Intelligent Digital Redesign for Nonlinear Interconnected Systems using Decentralized Fuzzy Control

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Abstract – In this paper, a novel intelligent digital redesign (IDR) technique is proposed for the nonlinear interconnected systems which can be represented by a Takagi-Sugeno (T-S) fuzzy model. The IDR technique is to convert a pre-designed analog controller into an equivalent digital one. To develop this method, the discretized models of the analog and digital closed-loop system with the decentralized controller are presented, respectively. Using these discretized models, the digital decentralized control gain is obtained to minimize the norm between the state variables of the analog and digital closed-loop systems and stabilize the digital closed-loop system. Its sufficient conditions are derived in terms of linear matrix inequalities (LMIs). Finally, a numerical example is provided to verify the effectiveness of the proposed technique.

Keywords: Nonlinear interconnected systems, Takagi-Sugeno (T-S) fuzzy model, Decentralized control, Intelligent digital redesign (IDR), Linear matrix inequality (LMI)

1. Introduction

Recently, many practical systems, such as power systems, communication networks, and economic models, have the high dimensionality and nonlinearity. In order to control and analyze these systems, many researchers have concerned about stability and stabilization of nonlinear inter-connected systems [1-9, 18, 19]. Because of the high dimensionality and the structural constraint, the traditionally centralized control method is not suitable for the interconnected systems. To solve this problem, various control approaches have been developed for the nonlinear interconnected systems. Among them, the decentralized fuzzy control technique using the Takagi-Sugeno (T-S) fuzzy model is regarded as a powerful technique and has gathered many research interests [4-6].

Apart from the nonlinearity issue, because of the ubiquitousness and its application, there are many cases to control the interconnected system using the digital device such as a computer. Thus, the study for the digital control is essential for the interconnected system. Among many digital control methods, a digital redesign (DR) is widely used as the predominant digital controller design technique [10-12]. It executes as a tool which converts an original analog controller into a new digital one maintaining the state-matching condition.

In order to apply DR method in the nonlinear system, an intelligent digital redesign (IDR), which is merged with the fuzzy control scheme and the DR method, was proposed in [13]. Also, the robust IDR problem was studied for nonlinear systems with parametric uncertainties [14]. However, these papers dealt with the local state-matching without concerning the global one. The global approaches of the IDR problem were presented in [15-17], but they did not consider the interconnection of the nonlinear systems. The decentralized digital controllers for the interconnected systems were proposed in [18-20]. But there were no studies about the IDR problem for the nonlinear interconnected systems.

In this paper, we propose a novel IDR method for the nonlinear interconnected systems which can be modelled by T-S fuzzy systems. The discretized models of the analog and digital closed-loop system with the decentralized controller are presented, respectively. Based on these discretized models, sufficient conditions are achieved for both the stability of the digital closed-loop system and the state-matching condition between analog and digital closed-loop systems. Its constructive conditions are presented in terms of linear matrix inequalities (LMIs). Finally, it shows the validity of the proposed ideas, techniques and procedures, through simple example.

This paper is organized as follows: Section 2 describes the discretized models of the T-S fuzzy interconnected systems. The stability and state-matching conditions are proposed with the LMI form in Section 3. In Section 4, simulation example is provided to demonstrate the design procedures. Finally, the conclusions are given in Section 5.

2. T-S Fuzzy Modeling and Discretization for Nonlinear Interconnected System

Consider an interconnected T-S fuzzy system consisting of \( q \) systems, in which the \( i \)th IF-THEN rule of the \( k \)th
subsystem is represented by the following form:

\[ R_{\mu}^i : \text{IF } z_{\mu}^i \text{ is } \Gamma_{\mu_1}^i \text{ and } \cdots \text{ and } z_{\mu}^i \text{ is } \Gamma_{\mu_p}^i, \]

THEN \( \dot{x}_i(t) = A_i^x x_i(t) + B_i^u u_i(t) + \sum_{j=1}^q A_i^{\mu j} x_j(t) \) \hspace{1cm} (1)

where \( z_{\mu}^i \), \( h \in I_{\mu} \) is the premise variable; \( x_i \in \mathbb{R}^n \) and \( u_i \in \mathbb{R}^m \) are the state and control input for the \( k \) th subsystem, respectively; \( \Gamma_{\mu}^i \), \((i, h, k) \in I_x \times I_{\mu} \times I_k \) is a fuzzy set for \( z_{\mu}^i \); \( A_i^x \) and \( B_i^k \) denote nominal system matrices with appropriate dimensions for the \( i \) th rule in the \( k \) th subsystem and \( A_{\mu k}^i \) is the interconnection matrix with the \( k \) th and the \( i \) th subsystem.

Using the center-average defuzzification, product inference, and singleton fuzzifier, the global dynamics is inferred as

\[ \dot{x}_i(t) = \sum_{i=1}^r \mu_i^k(t) \left( A_i^x x_i(t) + B_i^u u_i(t) + \sum_{j=1}^q A_i^{\mu j} x_j(t) \right) \] \hspace{1cm} (2)

where

\[ \mu_i^k(t) = \omega_i^k(z_i^k(t))/\sum_{i=1}^r \omega_i^k(z_i^k(t)), \]

\[ o_i^k(z_i^k(t)) = \prod_{h=1}^H \Gamma_{\mu}^i(z_{\mu h}^i(t)) \]

in which \( \Gamma_{\mu}^i(z_{\mu h}^i(t)) \) is the fuzzy membership grade of \( z_{\mu h}^i \) in \( \Gamma_{\mu h}^i \).

Suppose a pre-designed analog fuzzy controller for \( k \) th subsystem (2)

\[ u_i^k(t) = \sum_{i=1}^r \mu_i^k(t) K_i^x x_i(t) \] \hspace{1cm} (3)

where \( K_i^x \) denotes the analog control gain and the subscript 'c' means the analog control. Substituting (3) into (2), \( k \) th sub-closed-loop system is written as

\[ \dot{x}_i(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i^k(t) \mu_j^k(t) \]

\[ \times \left( \left( A_i^x + B_i^k K_i^x \right) x_i(t) + \sum_{j=1}^q A_i^{\mu j} x_j(t) \right) \] \hspace{1cm} (4)

Now, consider a digital fuzzy controller for (2)

\[ u_{d(i)}(t) = u_{d(i)}(nT) = \sum_{i=1}^r \mu_i^k(nT) K_i^x x_i(nT) \] \hspace{1cm} (5)

where \( u_{d(i)}(t) \) is the digital control input to be determined in time interval \( t \in [nT, nT + T) \), \( n \in \mathbb{Z}_{\geq 0} \) and \( T \in \mathbb{R}_{> 0} \) is a sample-ling period and the subscript 'd' denotes the digital control. Substituting (5) into (2), the \( k \) th sub-closed-loop system is obtained by

\[ \dot{x}_i(t) = \sum_{i=1}^r \sum_{j=1}^r \mu_i^k(t) \mu_j^k(t) \]

\[ \times \left( A_i^x x_i(t) + B_i^k K_i^x x_i(nT) + \sum_{j=1}^q A_i^{\mu j} x_j(t) \right) \] \hspace{1cm} (6)

Supposing the sampling period \( T \), closed-loop fuzzy system (4) and (6) are approximately discretized as

\[ x_i(nT + t) \cong \sum_{i=1}^r \sum_{j=1}^r \mu_i^k(t) \mu_j^k(nT) \]

\[ \times \left[ \Phi_i^k x_i(nT) + \sum_{l=1}^q \tilde{G}_l^{\mu j} x_i(nT) \right] \]

= \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r \mu_i^k(nT) \mu_j^k(nT) \]

\[ \times \left[ \frac{1}{T} \Phi_i^k \right] \left[ \tilde{G}_l^{\mu j} \right] x_i(nT) \] \hspace{1cm} (7)

where

\[ \Phi_i^k = \exp((A_i^x + B_i^k K_i^x)T), \]

\[ \tilde{G}_l^{\mu j} = (\Phi_i^k - I)(A_i^x + B_i^k K_i^x)^{-1} A_i^{\mu j} \]

and

\[ x_i(nT + t) \cong \sum_{i=1}^r \sum_{j=1}^r \mu_i^k(t) \mu_j^k(nT) \]

\[ \times \left[ \left( G_i^k + H_i^k K_i^x \right) x_i(nT) + \sum_{l=1}^q G_l^{\mu j} x_i(nT) \right] \]

\[ = \sum_{i=1}^r \sum_{j=1}^r \sum_{l=1}^r \mu_i^k(nT) \mu_j^k(nT) \]

\[ \times \left[ \frac{1}{T} \left( G_i^k + H_i^k K_i^x \right) \right] \left[ G_l^{\mu j} \right] x_i(nT) \] \hspace{1cm} (8)

where

\[ G_i^k = \exp(A_i^x T), \]

\[ H_i^k = (G_i^k - I)(A_i^x)^{-1} B_i^k, \]

\[ G_l^{\mu j} = (G_i^k - I)(A_i^x)^{-1} A_i^{\mu j}. \]

Remark 1: There are fruitful research works [3-6] focusing on the design of the decentralized fuzzy controller in the continuous-time or the discrete-time domains. However, the study of IDR for the interconnected fuzzy system has not been sufficiently achieved.
3. New IDR Based on Discretized T-S Fuzzy System

In this paper, our main problem is addressed as follows:

Problem 1: Suppose that the equilibrium point \( x_i(t) = 0 \) of (4) is asymptotically stable by a well-constructed analog controller (3). Then digital controller (5) has to satisfy the following conditions:

- The state-matching error between \( x_i(nT) \) of (7) and \( x_i(nT) \) of (8) is minimized for any \( k \in \mathbb{Z}_{\geq 0} \).
- The digital closed-loop system (6) and its discretized system (8) are globally asymptotically stable.

Before proceeding to main results, the following matrix inequality and proposition will be needed throughout the proof:

**Lemma 1([21]):** Given any matrices \( Y \) and \( P = P^T > 0 \), we have

\[
-Y^T P^{-1} Y \leq P - Y^T Y.
\]

**Proposition 1:** In (6), there exists some constant \( \eta > 0 \) such that

\[
\sum_{k=1}^{\infty} \| x_{i_k}(t) \| \leq \eta \sum_{k=1}^{\infty} \| x_{i_k}(nT) \|
\]

where

\[
\eta = \sum_{k=1}^{\infty} (1 + T \beta_k) e^{(\alpha_k - (q - 1) \gamma_k) T}, \quad \alpha_k = \sup_{(i,j) \in \mathcal{T}, t \in [0,T]} A^i_{j_k} \quad \beta_k = \sup_{(i,j) \in \mathcal{T}, t \in [0,T]} B^i_j K^j_k \quad \gamma_k = \sup_{(i,j) \in \mathcal{T}, t \in [0,T]} \| K^j_k \|
\]

for \( t \in [nT, nT + T) \).

**Proof:** Integrating (6) from \( nT \) to \( t \), the solution is given by

\[
x_{i_k}(t) = x_{i_k}(nT) + \int_{nT}^{t} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \mu_i^j(t) \mu_j^i(nT) \times \left( A^i_{j_k} x_{i_k}(\tau) + B^i_j K^j_k x_{i_k}(nT) + \sum_{l,j=1}^{\infty} A^i_{jl} x_{i_l}(\tau) \right) d\tau
\]

for \( t \in [nT, nT + T) \). Taking the norms and the sum of all subsystems on both sides yields

\[
\sum_{k=1}^{\infty} \| x_{i_k}(t) \| = \sum_{k=1}^{\infty} \| x_{i_k}(nT) \| + \sum_{k=1}^{\infty} \int_{nT}^{t} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \mu_i^j(t) \mu_j^i(nT) \times \left( A^i_{j_k} x_{i_k}(\tau) + B^i_j K^j_k x_{i_k}(nT) + \sum_{l,j=1}^{\infty} A^i_{jl} x_{i_l}(\tau) \right) d\tau
\]

\[
\leq \sum_{k=1}^{\infty} \| x_{i_k}(nT) \| + \sum_{k=1}^{\infty} \int_{nT}^{t} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \mu_i^j(t) \mu_j^i(nT) \times \left( A^i_{j_k} x_{i_k}(\tau) + B^i_j K^j_k x_{i_k}(nT) + \sum_{l,j=1}^{\infty} A^i_{jl} x_{i_l}(\tau) \right) d\tau
\]

\[
= \sum_{k=1}^{\infty} \| x_{i_k}(nT) \| + \sum_{k=1}^{\infty} \int_{nT}^{t} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \mu_i^j(t) \mu_j^i(nT) \times \left( A^i_{j_k} x_{i_k}(\tau) + B^i_j K^j_k x_{i_k}(nT) + \sum_{l,j=1}^{\infty} A^i_{jl} x_{i_l}(\tau) \right) d\tau
\]

An application of the Gronwall-Bellman inequality to \( \sum_{k=1}^{\infty} \| x_{i_k}(t) \| \) results in

\[
\sum_{k=1}^{\infty} \| x_{i_k}(t) \| \leq \eta \sum_{k=1}^{\infty} \| x_{i_k}(nT) \| \quad \square
\]

The main results are summarized to solve Problem 1 in the following theorem:

**Theorem 1:** If there exist some symmetric and positive matrices \( P^k, P^\ell \), some symmetric matrices \( a^k_{ij}, b^k_{ij}, c^k_{ij} \), and some matrices \( a^\ell_{ij}, b^\ell_{ij}, c^\ell_{ij}, d^\ell_{ij}, M^k, Y^k \), such that the following LMIs have optimal solutions, then, \( x_i(nT) \) of (8) closely matches \( x_i(nT) \) of (7), and origin of (8) is globally asymptotically stable.

\[
\begin{align*}
\text{minimize} \quad & \gamma \\
\text{subject to} \quad & \begin{bmatrix}
-\gamma P^k & * & * \\
0 & -\gamma P^\ell & * \\
\frac{1}{\gamma^2} (\Psi^k Y^k - \Psi^\ell Y^\ell) & G^k Y^k - G^\ell Y^\ell & I - \gamma Y^k - \gamma (Y^k)^T \\
-\omega P^k + a^k_{ij} & * & * \\
0 & -\omega P^\ell + c^\ell_{ij} & * \\
\frac{1}{\omega} (\Psi^k Y^k + \Psi^\ell Y^\ell) & G^k Y^k + G^\ell Y^\ell & P^k - Y^k - (Y^k)^T \\
-\frac{4\omega P^k + 2a^k_{ij}}{\omega} + 2(a^k_{ij})^T & * & * \\
2b^k_{ij} + 2(d^k_{ij})^T & -\omega P^k + 2c^k_{ij} + 2(c^k_{ij})^T & * \\
\frac{(\Psi^k Y^k + \Psi^\ell Y^\ell)}{\omega} & G^k Y^k + G^\ell Y^\ell & P^k - Y^k - (Y^k)^T \\
\end{bmatrix} < 0
\end{align*}
\]

where

\[
\begin{align*}
\Psi^k &= G^k Y^k + H^k M^k, \\
\Psi^\ell &= G^\ell Y^\ell + H^\ell M^\ell, \\
\omega &= \frac{1}{\gamma^2}, \\
X^k &= (X^k)^T = \begin{bmatrix} a^k_{ij} & b^k_{ij} & c^k_{ij} \\ b^k_{ij} & d^k_{ij} & c^k_{ij} \\ c^k_{ij} & d^k_{ij} & c^k_{ij} \end{bmatrix}, \\
X^\ell &= (X^\ell)^T = \begin{bmatrix} a^\ell_{ij} & b^\ell_{ij} & c^\ell_{ij} \\ b^\ell_{ij} & d^\ell_{ij} & c^\ell_{ij} \\ c^\ell_{ij} & d^\ell_{ij} & c^\ell_{ij} \end{bmatrix} \\
\end{align*}
\]

for \( \forall (i, j, k) \in \mathcal{I} \times \mathcal{I} \times \mathcal{I} \) with \( 1 \leq i \leq j \leq r \).
Proof: In order to closely match $x_n(t)T$ of (7) to $x_n(nT)$ of (8), consider the norm minimization problem in the following form:

$$\left\| \frac{1}{\gamma} \Phi_{\theta} - \hat{G}_{\theta} + H^T K_{\theta} + H^T K_{\theta}^T \right\|_{\mathcal{F}} \leq \gamma$$

(14)

for some $\gamma > 0$. Using the norm property, the above inequality holds

$$\left[ \frac{1}{\gamma} \Phi_{\theta} - G_{\theta} - H^T K_{\theta} \right] \hat{G}_{\theta} - G_{\theta}^T$$

(15)

where $\gamma > 0$ and $P^j = (P^j)^T > 0$ with $\gamma I \prec \gamma I = \gamma I$. Taking the Schur complement and the congruence transform with $\mathcal{D}$, (15) yields

$$\left[ \begin{array}{cc} I & 0 \\ 0 & 0 \\ 0 & Y^T \\ 0 & I & \ast \\ 0 & I & \ast \\ Y^T \\ 0 & Y \end{array} \right] \begin{bmatrix} -\gamma P^j & \ast & \ast \\ \ast & 0 & \ast \\ \ast & \ast & -\gamma P^j \end{bmatrix}$$

(16)

where $Y^T$ is any full rank matrix with appropriate dimension. Applying Lemma Lemma 1 and denoting $K_{nT}, \gamma = M_{nT}$, inequality (16) yields LMI (10).

Apart from the state-matching problem, for solving the stability problem, suppose the Lyapunov function $V(nT) = \sum_{i=1}^q x_{i}(nT)P^i x_{i}(nT)$. The rate of increase of $V(nT)$ along (8) is then

$$V(nT+T) - V(nT) \leq -\frac{1}{4} \sum_{i=1}^q \sum_{j=1}^r \sum_{k=1}^q \mu^j_k (nT) \mu^j_k (nT)$$

(17)

$$\left[ \begin{array}{c} (G_{i} + G_{j} + H^T K_{i} + H^T K_{j}^T) x_{i} + \sum_{k=1}^q (G_{i} + G_{j}) x_{k} \\ (G_{i} + G_{j} + H^T K_{i} + H^T K_{j}^T) x_{i} + \sum_{k=1}^q (G_{i} + G_{j}) x_{k} \end{array} \right] \left[ \begin{array}{c} P^i x_{i} \\ 0 \end{array} \right]$$

$$-4x_{i}P^i x_{i}$$

(18)

We have used $\alpha \sum_{i=1}^q \sum_{j=1}^r \sum_{k=1}^q \mu^j_k (nT) \mu^j_k (nT)$ with $\alpha \in \mathbb{R}_+$, then

$$V(nT+T) - V(nT) \leq -\frac{1}{4} \sum_{i=1}^q \sum_{j=1}^r \sum_{k=1}^q \mu^j_k (nT) \mu^j_k (nT)$$

(19)

$$\begin{bmatrix} x_{i} \\ x_{i} \end{bmatrix}$$

where

$$\Theta_{i} = \left[ \begin{array}{c} \gamma_{i}^T, \gamma_{i}^T, \ldots, \gamma_{i}^T \end{array} \right]$$

$$\hat{\gamma}_{i} = \text{diag}(\theta_{i}, \ldots, \theta_{i}) \in \mathbb{R}^{2q \times 2q}$$
\[ \hat{P}^0 = \text{diag}(P^0, P^0) \]
\[ \hat{P}^0 = \text{diag}(\hat{P}^0, \ldots, \hat{P}^0) \in \mathbb{R}^{2m_r r, 2m_r r} \]

Applying Schur complement to (17) and the congruence transformation with diag\{I, I, Y^4\}, we obtain
\[
\begin{align*}
& -\left(\frac{1}{m_r} + \frac{\mu}{a} \right)P^0 + a_0^t \quad * \quad * \\
& b_0^t \quad * \\
& \frac{1}{\sqrt{m_r}} (G_i Y^2 + H_i K_i^0 Y^4) \quad G_i^2 Y^2 - (Y^4)'(P^0)'Y^4 < 0
\end{align*}
\]

Applying Lemma 1 and denoting \( K_i^0 Y^4 = M_i^0 \) result in (11).

We can again establish a similar argument to (18), in order to obtain (11), as follows:
\[
\begin{align*}
& \left(\frac{1}{m_r} - \alpha \right)P^0 + 2a_0^t + 2a_0^t \\
& 2b_0^t + 2b_0^t \quad -\alpha P^0 + 2a_0^t + 2a_0^t \\
& \frac{1}{\sqrt{m_r}} (\Lambda_0^t + \Lambda_0^t) \quad G_i^2 Y^2 + G_i^2 Y^2 - (Y^4)'(P^0)'Y^4
\end{align*}
\]

\( \L_i^0 = G_i^2 Y^2 + H_i^2 K_i^0 Y^4 \)

Remark 2: From (9), we can know that, if state \( x_i(nT) \) converges to origin, then state \( x_i(t) \) also tends to origin. In other words, if the discretized system (8) is asymptotically stable, then the original digital system is also asymptotically stable. Thus, we can obtain the state-matching condition between \( x_i(nT) \) and \( x_i(nT) \) and stability condition of (6) by Proposition 1 and Theorem 1.

4. New IDR Based on Discretized T-S Fuzzy System

In this section, two examples are given to validate the proposed method. First example is two inverted pendulums connected by a spring mounted on two carts and second example is two flexible joint robot arms connected by a spring.

4.1 Example 1

Suppose the interconnected system which is two inverted pendulums connected by a spring mounted on two carts [22]. The dynamic equation of the system is described by
\[
\dot{\theta}_i(t) - \{(g / c) - (\kappa(a - c) / \text{cm}^2)\} \theta_i(t) + (m / M) \sin(\theta_i(t)) \dot{\theta}_i(t)^2 = (\kappa(a - c) / \text{cm}^2) \theta_i(t) + (1 / \text{cm}^2) u_i(t)
\]

where \( \{(k, l) \in \mathcal{I}_r | k \neq l\} \); \( \theta_i(t) \) is the angle of the \( k \) th pendulum; \( c = m(m+M) \); \( m = 1 \)kg is the mass of pendulum; \( M = 5 \)kg is the mass of the cart; \( a = 0.2 \)m is the length from the cart to the spring; \( l = 1 \)m is the length of the pendulum; \( \kappa = 1N / m \) is the spring constant; \( g = 9.8 \text{m/s}^2 \) is the gravity constant.

Assuming \( \dot{\theta}_i(t) \in [\Omega_i, \Omega_i] \) with \( \Omega_i > 0 \) and choosing \( x_i(t) = [x_i(t), x_i(t)] = [\dot{\theta}_i(t), \dot{\theta}_i(t)] \), the T-S fuzzy system of the \( k \)th subsystem can be constructed as follows:
\[
\dot{x}_i(t) = \sum_{i=1}^{4} \mu_i^t(x_i, x_i)(A_i^t x_i(t) + B_i^t u_i(t) + \sum_{i=1}^{4} A_i^t x_i(t))
\]

where
\[
A_i^t = \begin{bmatrix} 0 & 1 \\ \frac{\kappa(a - c)}{\text{cm}^2} & -\frac{m}{M} \Omega_i & 0 \\ 0 & 0 & 1 \\ \frac{\kappa(a - c)}{\text{cm}^2} & -\frac{m}{M} \Omega_i & 0 \end{bmatrix}
\]
\[
B_i^t = \begin{bmatrix} 0 \\ 0 \\ \frac{\kappa(a - c)}{\text{cm}^2} \\ 0 \end{bmatrix}
\]

for \( (i, k) \in \mathcal{I}_r \times \mathcal{I}_r \). The membership functions are
\[
\mu_i^t(x_i, x_i) = \frac{x_i(t) + \sin(x_i(t))}{2x_i(t)} \times \frac{\Omega_i - x_i(t)}{\Omega_i}
\]
\[
\mu_i^t(x_i, x_i) = \frac{x_i(t) + \sin(x_i(t))}{2x_i(t)} \times \frac{\Omega_i - x_i(t)}{\Omega_i}
\]
\[
\mu_i^t(x_i, x_i) = \frac{x_i(t) + \sin(x_i(t))}{2x_i(t)} \times \frac{\Omega_i - x_i(t)}{\Omega_i}
\]
\[
\mu_i^t(x_i, x_i) = \frac{x_i(t) + \sin(x_i(t))}{2x_i(t)} \times \frac{\Omega_i - x_i(t)}{\Omega_i}
\]

We assume \( \Omega_i = 4.9 \), \( \Omega_i = 5 \), the initial state conditions \( x_i(0) = [\pi / 3, 0] \) and a sampling time \( T = 0.2 \) s. By using Theorem 1 and solving the corresponding LMIs, we obtain the control gains as followings:
\[
K_i^0 = [-12.8249, -1.5232], \quad K_i^0 = [-11.8367, -1.4769],
\]
\[
K_i^0 = [-12.8249, -1.5232], \quad K_i^0 = [-11.8367, -1.4752],
\]
\[
K_i^0 = [-12.3014, -1.5235], \quad K_i^2 = [-12.7911, -1.5623].
\]

As time responses for each subsystem are shown in Figs. 1, 2, 3 and 4, we can observe the discrepancy between the
4.2 Example 2

We consider the system of two flexible joint robot arms connected by a spring. The \( k \) th arm system is given by

\[
I_k \ddot{\theta}_k(t) + M_k g l_k \sin(\theta_k(t)) + \kappa_k (\theta_{k1}(t) - \theta_{k2}(t)) + \kappa_{kl}(\theta_{k1}(t) - \theta_{l1}(t)) = 0
\]

\[
J_k \ddot{\theta}_{l1}(t) - \kappa_l (\theta_{k1}(t) - \theta_{l2}(t)) = u_{l1}(t)
\]

where \( \kappa_k \) is the spring constants of each arms, \( \kappa_{kl} \) is the constant of coupling spring between the arms, \( \theta_{k1} \) is the link angle, \( \theta_{k2} \) is the motor angle, \( I_k \) is the rotational inertia about the axis of rotations, \( J_k \) is the rotor inertia of the actuator shafts, \( M_k \) is the total mass of \( k \) th arm, \( l_k \) is the distance to the joint from the mass centers of the axis of rotations, and \( g \) is the gravity constant.

Choosing \( x_k \) as \( [\theta_{k1} \ \dot{\theta}_{k1} \ \theta_{k2} \ \dot{\theta}_{k2}]^T \), the \( k \) th T-S fuzzy system is represented as follows:

\[
\dot{x}_k(t) = \sum_{i=1}^{2} \mu_i^k(x_{ki}(t)) \left( A_i^k x_k(t) + B_i^k u_i(t) + \sum_{j \neq i, j \neq k} \delta_{ji} A_{ji}^k x_j(t) \right)
\]

where
Intelligent Digital Redesign for Nonlinear Interconnected Systems using Decentralized Fuzzy Control

$$A_i^k = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-M_i g l_i}{I_k} - \frac{\kappa_i}{I_k} - \frac{\kappa_k}{I_k} & 0 & \frac{\kappa_i}{I_k} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\kappa_k}{J_k} & 0 & -\frac{\kappa_k}{J_k} & 0 \end{bmatrix},$$

$$A_i^l = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-aM_i g l_i}{I_k} - \frac{\kappa_i}{I_k} - \frac{\kappa_k}{I_k} & 0 & \frac{\kappa_i}{I_k} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\kappa_k}{J_k} & 0 & -\frac{\kappa_k}{J_k} & 0 \end{bmatrix},$$

$$B_i^l = B_i^k = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_k} \end{bmatrix}, A_i^i = A_i^l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\kappa_i}{I_k} & 0 & 0 \end{bmatrix}$$

for $k,l \in \mathcal{I}_2$, $k \neq l$, $i \in \mathcal{I}_2$ and $a = -1$.

The membership functions for each system are

$$\mu_i^k(x) = x_i(t) + \sin(x_i(t)), \quad \mu_i^l(x) = x_i(t) - \sin(x_i(t))$$

The parameter values of each subsystem are determined in Table 1 and $\kappa_i = 10(Nm/rad)$ and $g = 9.8(m/sec^2)$.

Table 1. The parameter values of each subsystems

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Subsystem 1</th>
<th>Subsystem 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_i(kg m^2)$</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>$J_i(kg m^2)$</td>
<td>0.004</td>
<td>0.005</td>
</tr>
<tr>
<td>$M_i(kg)$</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>$l_i(m)$</td>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>$\kappa_i(Nm/rad)$</td>
<td>31</td>
<td>30</td>
</tr>
</tbody>
</table>

The initial condition is set to $x_i(0) = x_i(0) = \left[ \frac{\xi}{T_0} \frac{\xi}{T_0} \frac{\xi}{T_0} \right]^T$ and we can obtain the controller gain matrices $K_{cl}$ using Theorem 1 and solving the corresponding LMI:

$$K_{cl} = [-43.4522, -3.9755, -49.6651, -0.6678],$$

$$K_{cl} = [-93.7244, -6.6560, -65.4239, -0.8222],$$

$$K_{cl} = [-48.5207, -4.0162, -49.8484, -0.6666],$$

$$K_{cl} = [-104.1431, -6.7485, -65.4641, -0.8158].$$

The time responses of two flexible joint robot arms with controllers are shown in Fig. 5, 6, 7 and 8 with $T=0.01$s.
Fig. 8. The time response $x_{i_1}$ of each controlled subsystem: analog $x_{i_1}$ (dashed), proposed $x_{i_1}$ (solid), analog $x_{i_2}$ (dotted), proposed $x_{i_2}$ (dash-dotted).

As the results in the example 1, we can know that the proposed IDR technique satisfies the stability and the state-matching condition. Also, example 2 shows the good performance of the proposed method in the complex system.

5. Conclusion

This paper has established the IDR method for the nonlinear interconnected systems. Using the T-S fuzzy model, we have presented the analog and digital closed-loop fuzzy systems and their discretized systems. Based on these systems, it was shown that the IDR technique can find the digital decentralized fuzzy gains to minimize the norm distance between the states of each system and stabilize the digital closed-loop system. Also, the sufficient design conditions were derived and formulated in the LMI format, and were therefore easily tractable by convex optimization. Finally a simulation example has shown that the results of this paper are effective and valuable.

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