} + 2b_{ji}^{kl} & -\alpha P^l + 2c_{ij}^{kl} + 2c_{ji}^{kl} & * \\ \frac{1}{\sqrt{q-1}}(\Lambda_{ij}^k + \Lambda_{ji}^k) & G_i^{kl} Y^k + G_j^{kl} Y^k & -(Y^k)^T (P^k)^{-1} Y^k \end{bmatrix}$$
$\Leftrightarrow$ (12) .

where

$$\Lambda_{ij}^k = G_i^k Y^k + H_i^k K_{d_j}^k Y^k$$ □

Remark 2: From (9), we can know that, if state $x_{d_k}(nT)$ converges to origin, then state $x_{d_k}(t)$ also tends to origin. In other words, if the discretized system (8) is asymptotically stable, then the original digital system is also asymptotically stable. Thus, we can obtain the state-matching condition between $x_{c_k}(nT)$ and $x_{d_k}(nT)$ and stability condition of (6) by Proposition 1 and Theorem 1.

4. New IDR Based on Discretized T-S Fuzzy System

In this section, two examples are given to validate the proposed method. First example is two inverted pendulums connected by a spring mounted on two carts and second example is two flexible joint robot arms connected by a spring.

4.1 Example 1

Suppose the interconnected system which is two inverted pendulums connected by a spring mounted on two carts [22]. The dynamic equation of the system is described by

$$\ddot{\theta}_k(t) - \{(g/cl) - (\kappa a(a-cl)/cml^2)\}\theta_k(t) + (m/M)sin(\theta_k(t))\dot{\theta}_k(t)^2$$
$$= (\kappa a(a-cl)/cml^2)\theta_l(t) + (1/cml^2)u_k(t)$$

where $\{(k,l) \in \mathcal{I}_2 \mid k \neq l\}$; $\theta_k(t)$ is the angle of the k th pendulum; c=m/(m+M) ; m=1kg is the mass of pendulum; M=5kg is the mass of the cart; a=0.2m is the length from the cart to the spring; l=1m is the length of the pendulum; $\kappa = 1N/m$ is the spring constant; $g = 9.8m/s^2$ is the gravity constant.

Assuming $\dot{\theta}_k(t) \in [-\Omega_k, \Omega_k]$ with $\Omega_k > 0$ and choosing $x_k(t) = [x_{k_1}(t) \quad x_{k_2}(t)]^T = [\theta_k(t) \quad \dot{\theta}_k(t)]^T$, the T-S fuzzy system of the k th subsystem can be constructed as follows:

$$\dot{x}_k(t) = \sum_{i=1}^{4} \mu_i^k(x_{k_1}, x_{k_2})\left(A_i^k x_k(t) + B_i^k u_k(t) + \sum_{l=1, l\neq k}^{2} A_i^{kl} x_l(t)\right)$$

where

$$A_1^k = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{\kappa a(a-cl)}{cml^2} & 0 \end{bmatrix}, \quad A_2^k = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{\kappa a(a-cl)}{cml^2} - \frac{m}{M}\Omega_k^2 & 0 \end{bmatrix},$$
$$A_3^k = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{\kappa a(a-cl)}{cml^2} & 0 \end{bmatrix}, \quad A_4^k = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{\kappa a(a-cl)}{cml^2} + \frac{m}{M}\Omega_k^2 & 0 \end{bmatrix},$$
$$B_i^k = \begin{bmatrix} 0 \\ \frac{1}{cml^2} \end{bmatrix}, \quad A_i^{kl} = \begin{bmatrix} 0 & 0 \\ \frac{\kappa a(a-cl)}{cml^2} & 0 \end{bmatrix}$$

for $(i,k) \in \mathcal{I}_r \times \mathcal{I}_n$. The membership functions are

$$\mu_1^k(x_{k_1}, x_{k_2}) = \frac{x_{k_1}(t) + sin(x_{k_1}(t))}{2x_{k_1}(t)} \times \frac{\Omega_k^2 - x_{k_2}(t)^2}{\Omega_k^2}$$
$$\mu_2^k(x_{k_1}, x_{k_2}) = \frac{x_{k_1}(t) + sin(x_{k_1}(t))}{2x_{k_1}(t)} \times \frac{x_{k_2}(t)^2}{\Omega_k^2}$$
$$\mu_3^k(x_{k_1}, x_{k_2}) = \frac{x_{k_1}(t) - sin(x_{k_1}(t))}{2x_{k_1}(t)} \times \frac{\Omega_k^2 - x_{k_2}(t)^2}{\Omega_k^2}$$
$$\mu_4^k(x_{k_1}, x_{k_2}) = \frac{x_{k_1}(t) - sin(x_{k_1}(t))}{2x_{k_1}(t)} \times \frac{x_{k_2}(t)^2}{\Omega_k^2}$$

We assume $\Omega_1 = 4.9$, $\Omega_2 = 5$, the initial state conditions $x_k(0) = [\pi/3, 0]^T$ and a sampling time T=0.2 s. By using Theorem 1 and solving the corresponding LMIs, we obtain the control gains as followings:

$$K_{d_1}^1 = [-12.2849 \quad -1.5232], \quad K_{d_2}^1 = [-11.8367 \quad -1.4769],$$
$$K_{d_3}^1 = [-12.2849 \quad -1.5232], \quad K_{d_4}^1 = [-12.7825 \quad -1.5627],$$
$$K_{d_1}^2 = [-12.3014 \quad -1.5235], \quad K_{d_2}^2 = [-11.8367 \quad -1.4752],$$
$$K_{d_3}^2 = [-12.3014 \quad -1.5235], \quad K_{d_4}^2 = [-12.7911 \quad -1.5623].$$

As time responses for each subsystem are shown in Figs. 1, 2, 3 and 4, we can observe the discrepancy between the

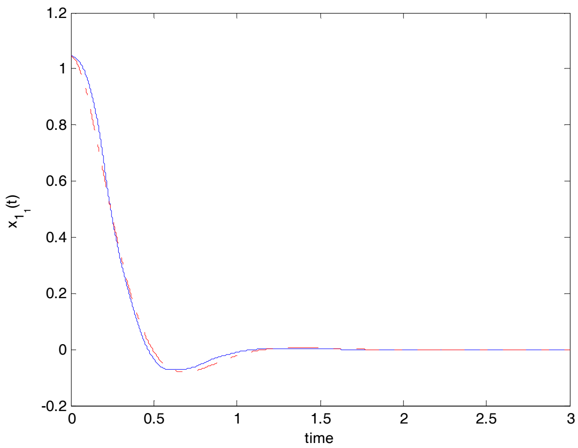

Fig. 1. The time response x_{1_1} of controlled subsystem 1 for $T = 0.2$ s: analog (dash-dotted), proposed (solid).

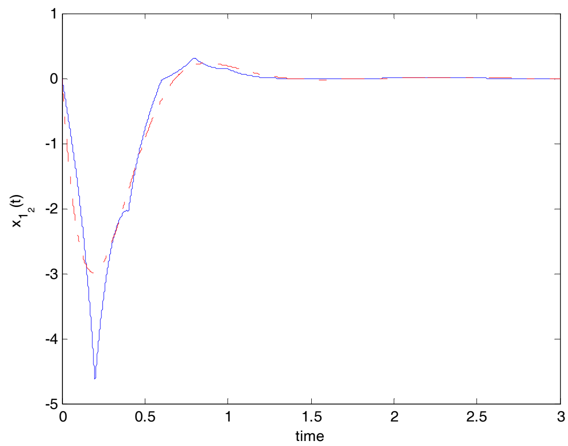

Fig. 2. The time response x_{1_2} of controlled subsystem 1 for $T = 0.2$ s: analog (dash-dotted), proposed (solid).

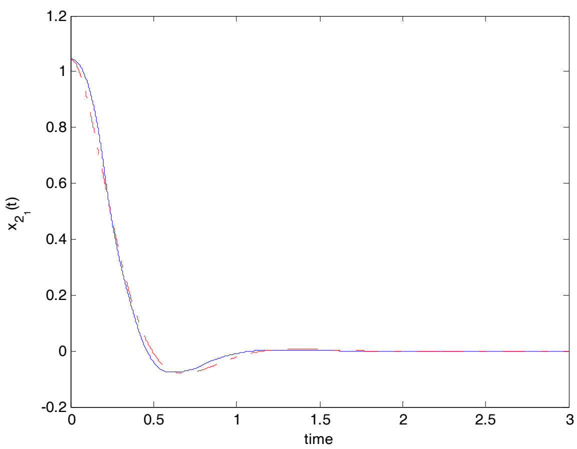

Fig. 3. The time response x_{2_1} of controlled subsystem 1 for $T = 0.2$ s: analog (dash-dotted), proposed (solid).

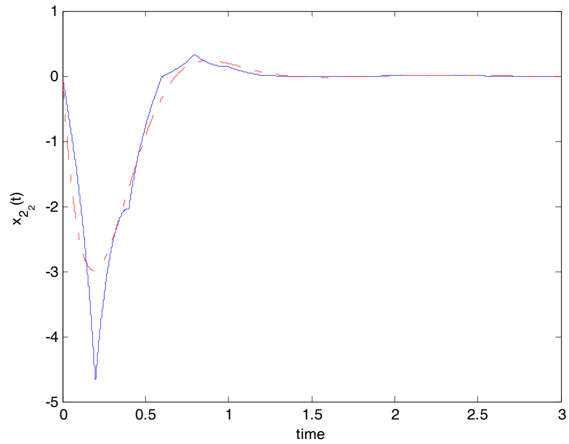

Fig. 4. The time response x_{2_2} of controlled subsystem 1 for $T = 0.2$ s: analog (dash-dotted), proposed (solid).

analog and proposed results during the initial transient period. This overshoot is based on the structural difference between the interconnection terms $\hat{G}_{ij}^{kl}$ and G_i^{kl} by the discretization. Except the overshoot in the early times, the controlled state variables by IDR method follow the controlled analog ones and converge to the origin.

From the result of the inverted pendulum example, we can know that the proposed IDR technique satisfies an asymptotic stability of digitally closed-loop interconnected system and the state-matching requirement with the originally closed-loop system.

4.2 Example 2

We consider the system of two flexible joint robot arms connected by a spring. The k th arm system is given by

$$I_k\ddot{\theta}_{k1}(t)+M_kgl_k\sin(\theta_{k1}(t))+\kappa_k(\theta_{k1}(t)-\theta_{k2}(t))+\kappa_{kl}(\theta_{k1}(t)-\theta_{l1}(t))=0$$
$$J_k\ddot{\theta}_{k2}(t)-\kappa_k(\theta_{k1}(t)-\theta_{k2}(t))=u_{k1}(t)$$

where κ_k is the spring constants of each arms, κ_{kl} is the constant of coupling spring between the arms, θ_{k1} is the link angle, θ_{k2} is the motor angle, I_k is the rotational inertias about the axis of rotations, J_k is the rotor inertias of the actuator shafts, M_k is the total mass of k th arm, l_k is the distance to the joint from the mass centers of the axis of rotations, and g is the gravity constant.

Choosing x_k as $[\theta_{k1} \quad \dot{\theta}_{k1} \quad \theta_{k2} \quad \dot{\theta}_{k2}]^T$, the k th T-S fuzzy system is represented as follows:

$$\dot{x}_k(t)=\sum_{i=1}^{2}\mu_i^k(x_{k1}(t))\Big(A_i^kx_k(t)+B_i^ku_k(t)+\sum_{k=1,k\neq l}^{2}A_i^{kl}x_l(t)\Big)$$

where

$$A_1^k = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{M_k g l_k}{I_k} - \frac{\kappa_k}{I_k} - \frac{\kappa_{kl}}{I_k} & 0 & \frac{\kappa_k}{I_k} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\kappa_k}{J_k} & 0 & -\frac{\kappa_k}{J_k} & 0 \end{bmatrix},$$

$$A_2^k = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{a M_k g l_k}{I_k} - \frac{\kappa_k}{I_k} - \frac{\kappa_{kl}}{I_k} & 0 & \frac{\kappa_k}{I_k} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\kappa_k}{J_k} & 0 & -\frac{\kappa_k}{J_k} & 0 \end{bmatrix},$$

$$B_1^k = B_2^k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_k} \end{bmatrix}, \quad A_1^{kl} = A_2^{kl} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{\kappa_{kl}}{I_k} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

for $k,l \in \mathcal{I}_2$, $k \neq l$, $i \in \mathcal{I}_2$ and $a = -1$.

The membership functions for each system are

$$\mu_1^k(x_{k_1}) = \frac{x_{k_1}(t) + sin(x_{k_1}(t))}{2x_{k_1}(t)}, \quad \mu_2^k(x_{k_1}) = \frac{x_{k_1}(t) - sin(x_{k_1}(t))}{2x_{k_1}(t)}$$

The parameter values of each subsystem are determined in Table 1 and $\kappa_{kl} = 10(Nm/rad)$ and $g = 9.8(m/sec^2)$.

Table 1. The parameter values of each subsystems

Parameters	Subsystem 1	Subsystem 2
$I_k(kgm^2)$	0.03	0.04
$J_k(kgm^2)$	0.004	0.005
$M_k(kg)$	0.2	0.3
$l_k(m)$	1	0.9
$\kappa_k(Nm/rad)$	31	30

The initial condition is set to $x_1(0) = x_2(0) = [\frac{\pi}{2}\ -\frac{\pi}{6}\ -\frac{\pi}{2}\ \frac{\pi}{6}]^T$ and we can obtain the controller gain matrices $K_{d_i}^k$ using Theorem 1 and solving the corresponding LMI:

$$K_{d_1}^1 = [-43.4522 \quad -3.9755 \quad -49.6651 \quad -0.6678],$$
$$K_{d_2}^1 = [-93.7244 \quad -6.6560 \quad -65.4239 \quad -0.8222],$$
$$K_{d_1}^2 = [-48.5207 \quad -4.0162 \quad -49.8484 \quad -0.6666],$$
$$K_{d_2}^2 = [-104.1431 \quad -6.7485 \quad -65.4641 \quad -0.8158].$$

The time responses of two flexible joint robot arms with controllers are shown in Fig. 5, 6, 7 and 8 with T=0.01 s.

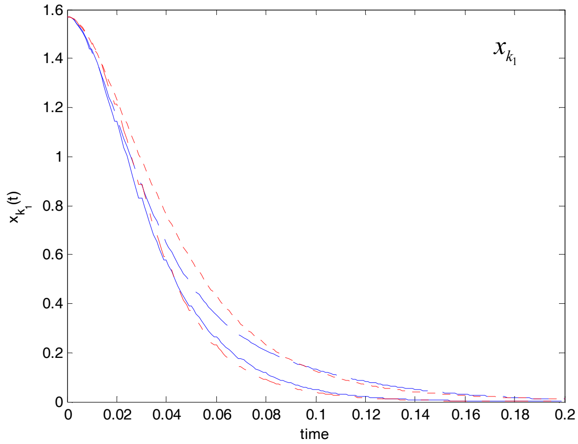

Fig. 5. The time response x_{k_1} of each controlled subsystem: analog x_{1_1} (dashed), proposed x_{1_1} (solid), analog x_{2_1} (dotted), proposed x_{2_1} (dash-dotted).

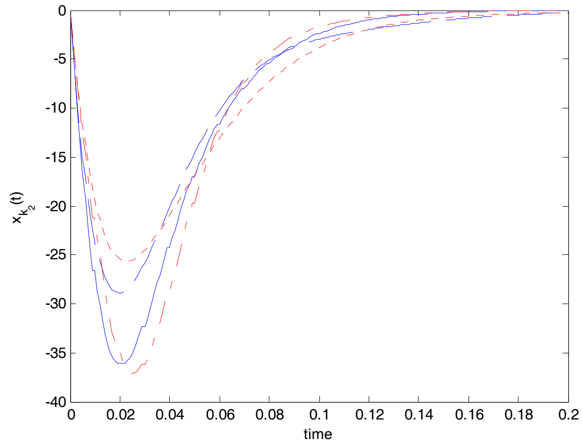

Fig. 6. The time response x_{k_2} of each controlled subsystem: analog x_{1_2} (dashed), proposed x_{1_2} (solid), analog x_{2_2} (dotted), proposed x_{2_2} (dash-dotted).

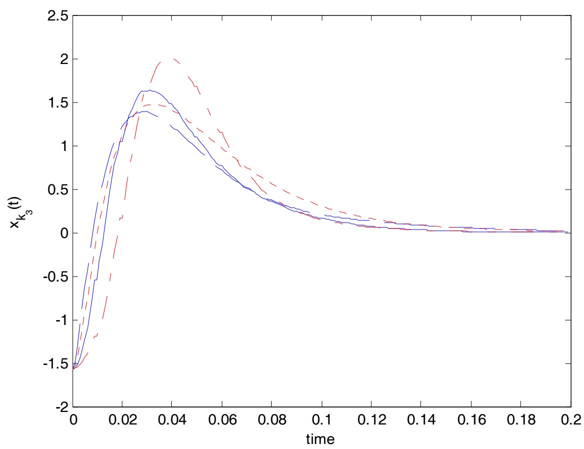

Fig. 7. The time response x_{k_3} of each controlled subsystem: analog x_{1_3} (dashed), proposed x_{1_3} (solid), analog x_{2_3} (dotted), proposed x_{2_3} (dash-dotted).

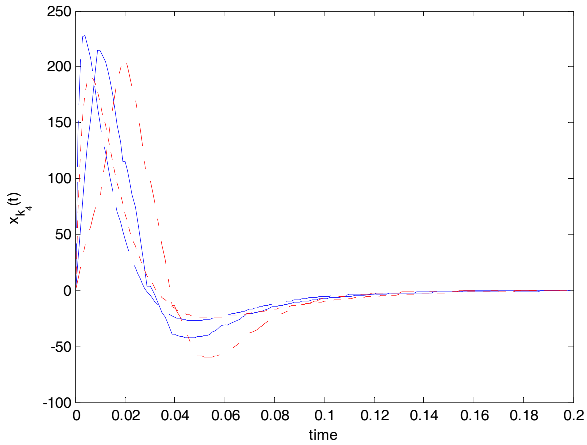

Fig. 8. The time response x_{k_4} of each controlled subsystem: analog x_{1_4} (dashed), proposed x_{1_4} (solid), analog x_{2_4} (dotted), proposed x_{2_4} (dash-dotted).

As the results in the example 1, we can know that the proposed IDR technique satisfies the stability and the state-matching condition. Also, example 2 shows the good performance of the proposed method in the complex system.

5. Conclusion

This paper has established the IDR method for the nonlinear interconnected systems. Using the T-S fuzzy model, we have presented the analog and digital closed-loop fuzzy systems and their discretized systems. Based on these systems, it was shown that the IDR technique can find the digital decentralized fuzzy gains to minimize the norm distance between the states of each system and stabilize the digital closed-loop system. Also, the sufficient design conditions were derived and formulated in the LMI format, and were therefore easily tractable by convex optimization. Finally a simulation example has shown that the results of this paper are effective and valuable.

Acknowledgements

This work was financially supported by the Ministry of Education, Science Technology(MEST) and National Research Foundation of Korea(NRF) through the Human Resource Training Project for Regional Innovation and the Human Resources Development of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Knowledge Economy.(*No.* 20104010100590).

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